



mastering financial calculations

a step-by-step guide to the
mathematics of financial
market instruments

ROBERT STEINER

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market editions

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of financial market instruments

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Robert was previously senior consultant with HongKong and Shanghai Banking Corporation, where he worked in London and New York in the dealing area as consultant to major US and European companies on treasury management. He has also been treasurer and fund manager of H P Bulmer Holdings PLC and English and American Insurance Group PLC, active in currency and interest rate management. He has thus been closely involved in treasury management as consultant and practitioner. He has also worked in the Overseas Department of the Bank of England, and with the European Commission in Brussels.

Robert spends a considerable amount of time training bankers, systems staff, corporate staff and others involved in the financial markets. In particular, he personally runs courses for several of the ACI exams – the ACI Diploma, the Dealing Certificate, Repo Markets and the Settlements Certificate. He is the author of *Mastering Repo Markets*, also published by Financial Times Prentice Hall.

His academic background is an honours degree in mathematics from Cambridge University followed by further studies in economics with London University. He is an associate member of ACI UK, and a member of the Association of Corporate Treasurers.

TO MY PARENTS

without whom none of this would have been possible

AND TO MY FAMILY

who have suffered while I've been writing it

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FOREWORD

ACI Education (formerly The ACI Institute) is the educational division of ACI – the Financial Markets Association, the primary international professional body for the international financial markets, founded in 1955 and now represented in 59 countries internationally with over 24,000 members.

The task of ACI Education is simple to state and is embodied in the Charter of ACI. It is to ensure that educational programmes that reflect the constantly changing nature of the industry are made available to both new entrants to the profession and seasoned professionals, and to set professional standards for the industry globally. This task is both challenging and exciting. Recent innovations include the decision to convert the examinations to an electronically delivered format, a change that will facilitate the availability of ACI's professional testing programme, on a daily basis via a network of over 2,000 test centres globally.

One of the component ACI examinations is the Level 2 ACI Diploma that has been running successfully since 1990. Research into ACI's large database of results yields some interesting findings. Among these it can be seen that one of the best predictors of overall success in the examination has been a good performance on the part of the candidate in the financial calculations section. Conversely, candidates performing badly in this section have often performed poorly, overall, in the examination.

ACI, as part of its educational programmes encourages authors, publishers and tutors to provide a wide variety of routes to its examinations as possible. For this reason it is delighted that Robert Steiner and Financial Times Prentice Hall have produced *Mastering Financial Calculations*. It captures the professional knowledge necessary to appreciate fully the topic, and provides an excellent additional source of study for both the ACI Dealing Certificate and key sections of the ACI Diploma. It also believes that the publication will be appreciated not only by examination candidates but also by the market professionals seeking an 'easy-to-read' reference manual on this highly important subject.

Heering Lighthart
President

ACI – The Financial Markets Association

INTRODUCTION

The aim of the book

This book is aimed at the very many people in financial institutions, universities and elsewhere who will benefit from a solid grounding in the calculations behind the various financial instruments – dealers, treasurers, investors, programmers, students, auditors and risk managers – but who do not necessarily do complex mathematical puzzles for fun. There is also a strong emphasis on the mechanics, background and applications of all the various instruments.

Many dealers would claim to be able to perform their job – essentially buying and selling – very profitably, without using much maths. On the other hand, the different dealing areas in a bank overlap increasingly and it is essential to understand the basis for pricing each instrument, in order to understand how the bank's position in one relates to a position in another.

Almost all the concepts in this book regarding pricing the different instruments can be reduced to only two basic ideas. The first concerns the 'time value of money': a hundred dollars is worth more if I have it in my hand today than if I do not receive it until next year; or alternatively, for a given amount of cash which I will receive in the future, I can calculate an equivalent 'present value' now. The second is the 'no free lunch' principle: in theory, it should not generally be possible to put together a series of simultaneous financial transactions which lock in a guaranteed no-risk profit. For example, if a dealer buys an interest rate futures contract and simultaneously reverses it with an FRA, he will in general make no profit or loss if we ignore minor mechanical discrepancies and transaction costs. This concept links the pricing calculations for the two instruments. If the reader has a clear grasp by the end of the book of how to apply these two crucial concepts, he/she will be well equipped to cope. Much of the difficulty lies in seeing through the confusion of market terminology and conventions.

There is a significant jump between the arithmetic needed for most instruments, and the complex mathematics needed to construct option pricing formulas from first principles. I have deliberately excluded much of the latter from this book, because many readers are more likely to be discouraged than helped, and because that subject alone warrants an entire book. On the other hand, I have discussed the ideas behind option pricing, particularly volatility, which introduce the concepts without being mathematically frightening. I have also included the Black–Scholes formula for reference and mentioned standard books on the subject for those interested.

The book has been conceived as a workbook, as well as a reference book, with an emphasis on examples and exercises. It has grown out of workshops on financial mathematics which I have run for many years, including some for the ACI's former Financial Calculations exam. The book is therefore also suitable for the student who chooses to take exams without any formal tuition course. There is however considerably more background material than is required for any exam aimed specifically at calculations. The intention is to benefit the reader technically as well as making the book more readable.

The structure of the book

The book is set out in five parts:

The basics

This covers the fundamental concepts of time value of money, discounting, present values and calculations with interest rates.

Interest rate instruments

This looks at the range of instruments based on interest rates or yields in both the money markets and capital markets – simple deposits, CDs, bills and CP, FRAs and futures, and bonds. The market background and use for each instrument is given as a structure within which to see the calculations. This section also covers zero coupon rates, yield curves, duration and convexity.

Foreign exchange

This looks at all the foreign exchange instruments from basic spot, through swaps, outright, short dates, forward-forwards and time options, to SAFEs. Again, the context of each instrument in the market is explained, together with the link with the money markets.

Swaps and options

This part explains both interest rate and currency swaps, and options. Examples are given of the basic uses as well as some of the combination option strategies. It also introduces option pricing concepts and the 'Greeks'.

Further exercises, hints and answers

This section begins with some further exercises covering several areas of the book for readers who would like more practice. These are followed by hints and answers to the exercises which are found throughout the book.

Key features of the book

Key points

Throughout the book, key points have been highlighted for emphasis.

Calculation summaries

The procedures for all the necessary calculations have been summarized in the text to ensure the reader's understanding. These have also been collected together into a reference section in Appendix 3.

Glossary

The market terminology is explained throughout the text, with a glossary for reference in Appendix 4.

Market interest conventions

A constant source of confusion is the range of different conventions used for interest and coupon calculations in different markets. These are also summarized in Appendix 2 for various countries' markets.

Examples

Many examples to help the reader's understanding have been included throughout the text.

Exercises

Fully worked answers are provided at the end of the book in Chapter 11 for each chapter's exercises and for the further exercises in Chapter 10. The exercises form an important part of the book as they provide further examples of the calculations covered in the text. Readers who do not wish to attempt them as exercises should therefore read them as worked examples. For the reader who is struggling with a particular question but who wishes to persevere, there is a section of hints, which will lead him/her through the procedure to be followed for each question.

Calculator keystrokes

Many of the examples and answers include the full keystroke procedures necessary using a Hewlett Packard calculator. The use of an HP calculator is described in detail in Appendix 1.

A final word

However hard one tries, there are always mistakes. I apologize for these and welcome comments or suggestions from readers – particularly on other areas which might usefully be included in the book without making it too boffin orientated. As always, I am hugely indebted to my wife for her support throughout.

The Basics



“The fundamental principle behind market calculations is the time value of money: as long as interest rates are not negative, any given amount of money is worth more sooner than it is later because you can place it on deposit to earn interest.”

Financial Arithmetic Basics

Some opening remarks on formulas

Use of an HP calculator

Simple and compound interest

Nominal and effective rates

Future value/present value; time value of money

Discount factors

Cashflow analysis

Interpolation and extrapolation

Exercises

SOME OPENING REMARKS ON FORMULAS

There are three important points concerning many of the formulas in this book. These have nothing to do with the business of finance, but rather with how mathematical formulas are written in all areas of life.

First, consider the expression $1 + 0.08 \times \frac{92}{365}$. In this, you must do the multiplication *before* the addition. It is a convention of the way mathematical formulas are written that any exponents (5^4 , x^2 , $4.2^{\frac{1}{4}}$ etc.) must be done first, followed by multiplication and division (0.08×92 , $x \div y$, $\frac{17}{38}$ etc.) and addition and subtraction last. This rule is applied first to anything inside brackets “(...)” and then to everything else. This means that $1 + 0.08 \times \frac{92}{365}$ is the same as $1 + (0.08 \times \frac{92}{365})$ and is equal to 1.0202. This is *not* the same as $(1 + 0.08) \times \frac{92}{365}$, which is equal to 0.2722. If I mean to write this latter expression, I must write the brackets. If I mean to write the first expression, I do not need to write the brackets and it is in fact usual to leave them out.

Second, the expression “per cent” means “divided by 100.” Therefore “4.7%”, $\frac{4.7}{100}$ and “0.047” are all the same. When writing a formula which involves an interest rate of 4.7 percent for example, this usually appears as “0.047” rather than $\frac{4.7}{100}$, simply because “0.047” is neater. Similarly, when we speak of an “interest rate i ,” we mean a decimal such as 0.047, not a number such as 4.7.

Third, the symbol Σ is shorthand for “the sum of all.” Thus for example, $\Sigma (\text{cashflow})_i$ is shorthand for “ $\text{cashflow}_1 + \text{cashflow}_2 + \text{cashflow}_3 + \dots$ ” Where there are many cashflows, this is a very useful abbreviation.

USE OF AN HP CALCULATOR

Some of the calculations used in financial arithmetic are mostly easily performed using a specialist calculator. The ones most widely used in the markets are probably Hewlett Packard calculators. In many of the worked examples and exercises in this book, as well as showing the method of calculation, we have therefore also given the steps used on an HP. It is important to understand that you do not *need* an HP in order to work through these examples. An ordinary non-financial calculator is fine for many calculations, and you can use an alternative specialist calculator – or a computer spreadsheet – for the rest. We have chosen to show the steps for an HP simply because many readers will already have access to one, and because it is recommended for use in the ACI Financial Calculations exam.

In order to make these steps intelligible to anyone new to an HP, we have explained the basics of operating the calculator in Appendix 1. We have certainly not tried to give full instructions for using an HP – the calculator’s own instruction manual is clearly the best place for that – but have set out only those operations which are necessary for our examples.

SIMPLE AND COMPOUND INTEREST

Simple interest

On short-term financial instruments, interest is usually “simple” rather than “compound”. Suppose, for example, that I place £1 on deposit at 8 percent for 92 days. As the 8 percent is generally quoted as if for a whole year rather than for only 92 days, the interest I expect to receive is the appropriate proportion of 8 percent:

$$£0.08 \times \frac{92}{365}$$

The total proceeds after 92 days are therefore the return of my principal, plus the interest:

$$£(1 + 0.08 \times \frac{92}{365})$$

If I place instead £73 on deposit at 8 percent for 92 days, I will receive a total of:

$$£73 \times (1 + 0.08 \times \frac{92}{365})$$

$$\text{Total proceeds of short-term investment} = \text{principal} \times \left(1 + \text{interest rate} \times \frac{\text{days}}{\text{year}}\right)$$

**Calculation
summary**

By “ $\frac{\text{days}}{\text{year}}$ ” we mean “ $\frac{\text{number of days in the period}}{\text{number of days in the year}}$ ”.

In this chapter “number of days in the year” generally means 365. In some markets, however, (as explained in Chapter 2) this number is 360 by convention. Where this might be the case, we have therefore used “year” to cover either situation.

Compound interest

Now consider an investment of 1 made for two years at 10 percent per annum. At the end of the first year, the investor receives interest of 0.10. At the end of the second year he receives interest of 0.10, plus the principal of 1. The total received is $0.10 + 0.10 + 1 = 1.20$. However, the investor would in practice reinvest the 0.10 received at the end of the first year, for a further year. If he could do this at 10 percent, he would receive an extra 0.01 (= 10 percent \times 0.10) at the end of the second year, for a total of 1.21.

In effect, this is the same as investing 1 for one year at 10 percent to receive $1 + 0.10$ at the end of the first year and then reinvesting this whole $(1 + 0.10)$ for a further year, also at 10 percent, to give $(1 + 0.10) \times (1 + 0.10) = 1.21$.

The same idea can be extended for any number of years, so that the total return after N years, including principal, is:

$$\text{Principal} \times (1 + \text{interest rate})^N$$

This is “compounding” the interest, and assumes that all interim cashflows can be reinvested at the same original interest rate. “Simple” interest is when the interest is not reinvested.

**Calculation
summary**

$$\begin{aligned} \text{Total proceeds of long-term investment for } N \text{ years} \\ = \text{principal} \times (1 + \text{interest rate})^N \end{aligned}$$

Nominal rates and effective rates

Now consider a deposit of 1 made for only one year at 10 percent per annum, but with quarterly interest payments. This would mean interest of 0.025 received each quarter. As above, this could be reinvested for a further quarter to achieve interest-on-interest of $0.025 \times 0.025 = 0.000625$. Again, this is the same as investing 1 for only three months at 10 percent per annum (that is, 2.5 percent per quarter) to receive $(1 + 0.025)$ at the end of the first quarter and then reinvesting this whole $(1 + 0.025)$ for a further quarter, also at the same interest rate, and so on for a total of four quarters. At the end of this, the total return, including principal is:

$$(1 + 0.025) \times (1 + 0.025) \times (1 + 0.025) \times (1 + 0.025) = (1 + 0.025)^4 = 1.1038$$

The same idea can be extended for monthly interest payments or semi-annual (6-monthly) interest payments, so that in general the proceeds at the end of a year including principal, with n interest payments per year, are:

$$\text{Principal} \times \left(1 + \frac{\text{interest rate}}{n}\right)^n$$

It is often useful to compare two interest rates which are for the same investment period, but with different interest payment frequency – for example, a 5-year interest rate with interest paid quarterly compared with a 5-year rate with interest paid semi-annually. To do this, it is common to calculate an equivalent annualized rate. This is the rate with interest paid annually which would give the same compound return at the end of the year as the rate we are comparing. From this it follows that:

$$\text{Principal} \times \left(1 + \frac{\text{interest rate}}{n}\right)^n = \text{Principal} \times (1 + \text{equivalent annual rate})$$

Thus if the interest rate with n payments per year is i , the equivalent annual interest rate i^* is:

**Calculation
summary**

$$i^* = \left[\left(1 + \frac{i}{n}\right)^n - 1\right]$$

This equivalent annual interest rate i^* is known as the “effective” rate. The rate i from which it is calculated is known as the “nominal” rate. It follows that:

$$i = \left[(1 + i^*)^{\frac{1}{n}} - 1 \right] \times n$$

**Calculation
summary**

Example 1.1

8% is the nominal interest rate quoted for a 1-year deposit with the interest paid in quarterly instalments. What is the effective rate (that is, the equivalent rate quoted when all the interest is paid together at the end of the year)?

$$\left[\left(1 + \frac{0.08}{4} \right)^4 - 1 \right] = 8.24\%$$

Answer: 8.24%

Using an HP-19BII

.08 ENTER

4 ÷ 1 +

4 □ ^

1 -

Example 1.2

5% is the nominal interest rate quoted for a 1-year deposit when the interest is paid all at maturity. What is the quarterly equivalent?

$$\left[(1.05)^{\frac{1}{4}} - 1 \right] \times 4 = 4.91\%$$

Answer: 4.91%

1.05 ENTER

4 □ 1/x □ ^

1 - 4 x

In the same way, we might for example wish to compare the rate available on a 40-day investment with the rate available on a 91-day investment. One approach, as before, is to calculate the effective rate for each. In this case, the effective rate formula above can be extended by using the proportion $\frac{365}{\text{days}}$ instead of the interest rate frequency n .

Example 1.3

The interest rate for a 5-month (153-day) investment is 10.2%. What is the effective rate?

$$\begin{aligned}\text{Effective rate} &= \left(1 + i \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1 \\ &= \left(1 + 0.102 \times \frac{153}{365}\right)^{\frac{365}{153}} - 1 = 10.50\%\end{aligned}$$

.102 ENTER 153 x 365 ÷ 1 +
365 ENTER 153 ÷ □ ^ 1 -

**Calculation
summary**

$$\text{Effective rate} = \left(1 + i \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1$$

Continuous compounding

The effect of compounding increases with the frequency of interest payments, because there is an increasing opportunity to earn interest on interest. As a result, an annual rate will always be greater than the semi-annual equivalent, which in turn will always be greater than the monthly equivalent, etc. In practice, the limit is reached in considering the daily equivalent rate. Thus the equivalent rate with daily compounding for an annual rate of 9.3 percent is:

$$\left[(1 + 0.093)^{\frac{1}{365}} - 1\right] \times 365 = 8.894\%$$

1.093 ENTER 365 □ 1/x □ ^ 1 - 365 x

In theory, however, the frequency could be increased indefinitely, with interest being compounded each hour, or each minute. The limit of this concept is where the frequency is infinite – that is, “continuous compounding.” The equivalent interest rate in this case can be shown to be $\text{LN}(1.093)$, where LN is the natural logarithm \log_e . The number e – approximately 2.7183 – occurs often in mathematical formulas; e^x and $\log_e x$ can both usually be found on mathematical calculators, sometimes as EXP and LN. Thus:

$$\text{Continuously compounded rate} = \text{LN}(1.093) = 8.893\%$$

1.093 □ MATHS LOG LN

In general:

The continuously compounded interest rate r is:

$$r = \frac{365}{\text{days}} \times \text{LN}\left(1 + i \times \frac{\text{days}}{\text{year}}\right) \text{ where } i \text{ is the nominal rate for that number of days.}$$

In particular:

$$r = \text{LN}(1 + i) \text{ where } i \text{ is the annual effective rate.}$$

**Calculation
summary**

These relationships can be reversed to give:

$$i = \frac{\text{year}}{\text{days}} \times \left(e^{r \times \frac{\text{days}}{365}} - 1 \right)$$

**Calculation
summary**

Example 1.4

The 91-day interest rate is 6.4%. What is the continuously compounded equivalent?

$$\begin{aligned} r &= \frac{365}{\text{days}} \times \text{LN}\left(1 + i \times \frac{\text{days}}{\text{year}}\right) \\ &= \frac{365}{91} \times \text{LN}\left(1 + 0.064 \times \frac{91}{365}\right) = 6.35\% \end{aligned}$$

.064 ENTER 91 × 365 ÷ 1 + □ MATH LOGS LN 365 × 91 ÷

Example 1.5

The continuously compounded rate is 7.2%. What is the effective rate?

$$i = e^r - 1 = e^{0.072} - 1 = 7.47\%$$

.072 □ MATH LOGS EXP 1 -

Reinvestment rates

The assumption we have used so far for compounding interest rates, calculating effective rates, etc. is that interest cashflows arising during an investment can be reinvested at the same original rate of interest. Although these calculations are very important and useful, reinvestment of such cashflows is in practice likely to be at different rates.

To calculate the accumulated value by the end of the investment in this case, account must be taken of the different rate paid on the interim cashflows.

Example 1.6

£100 is invested for 3 years at 3.5% (paid annually). By the end of the first year, interest rates for all periods have risen to 4.0% (paid annually). By the end of the second year, rates have risen to 5.0% (paid annually). Whenever an interest payment is received, it is reinvested to the end of the 3-year period. What is the total cash returned by the end of the third year?

The cashflows received from the original investment are:

1 year: + £3.50
 2 years: + £3.50
 3 years: + £103.50

At the end of year 1, the £3.50 is reinvested at 4.0% to produce the following cashflows:

2 years: + £3.50 × 4% = £0.14
 3 years: + £3.50 × (1 + 4%) = £3.64

At the end of year 2, the £3.50 from the original investment plus the £0.14 arising from reinvestment of the first year's interest, is reinvested at 5.0% to produce the following cashflow:

3 years: + £3.64 × (1 + 5%) = £3.82

The total return is therefore £103.50 + £3.64 + £3.82 = £110.96

Note that the result would be slightly different if the interim interest payments were each reinvested only for one year at a time (and then rolled over), rather than reinvested to the maturity of the original investment.

```
3.5 ENTER .04 x
3.5 +
.05 ENTER 1 + x
.04 ENTER 1 + 3.5 x +
103.5 +
```

NOMINAL AND EFFECTIVE RATES

We have seen that there are three different elements to any interest rate. Confusion arises because the words used to describe each element can be the same.

1. The period for which the investment/loan will last

In this sense, a “6-month” interest rate is one for an investment which lasts six months and a “1-year” interest rate is one for an investment which lasts one year.

2. The absolute period to which the quoted interest rate applies

Normally, this period is always assumed to be one year, regardless of the actual period of investment. Thus, if the interest rate on a 6-month invest-

ment is quoted as 10 percent, this does not mean that the investor in fact receives 10 percent after six months; rather, the investor receives only 5 percent, because the quoted rate is expressed as the 1-year *simple* equivalent rate, even though there is no intention to hold the investment for a year. Similarly, a 5-year zero-coupon rate of 10 percent means that the investor will in fact receive 61.05 percent after five years. Again, the quoted rate is the *decompounded* one-year equivalent, even though the investment is fixed for five years and pays no actual interest at the end of one year.

The reason for this “annualizing” of interest rates is to make them approximately comparable. If this were not done, one would not be able immediately to compare rates quoted for different periods. Instead, we would be trying to compare rates such as 0.0274 percent for 1 day, 5 percent for 6 months and 61.05 percent for 5 years (all in fact quoted as 10 percent). A yield curve drawn like this would be difficult to interpret at a glance.

One exception is the interest paid on personal credit cards, which is sometimes quoted for a monthly period rather than annualized (for cosmetic reasons). In the UK, for example, credit card companies are obliged also to quote the effective rate (known as APR, annualized percentage rate).

3. The frequency with which interest is paid

If interest on a 1-year deposit is paid each 6 months, the total interest accumulated at the end of the year, assuming reinvestment of the interim interest payment, will be greater than the interest accumulated if the deposit pays the same quoted rate, but all in one amount at the end of the year. When the interest rate on a money market investment is quoted, it is generally quoted on the basis of the frequency which the investment does actually have. For example, if a 1-year deposit pays 2.5 percent each 3 months, the interest is quoted as 10 percent; the rate is quoted on the basis that all parties know the payment frequency is 3 months. Similarly, if a 1-year deposit pays 5 percent each 6 months, or 10 percent at the end, the rate is quoted as 10 percent on the basis that all parties know the payment frequency is 6 months, or 1 year. The economic effect is different in each case, but no adjustment is made to make the quotations comparable; they are simply stated as “10 percent quarterly” or “10 percent semi-annual” or “10 percent annual.”

Yields for financial instruments are generally quoted in the market in this *simple* way, regardless of whether they are short-term money market instruments such as treasury bills and deposits, or long-term capital markets instruments such as bonds and swaps. One exception is zero-coupon rates, where the interest rate is *compounded* downwards when expressed as an annualized rate. Other exceptions may arise in specific markets – for example treasury bills in Norway are quoted on the basis of an *effective* annual equivalent yield. Furthermore, because an investor does need to be able to compare rates, they are often all converted by the investor to a common effective basis. In this way, “10 percent quarterly” is converted to an effective “10.38 annual equivalent”, or “10 percent annual” to an effective “9.65 percent quarterly equivalent” or “9.532 percent daily equivalent”.

FUTURE VALUE / PRESENT VALUE; TIME VALUE OF MONEY

This section is probably the most important in this book, as present value calculations are the key to pricing financial instruments.

Short-term investments

If I deposit 100 for 1 year at 10 percent per annum, I receive:

$$100 \times (1 + 0.10) = 110$$

at the end of the year. In this case, 110 is said to be the “future value” (or “accumulated value”) after 1 year of 100 now. In reverse, 100 is said to be the “present value” of 110 in a year’s time. Future and present values clearly depend on both the interest rate used and the length of time involved.

Similarly, the future value after 98 days of £100 now at 10 percent per annum would be $100 \times (1 + 0.10 \times \frac{98}{365})$.

The expression above can be turned upside down, so that the present value now of 102.68 in 98 days’ time, using 10 percent per annum, is:

$$\frac{102.68}{(1 + 0.10 \times \frac{98}{365})} = 100$$

In general, the present value of a cashflow C, using an interest rate i is:

$$\frac{C}{(1 + i \times \frac{\text{days}}{\text{year}})}$$

We can therefore now generate a future value from a present value, and vice versa, given the number of days and the interest rate. The third calculation needed is the answer to the question: if we know how much money we invest at the beginning (= the present value) and we know the total amount at the end (= the future value), what is our rate of return, or yield (= the interest rate) on the investment? This is found by turning round the formula above again, to give:

$$\text{yield} = \left(\frac{\text{future value}}{\text{present value}} - 1 \right) \times \frac{\text{year}}{\text{days}}$$

This gives the yield as normally expressed – that is, the yield for the period of the investment. This can of course then be converted to an effective rate by using:

$$\text{effective yield} = \left(1 + i \times \frac{\text{days}}{\text{year}} \right)^{\frac{365}{\text{days}}} - 1$$

Combining these two relationships gives:

$$\text{effective yield} = \left(\frac{\text{future value}}{\text{present value}} \right)^{\frac{365}{\text{days}}} - 1$$

The calculation of present value is sometimes known as “discounting” a future value to a present value and the interest rate used is sometimes known as the “rate of discount.”

In general, these calculations demonstrate the fundamental principles behind market calculations, the “time value of money.” As long as interest rates are not negative, any given amount of money is worth more sooner than it is later because if you have it sooner, you can place it on deposit to earn interest. The extent to which it is worthwhile having the money sooner depends on the interest rate and the time period involved.

For short-term investments

$$FV = PV \times \left(1 + i \times \frac{\text{days}}{\text{year}}\right)$$

$$PV = \frac{FV}{\left(1 + i \times \frac{\text{days}}{\text{year}}\right)}$$

$$i = \left(\frac{FV}{PV} - 1\right) \times \frac{\text{year}}{\text{days}}$$

$$\text{effective yield (annual equivalent)} = \left(\frac{FV}{PV}\right)^{\frac{365}{\text{days}}} - 1$$

**Calculation
summary**

Long-term investments

The formulas above are for investments where no compound interest is involved. For periods more than a year where compounding is involved, this compounding must be taken into account.

The future value in three years' time of 100 now, using 10 percent per annum is:

$$100 \times (1 + 0.10)^3 = 133.10$$

This expression can again be turned upside down, so that the present value now of 133.10 in three years' time, using 10 percent per annum, is:

$$\frac{133.10}{(1 + 0.10)^3} = 100$$

In general, the present value of a cashflow C in N years' time, using an interest rate i, is:

$$\frac{C}{(1 + i)^N}$$

**Calculation
summary****For long-term investments over N years**

$$FV = PV \times (1 + i)^N$$

$$PV = \frac{FV}{(1 + i)^N}$$

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{N}} - 1$$

Example 1.7

What is the future value in 5 years' time of £120 now, using 8% per annum?

$$120 \times (1.08)^5 = 176.32$$

(The interest rate is compounded because interest is paid each year and can be reinvested)

Answer: £176.32

1.08 ENTER
5 □ ^
120 x

Example 1.8

What is the future value in 92 days' time of £120 now, using 8% per annum?

$$120 \times \left(1 + 0.08 \times \frac{92}{365}\right) = 122.42$$

(Simple interest rate, because there is only one interest payment, at maturity)

Answer: £122.42

.08 ENTER
92 × 365 ÷
1 + 120 x

Example 1.9

What is the present value of £270 in 4 years' time, using 7% per annum?

$$\frac{270}{(1.07)^4} = 205.98$$

(The interest rate is compounded because interest is paid each year and can be reinvested)

Answer: £205.98

1.07 ENTER
 4 \square \wedge
 270 \square $\times \frac{\text{days}}{\text{year}}$ \div

Example 1.10

What is the present value of £270 in 180 days' time, using 7% per annum?

$$\frac{270}{\left(1 + 0.07 \times \frac{180}{365}\right)} = 260.99$$

(Simple interest rate, because there is only one interest payment, at maturity)

Answer: £260.99

.07 ENTER
 180 \times 365 \div
 1 +
 270 \square $\times \frac{\text{days}}{\text{year}}$ \div

Example 1.11

I invest £138 now. After 64 days I receive back a total (principal plus interest) of £139.58. What is my yield on this investment?

$$\text{yield} = \left(\frac{139.58}{138.00} - 1 \right) \times \frac{365}{64} = 0.0653$$

Answer: 6.53%

139.58 ENTER
 138 \div
 1 –
 365 \times
 64 \div

DISCOUNT FACTORS

So far, we have discounted from a future value to a present value by dividing by $(1 + i \times \frac{\text{days}}{\text{year}})$ for simple interest and $(1+i)^N$ for compound interest. An alternative way of expressing this, also commonly used, is to multiply by the reciprocal of these numbers, which are then usually called “discount factors.”

Example 1.12

What is the 92-day discount factor if the interest rate for the period is 7.8%? What is the present value of £100 in 92 days' time?

$$\text{discount factor} = \frac{1}{1 + 0.078 \times \frac{92}{365}} = 0.9807$$

$$£100 \times 0.9807 = £98.07$$

.078 ENTER 92 × 365 ÷ 1 + □ 1/x	(Discount factor)
100 ×	(Present value)

Example 1.13

What is the 3-year discount factor based on a 3-year interest rate of 8.5% compounded annually? What is the present value of £100 in 3 years' time?

$$\text{discount factor} = \frac{1}{(1 + 0.085)^3} = 0.7829$$

$$£100 \times 0.7829 = £78.29$$

1.085 ENTER 3 □ ∧ □ 1/x	(Discount factor)
100 ×	(Present value)

**Calculation
summary**

For simple interest

$$\text{Discount factor} = \frac{1}{1 + i \times \frac{\text{days}}{\text{year}}}$$

For compound interest

$$\text{Discount factor} = \left(\frac{1}{1 + i} \right)^N$$

Note that we know from an earlier section that using a continuously compounded interest rate r , $i = \frac{\text{year}}{\text{days}} \times (e^{r \times \frac{\text{days}}{365}} - 1)$.

Using a continuously compounded interest rate therefore, the discount factor

becomes:
$$\frac{1}{1 + i \times \frac{\text{days}}{\text{year}}} = \frac{1}{1 + (e^{r \times \frac{\text{days}}{365}} - 1)} = e^{-r \times \frac{\text{days}}{365}}$$

This way of expressing the discount factor is used commonly in option pricing formulas.

**Calculation
summary**

Using a continuously compounded interest rate

$$\text{Discount factor} = e^{-r \times \frac{\text{days}}{365}}$$

CASHFLOW ANALYSIS

NPV

Suppose that we have a series of future cashflows, some of which are positive and some negative. Each will have a present value, dependent on the time to the cashflow and the interest rate used. The sum of all the positive and negative present values added together is the net present value or NPV.

Example 1.14

What is the NPV of the following future cashflows, discounting at a rate of 7.5%?

After 1 year: + \$83
After 2 years: − \$10
After 3 years: +\$150

$$\frac{83}{(1.075)} - \frac{10}{(1.075)^2} + \frac{150}{(1.075)^3} = 189.30$$

Answer: +\$189.30

83 ENTER 1.075 ÷
10 ENTER 1.075 ENTER 2 \square \wedge ÷ −
150 ENTER 1.075 ENTER 3 \square \wedge ÷ +

NPV = sum of all the present values

Key Point

IRR

An internal rate of return (IRR) is the one single interest rate which it is necessary to use when discounting a series of future values to achieve a given net present value. This is equivalent to the interest rate which it is necessary to use when discounting a series of future values *and* a cashflow *now*, to achieve a *zero* present value. Suppose the following cashflows, for example, which might arise from some project:

Now: − 87
After 1 year: + 25
After 2 years: − 40
After 3 years: + 60
After 4 years: + 60

What interest rate is needed to discount +25, −40, +60 and +60 back to a net present value of +87? The answer is 5.6 percent. It can therefore be said that an initial investment of 87 produces a 5.6 percent internal rate of return if it

generates these subsequent cashflows. This is equivalent to saying that, using 5.6 percent, the net present value of $-87, +25, -40, +60$ and $+60$ is zero.

Calculating a NPV is relatively simple: calculate each present value separately and add them together. Calculating an IRR however requires a repeated “trial and error” method and is therefore generally done using a specially designed calculator.

Example 1.15

What is the IRR of the following cashflows?

Now:	−\$164
After 1 year:	+\$45
After 2 years:	+\$83
After 3 years:	+\$75

Answer: 10.59% (If you do not have a calculator able to calculate this, try checking the answer by working backwards: the NPV of all the future values, using the rate of 10.59%, should come to + \$164).

See below for an explanation of how to use the HP19 to solve this.

Key Point

The internal rate of return is the interest rate which discounts all the known future cashflows to a given NPV.

This is equivalent to the interest rate which discounts all the cashflows including any cashflow now to zero.

Annuity

An annuity is a regular stream of future cash receipts which can be purchased by an initial cash investment. The size of the future cash receipts is determined by the yield which can be obtained on the investment. In other words, the internal rate of return of the cashflows (initial outflow and subsequent inflows) is the yield.

Example 1.16

An investor puts £50,000 in a 20-year annuity, yielding 7.2%. The annuity returns an equal cash amount at the end of each year, with no additional amount at maturity. What is the annual cash amount?

Answer: £4,793.26. (If you do not have a calculator able to calculate this, you can check the answer by working backwards: the NPV at 7.2% of £4,793.26 each year for 20 years is £50,000.)

See below for an explanation of how to use the HP19 to solve this.

Using an HP calculator

There are various functions built into the HP calculator relating to the time value of money. These involve using five keys: N for the number of time periods involved, I%YR for the yield, PV for the initial cashflow or present value, PMT for a regular cashflow recurring at the end of each period and FV for an additional final cashflow or the future value. Any of the cashflows (PV, PMT, or FV) may be zero. Before any cashflow operation, we suggest setting the calculator by entering g END (for the HP12C) or selecting the FIN menu, selecting the TVM menu, selecting OTHER and then selecting END (for the other calculators). Note that you must always pay strict attention to the sign of each cashflow: a cash outflow is negative and a cash inflow is positive.

HP calculator example

What is the net present value of the following cashflows using an interest rate of 10%?

\$11 at the end of each year for 7 years.

\$80 at the end of the seventh year in addition to the \$11.

Answer: \$94.61

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
7 n 10 i 11 PMT 80 FV PV	Select FIN menu Select TVM menu 7 N 10 I%YR 11 PMT 80 FV PV	Select FIN menu Select TVM menu 7 N 10 I%YR 11 PMT 80 FV PV

HP calculator example

What is the internal rate of return of the following cashflows?

Outflow of \$94.6053 now.

Inflow of \$11 at the end of each year for seven years.

Inflow of \$80 at the end of the seventh year in addition to the \$11.

Answer: 10%

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
7 n 94.6053 CHS PV 11 PMT 80 FV i	Select FIN menu Select TVM menu 7 N 94.6053 +/- PV 11 PMT 80 FV I%YR	Select FIN menu Select TVM menu 7 N 94.6053 +/- PV 11 PMT 80 FV I%YR

HP calculator example

What regular cash inflow is needed at the end of each year for the next seven years to achieve an internal rate of return of 10%, assuming an additional final cash inflow of \$80 at the end of the seventh year and a net present value of \$94.6053?

Answer: \$11

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
7 n 10 i 94.6053 CHS PV 80 FV PMT	Select FIN menu Select TVM menu 7 N 10 I%YR 94.6053 +/- PV 80 FV PMT	Select FIN menu Select TVM menu 7 N 10 I%YR 94.6053 +/- PV 80 FV PMT

You can see that the last three examples all involve the same five data: the number of time periods ($N = 7$), the interest rate or internal rate of return per period ($I\%YR = 10$), the net present value ($PV = -94.6053$), the regular cash-flow each period ($PMT = 11$) and the additional final cashflow or future value ($FV = 80$). Once four of these five data have been entered, the fifth can be calculated by pressing the relevant key.

HP calculator example

What is the internal rate of return of the following cashflows?

Outflow of \$60 now.
 Inflow of \$10 at the end of each year for 10 years.

Answer: 10.5580%

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
10 n 60 CHS PV 10 PMT 0 FV i	Select FIN menu Select TVM menu 10 N 60 +/- PV 10 PMT 0 FV I%YR	Select FIN menu Select TVM menu 10 N 60 +/- PV 10 PMT 0 FV I%YR

For the TVM function to be appropriate, the time periods between each cashflow must be regular. They do not need to be 1 year. If the periods are shorter than a year however, the absolute interest rate for that period and the total number of periods (rather than years) must be entered.

HP calculator example

What is the net present value of the following cashflows using an interest rate of 10% per annum paid quarterly?

\$100 at the end of each quarter for 5 years.

\$1,000 at the end of the fifth year in addition to the \$100.

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
20 n 2.5 i 100 PMT 1000 FV PV	Select FIN menu Select TVM menu 20 N 2.5 I%YR 100 PMT 1000 FV PV	Select FIN menu Select TVM menu 20 N 2.5 I%YR 100 PMT 1000 FV PV

Answer: \$2,169.19

In the example above, instead of entering the interest rate per period as 2.5 percent, it is possible with the HP19BII to enter the interest rate per year

	HP19BII (RPN mode)	HP19BII (algebraic)
	Select FIN menu Select TVM menu OTHER 4 P/YR EXIT 20 N 10 I%YR 100 PMT 1000 FV PV	Select FIN menu Select TVM menu OTHER 4 P/YR EXIT 20 N 10 I%YR 100 PMT 1000 FV PV

as 10 percent and the number of payments per year as 4 as follows. This merely avoids dividing by 4.

The built-in TVM functions can be used when all the cashflows are the same except for the initial cashflow now or present value, and the additional final cashflow or future value. For bonds and swaps, this is often the case. In cases where the cashflows are irregular however, an alternative function must be used.

HP calculator example

What is the IRR of the following cashflows?

Now: −\$120
After 1 year: +\$20
After 2 years: +\$90
After 3 years: −\$10
After 4 years: +\$30
After 5 years: +\$30
After 6 years: +\$30
After 7 years: +\$40

Answer: 20.35%

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
f CLEAR REG 120 CHS g CFo 20 g Cfj 90 g Cfj 10 CHS g Cfj 30 g Cfj 3 g Nj 40 g Cfj f IRR	Select FIN menu Select CFLO menu GET ★NEW 120 +/- INPUT 20 INPUT INPUT 90 INPUT INPUT 10 +/- INPUT INPUT 30 INPUT 3 INPUT 40 INPUT INPUT CALC IRR%	Select FIN menu Select CFLO menu GET ★NEW 120 +/- INPUT 20 INPUT INPUT 90 INPUT INPUT 10 +/- INPUT INPUT 30 INPUT 3 INPUT 40 INPUT INPUT CALC IRR%

(If several successive cashflows are the same, the amount need be entered only once, followed by the number of times it occurs.)

HP calculator example

Using the same cashflows as above, what is the NPV of all the cashflows using an interest rate of 10%?

Answer: 41.63%

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
f CLEAR REG 120 CHS g Cfo 20 g Cfj 90 g Cfj 10 CHS g Cfj 30 g Cfj 3 g Nj 40 g Cfj 10 i f NPV	Select FIN menu Select CFLO menu GET ★NEW 120 +/- INPUT 20 INPUT INPUT 90 INPUT INPUT 10 +/- INPUT INPUT 30 INPUT 3 INPUT 40 INPUT INPUT CALC 10 I% NPV	Select FIN menu Select CFLO menu GET ★NEW 120 +/- INPUT 20 INPUT INPUT 90 INPUT INPUT 10 +/- INPUT INPUT 30 INPUT 3 INPUT 40 INPUT INPUT CALC 10 I% NPV

Solution to Example 1.15 using HP19BII

FIN CFLO GET ★NEW
 164 +/- INPUT
 45 INPUT INPUT
 83 INPUT INPUT
 75 INPUT INPUT
 CALC
 IRR%

Solution to Example 1.16 using HP19BII

FIN TVM
 20 N
 7.2 I%YR
 50,000 +/- PV
 0 FV
 PMT

INTERPOLATION AND EXTRAPOLATION

In the money market, prices are generally quoted for standard periods such as 1 month, 2 months, etc. If a dealer needs to quote a price for an “odd date” between these periods, he needs to “interpolate.”

Suppose for example that the 1-month rate (30 days) is 8.0 percent and that the 2-month rate (61 days) is 8.5 percent. The rate for 1 month and 9 days (39 days) assumes that interest rates increase steadily from the 1-month rate to the 2-month rate – a *straight line* interpolation. The increase from 30 days to 39 days will therefore be a $\frac{9}{31}$ proportion of the increase from 30 days to 61 days. The 39-day rate is therefore:

$$8.0\% + (8.5\% - 8.0\%) \times \frac{9}{31} = 8.15\%$$

The same process can be used for interpolating exchange rates.

Example 1.17

The 2-month (61 days) rate is 7.5% and the 3-month (92 days) rate is 7.6%.

What is the 73-day rate?

$$7.5 + (7.6 - 7.5) \times \frac{12}{31} = 7.5387$$

Answer: 7.5387%

7.6 ENTER 7.5 -
 12 x 31 ÷
 7.5 +

If the odd date required is just before or just after the known periods, rather than between them, the same principle can be applied (in this case “extrapolation” rather than interpolation).

Example 1.18

The 2-month (61 days) rate is 7.5% and the 3-month (92 days) rate is 7.6%.

What is the 93-day rate?

$$7.5 + (7.6 - 7.5) \times \frac{32}{31} = 7.6032$$

Answer: 7.6032%

7.6 ENTER 7.5 –
32 x 31 ÷
7.5 +

Calculation summary

$$i = i_1 + (i_2 - i_1) \times \frac{(d - d_1)}{(d_2 - d_1)}$$

where: i is the rate required for d days
 i_1 is the known rate for d_1 days
 i_2 is the known rate for d_2 days.



EXERCISES

1. What is the future value after 120 days of £43 invested at 7.5%?
2. You will receive a total of £89 after 93 days. What is the present value of this amount discounted at 10.1%?
3. You invest £83 now and receive a total of £83.64 back after 28 days. What is the yield on your investment?
4. What is the future value in 10 years' time of £36 now, using 9% per annum?
5. You have a choice between receiving DEM 1,000 now or DEM 990 in 3 months' time. Assuming interest rates of 8.0%, which do you choose?
6. If you invest £342 for 5 years at 6% per annum (interest paid annually), how much interest do you receive at the end of 5 years assuming that all interim cashflows can be reinvested also at 6%?
7. What is the present value, using a rate of discount of 11%, of a cashflow of DEM 98.00 in 5 years' time?
8. You place £1,000 in a 4-year investment which makes no interest payments but yields 5.4% per annum compound. How much do you expect to receive at the end of 4 years?
9. You invest £1,000 and receive back a total of £1,360.86 at the end of 7 years. There are no interest payments during the 7 years. What annual yield does this represent?
10. You deposit £1 million for 10 years. It accumulates interest at 6% quarterly for the first 5 years and 6.5% semi-annually for the next five years. The interest is automatically added to the capital at each payment date. What is the total accumulated value at the end of 10 years?
11. You place £1 million on deposit for 1 year at 8.5%. What total value will you have accumulated by the end of the year, assuming that the interest is paid quarterly and can be reinvested at the same rate? What would the total value be if the interest payments could be reinvested at only 8.0% paid quarterly?
12. You buy a 10-year annuity, with a yield of 9% per annum. How much must you invest in the annuity now to receive £5,000 at the end of each year?
13. You borrow £90,000 for 25 years at 7.25% per annum (interest paid monthly). You repay the loan by making equal payments which cover principal plus interest at the end of each month for the 25 years. How much are the monthly payments?

14. You receive 11.4% paid semi-annually. What is the effective rate (annual equivalent)?
15. You receive 12% paid annually. What are the equivalent quarterly rate and monthly rate?
16. If 7.0% is a continuously compounded interest rate, what is the total value accumulated at the end of a year at this rate, on a principal amount of £1 million, and what is the effective rate (annual equivalent)? If 9.0% is an effective (annual equivalent) rate, what is the equivalent continuously compounded rate?
17. You receive 6.5% per annum on a 138-day deposit. What is the effective rate? What is the daily equivalent rate? What is the 138-day discount factor?
18. The 30-day interest rate is 5.2% and the 60-day rate is 5.4%. Interpolate to find the 41-day rate.
19. What is the NPV of the following cashflows using an effective annual interest rate of 10% per annum?

Now	−\$105
6 months	−\$47
12 months	−\$47
18 months	−\$47
24 months	−\$93
36 months	+\$450
20. What is the IRR of the cashflows in the previous question?

Interest Rate Instruments



“For any instrument, the price an investor is prepared to pay is the present value of the future cashflows which he or she will receive because of owning it.”

The Money Market

Overview

Day/year conventions

Money market instruments

Money market calculations

Discount instruments

CDs paying more than one coupon

Exercises

OVERVIEW

The “money market” is the term used to include all *short-term* financial instruments which are based on an interest rate (whether the interest rate is actually paid or just implied in the way the instrument is priced).

The underlying instruments are essentially those used by one party (borrower, seller or issuer) to borrow and by the other party to lend (the lender, buyer or investor). The main such instruments are:

- Treasury bill – borrowing by government.
- Time deposit
- Certificate of deposit (CD) } borrowing by banks.
- Commercial paper (CP) – borrowing by companies (or in some cases, banks).
- Bill of exchange – borrowing by companies.

Each of these instruments represents an obligation on the borrower to repay the amount borrowed at maturity, plus interest if appropriate. As well as these underlying borrowing instruments, there are other money market instruments which are linked to these, or at least to the same interest rate structure, but which are not direct obligations on the issuer in the same way:

- Repurchase agreement – used to borrow short-term but using another instrument (such as a bond) as collateral.
- Futures contract
- Forward rate agreement (FRA) } used to trade or hedge short-term interest rates for future periods.

The money market is linked to other markets through arbitrage mechanisms. Arbitrage occurs when it is possible to achieve the same result in terms of position and risk through two alternative mechanisms which have a slightly different price; the arbitrage involves achieving the result via the cheaper method and simultaneously reversing the position via the more expensive method – thereby locking in a profit which is free from market risk (although still probably subject to credit risk). For example, if I can buy one instrument cheaply and simultaneously sell at a higher price another instrument or combination of instruments which has identical characteristics, I am arbitraging. In a completely free market with no other price considerations involved, the supply and demand effect of arbitrage tends to drive the two prices together.

For example, the money market is linked in this way to the forward foreign exchange market, through the theoretical ability to create synthetic deposits in one currency, by using foreign exchange deals combined with money market instruments. Similarly, it is linked to the capital markets (long-term financial instruments) through the ability to create longer-term instruments from a series of short-term instruments (such as a 2-year swap from a series of 3-month FRAs).

Eurocurrency Historically, the term “Euro” has been used to describe any instrument which is held outside the country whose currency is involved. The term does not imply “European” specifically. For example, a sterling deposit made by a UK resident in London is domestic sterling, but a sterling deposit made in New York is Eurosterling. Similarly, US dollar commercial paper issued outside the USA is Eurocommercial paper while US dollar commercial paper issued inside the USA is domestic commercial paper. Confusingly, this term has nothing whatever to do with the proposed European Union currency also called “euro.”

Coupon / yield A certificate of deposit pays interest at maturity as well as repaying the principal. For example, a CD might be issued with a face value of £1 million which is repaid on maturity together with interest of, say, 10 percent calculated on the number of days between issue and maturity. The 10 percent interest rate is called the “coupon.” The coupon is fixed once the CD is issued. This should not be confused with the “yield,” which is the current rate available in the market when buying and selling an instrument, and varies continually.

Discount 1. An instrument which does not carry a coupon is a “discount” instrument. Because there is no interest paid on the principal, a buyer will only ever buy it for less than its face value – that is “at a discount” (unless yields are negative!). For example, all treasury bills are discount instruments.

2. The word “discount” is also used in the very specialized context of a “discount rate” quoted in the US and UK markets on certain instruments. This is explained in detail below.

Bearer / registered A “bearer” security is one where the issuer pays the principal (and coupon if there is one) to whoever is holding the security at maturity. This enables the security to be held anonymously. A “registered” security, by contrast, is one where the owner is considered to be whoever is registered centrally as the owner; this registration is changed each time the security changes hands.

LIBOR “LIBOR” means “London interbank offered rate” – the interest rate at which one London bank offers money to another London bank of top creditworthiness as a cash deposit. LIBID means “London interbank bid rate” – the interest rate at which one London bank of top creditworthiness bids for money as a cash deposit from another. LIBOR is therefore always the higher side of a two-sided interest rate quotation (which is quoted high–low in some markets and low–high in others). LIMEAN is the average between the two sides. In practice, the offered rate for a particular currency at any moment is generally no different in London from any other major centre. LIBOR is therefore often just shorthand for “offered interest rate.”

Specifically, however, LIBOR also means the average offered rate quoted by a group of banks at a particular time (in London, usually 11:00 am) for a particular currency, which can be used as a benchmark



for agreeing rates between two parties. Similarly, PIBOR refers specifically to the interbank offered rate in Paris for French francs; STIBOR to the interbank offered rate in Stockholm for Swedish kronor; FIBOR in Frankfurt for Deutschemarks, etc.

DAY/YEAR CONVENTIONS

As a general rule in the money markets, the calculation of interest takes account of the exact numbers of days in the period in question, as a proportion of a year. Thus:

Key Point

Interest paid = interest rate quoted \times days in period/days in year

A variation between different money markets arises, however, in the conventions used for the number of days assumed to be in the base “year.” Domestic UK instruments, for example, assume that there are 365 days in a year, even when it is a leap year. Thus a sterling time deposit at 10 percent which lasts exactly one year, but includes 29 February in its period (a total of 366 days), will actually pay slightly more than 10 percent – in fact:

$$10\% \times \frac{366}{365} = 10.027\%$$

This convention is usually referred to as ACT/365 – that is, the actual number of days in the period concerned, divided by 365. Some money markets, however, assume that each year has a conventional 360 days. For example, a dollar time deposit at 10 percent which lasts exactly 365 days pays:

$$10\% \times \frac{365}{360} = 10.139\%$$

This convention is usually referred to as ACT/360. Most money markets assume a conventional year of 360 days. There are, however, some exceptions, which assume a year of 365 days. These include the Euromarkets and domestic markets in the following currencies:

- Sterling
- Irish pound
- Belgian / Luxembourg franc
- Portuguese escudo
- Greek drachma
- Hong Kong dollar

- Singapore dollar
- Malaysian ringgit
- Taiwan dollar
- Thai baht
- South African rand

and the domestic (but not Euro) markets in the following currencies:

- Japanese yen
- Canadian dollar
- Australian dollar
- New Zealand dollar
- Italian lira
- Finnish markka

In order to convert an interest rate i quoted on a 360-day basis to an interest rate i^* quoted on a 365-day basis:

$$i^* = i \times \frac{365}{360}$$

Similarly,

$$i^* = i \times \frac{360}{365}$$

$$\text{Interest rate on 360-day basis} = \text{interest rate on 365-day basis} \times \frac{360}{365}$$

$$\text{Interest rate on 365-day basis} = \text{interest rate on 360-day basis} \times \frac{365}{360}$$

**Calculation
summary**

Example 2.1

The yield on a security on an ACT/360 basis is 10.5%. What is the equivalent yield expressed on an ACT/365 day basis?

$$10.5 \times \frac{365}{360} = 10.6458$$

Answer: 10.6458%

10.5 ENTER
365 × 360 ÷

There are some exceptions to the general approach above. Yields on Swedish T-bills and some short-term German securities, for example, are calculated in the same way as Eurobonds (discussed in Chapter 5).

We have given a list of the conventions used in some important markets in Appendix 2.

Effective rates

The concept of “effective rate” discussed in Chapter 1 normally implies an annual equivalent interest rate on the basis of a calendar year – that is, 365 days. It is possible however to convert the result then to a 360-day basis.

Example 2.2

An amount of 83 is invested for 214 days. The total proceeds at the end are 92. What are the simple and effective rates of return on an ACT/360 basis?

$$\begin{aligned}\text{Simple rate of return (ACT/360)} &= \left(\frac{\text{total proceeds}}{\text{initial investment}} - 1 \right) \times \frac{\text{year}}{\text{days}} \\ &= \left(\frac{92}{83} - 1 \right) \times \frac{360}{214} = 18.24\%\end{aligned}$$

$$\begin{aligned}\text{Effective rate of return (ACT/365)} &= \left(\frac{\text{total proceeds}}{\text{initial investment}} \right)^{\frac{365}{\text{days}}} - 1 \\ &= \left(\frac{92}{83} \right)^{\frac{365}{214}} - 1 = 19.19\%\end{aligned}$$

$$\text{Effective rate of return (ACT/360)} = 19.19\% \times \frac{360}{365} = 18.93\%$$

92 ENTER 83 ÷ 1 – 360 × 214 ÷	(Simple rate ACT/360)
92 ENTER 83 ÷ 365 ENTER 214 ÷ □ ∧ 1 – 360 × 365 ÷	(Effective rate ACT/360)

Note that in Example 2.2 the effective rate on an ACT/360 basis is not $\left(\frac{92}{83} \right)^{\frac{360}{214}} - 1 = 18.91\%$.

This would instead be the equivalent 360-day rate (ACT/360 basis) – that is, the rate on a 360-day investment which is equivalent on a compound basis to 18.24 percent on a 214-day investment. The effective rate we want instead is the equivalent 365-day rate (ACT/360 basis) – that is, the rate on a 365-day investment which is equivalent on a compound basis to 18.24 percent (ACT/360) on a 214-day investment. The difference between these two is however usually not very significant.

MONEY MARKET INSTRUMENTS

Time deposit / loan

A time deposit or “clean” deposit is a deposit placed with a bank. This is not a security which can be bought or sold (that is, it is not “negotiable”), and it must normally be held to maturity.

<i>term</i>	from one day to several years, but usually less than one year
<i>interest</i>	usually all paid on maturity, but deposits of over a year (and sometimes those of less than a year) pay interest more frequently – commonly each six months or each year. A sterling 18-month deposit, for example, generally pays interest at the end of one year and then at maturity.
<i>quotation</i>	as an interest rate
<i>currency</i>	any domestic or Eurocurrency
<i>settlement</i>	generally same day for domestic, two working days for Eurocurrency
<i>registration</i>	there is no registration
<i>negotiable</i>	no

Certificate of deposit (CD)

A CD is a security issued to a depositor by a bank or building society, to raise money in the same way as a time deposit. A CD can however be bought and sold (that is, it is “negotiable”).

<i>term</i>	generally up to one year, although longer-term CDs are issued
<i>interest</i>	usually pay a coupon, although occasionally sold as a discount instrument. Interest usually all paid on maturity, but CDs of over a year (and sometimes those of less than a year) pay interest more frequently – commonly each six months or each year. Some CDs pay a “floating” rate (FRCD), which is paid and refixed at regular intervals.
<i>quotation</i>	as a yield
<i>currency</i>	any domestic or Eurocurrency
<i>settlement</i>	generally same day for domestic, two working days for Eurocurrency
<i>registration</i>	usually in bearer form
<i>negotiable</i>	yes

Treasury bill (T-bill)

Treasury bills are domestic instruments issued by governments to raise short-term finance.

<i>term</i>	generally 13, 26 or 52 weeks; in France also 4 to 7 weeks; in the UK generally 13 weeks
<i>interest</i>	in most countries non-coupon bearing, issued at a discount
<i>quotation</i>	in USA and UK, quoted on a “discount rate” basis, but in most places on a true yield basis
<i>currency</i>	usually the currency of the country; however the UK and Italian governments, for example, also issue ECU T-bills
<i>registration</i>	bearer security
<i>negotiable</i>	yes

Commercial paper (CP)

CP is issued usually by a company (although some banks also issue CP) in the same way that a CD is issued by a bank. CP is usually, however, not interest-bearing. A company generally needs to have a rating from a credit agency for its CP to be widely acceptable. Details vary between markets.

US CP

<i>term</i>	from one day to 270 days; usually very short-term
<i>interest</i>	non-interest bearing, issued at a discount
<i>quotation</i>	on a “discount rate” basis
<i>currency</i>	domestic US\$
<i>settlement</i>	same day
<i>registration</i>	in bearer form
<i>negotiable</i>	yes

Eurocommercial Paper (ECP)

<i>term</i>	from two to 365 days; usually between 30 and 180 days
<i>interest</i>	usually non-interest bearing, issued at a discount
<i>quotation</i>	as a yield, calculated on the same year basis as other money market instruments in that Eurocurrency
<i>currency</i>	any Eurocurrency but largely US\$
<i>settlement</i>	two working days
<i>registration</i>	in bearer form
<i>negotiable</i>	yes

Bill of exchange

A bill of exchange is used by a company essentially for trade purposes. The party owing the money is the “drawer” of the bill. If a bank stands as guarantor to the drawer, it is said to “accept” the bill by endorsing it appropriately, and is the “acceptor”. A bill accepted in this way is a “banker’s acceptance” (BA). In the UK, if the bank is one on a specific list of banks published by the Bank of England, the bill becomes an “eligible bill” which means it is eligible for discounting by the Bank of England, and will therefore command a slightly lower yield than an ineligible bill.

<i>term</i>	from one week to one year but usually less than six months
<i>interest</i>	non-interest bearing, issued at a discount
<i>quotation</i>	in USA and UK, quoted on a “discount rate” basis, but elsewhere on a true yield basis
<i>currency</i>	mostly domestic, although it is possible to draw foreign currency bills
<i>settlement</i>	available for discount immediately on being drawn
<i>registration</i>	none
<i>negotiable</i>	yes, although in practice banks often tend to hold the bills they have discounted until maturity

Repurchase agreement (repo)

A repo is an arrangement whereby one party sells a security to another party and simultaneously agrees to repurchase the same security at a subsequent date at an agreed price. This is equivalent to the first party borrowing from the second party against collateral, and the interest rate reflects this – that is, it is slightly lower than an unsecured loan. The security involved will often be of high credit quality, such as a government bond. A reverse repurchase agreement (reverse repo) is the same arrangement viewed from the other party’s perspective. The deal is generally a “repo” if it is initiated by the party borrowing money and lending the security and a “reverse repo” if it is initiated by the party borrowing the security and lending the money.

<i>term</i>	usually very short-term, although in principle can be for any term
<i>interest</i>	usually implied in the difference between the purchase and repurchase prices
<i>quotation</i>	as a yield
<i>currency</i>	any currency
<i>settlement</i>	generally cash against delivery of the security (DVP)
<i>registration</i>	n/a
<i>negotiable</i>	no

Repos provide a link between the money markets and the bond markets and we have considered an example in Chapter 5.

MONEY MARKET CALCULATIONS

For any instrument, the price an investor is prepared to pay is essentially the present value, or NPV, of the future cashflow(s) which he/she will receive because of owning it. This present value depends on the interest rate (the “yield”), the time to the cashflow(s) and the size of the cashflow(s).

Key Point

Price = present value

For an instrument such as a CD which has a coupon rate, the price in the secondary market will therefore depend not only on the current yield but also on the coupon rate, because the coupon rate affects the size of the cash-flow received at maturity.

Consider first a CD paying only one coupon (or in its last coupon period). The maturity proceeds of the CD are given by:

$$F \times \left(1 + \text{coupon rate} \times \frac{\text{days}}{\text{year}} \right)$$

where: F = face value of the CD
 days = number of days in the coupon period
 year = either 360 (e.g. in the USA) or 365 (e.g. in the UK).

The price P of this CD now is the investment needed at the current yield i to achieve this amount on maturity – in other words, the present value now of the maturity proceeds:

$$P = \frac{F \times \left(1 + \text{coupon rate} \times \frac{\text{days}}{\text{year}} \right)}{\left(1 + i \times \frac{d_{\text{pm}}}{\text{year}} \right)}$$

where: d_{pm} = number of days from purchase to maturity.

The price would normally be quoted based on a face value amount of 100.

We saw earlier that the simple return on *any* investment can be calculated as:

$$\left(\frac{\text{total proceeds at maturity}}{\text{initial investment}} - 1 \right) \times \frac{\text{year}}{\text{days held}}$$

Following this, the yield E earned on a CD purchased after issue and sold before maturity will be given by:

$$E = \left(\frac{\text{price achieved on sale}}{\text{price achieved on purchase}} - 1 \right) \times \left(\frac{\text{year}}{\text{days held}} \right)$$

From the previous formula, this is:

$$E = \left[\frac{\left(1 + i_p \times \frac{d_{pm}}{\text{year}} \right)}{\left(1 + i_s \times \frac{d_{sm}}{\text{year}} \right)} - 1 \right] \times \left(\frac{\text{year}}{\text{days held}} \right)$$

where: i_p = yield on purchase
 i_s = yield on sale
 d_{pm} = number of days from purchase to maturity
 d_{sm} = number of days from sale to maturity.

Example 2.3

A CD is issued for \$1 million on 17 March for 90 days (maturity 15 June) with a 6.0% coupon. On 10 April the yield is 5.5%. What are the total maturity proceeds? What is the secondary market price on 10 April?

$$\text{Maturity proceeds} = \$1 \text{ million} \times \left(1 + 0.06 \times \frac{90}{360} \right) = \$1,015,000.00$$

$$\text{Price} = \frac{\$1,015,000.00}{\left(1 + 0.055 \times \frac{66}{360} \right)} = \$1,004,867.59$$

On 10 May, the yield has fallen to 5.0%. What is the rate of return earned on holding the CD for the 30 days from 10 April to 10 May?

$$\text{Return} = \left[\frac{1 + 0.055 \times \frac{66}{360}}{1 + 0.050 \times \frac{36}{360}} - 1 \right] \times \frac{360}{30} = 6.07\%$$

.06 ENTER 90 × 360 ÷ 1 +	
1,000,000 ×	(Maturity proceeds)
.055 ENTER 66 × 360 ÷ 1 + ÷	(Secondary market cost)
.055 ENTER 66 × 360 ÷ 1 +	
.05 ENTER 36 × 360 ÷ 1 +	
÷	
1 −	
360 × 30 ÷	(Return over 30 days)

Calculation
summary

For a CD

Maturity proceeds = face value × $\left(1 + \text{coupon rate} \times \frac{\text{days from issue to maturity}}{\text{year}}\right)$

Secondary market price = $\frac{\text{maturity proceeds}}{\left(1 + \text{market yield} \times \frac{\text{days left to maturity}}{\text{year}}\right)}$

Return on holding a CD = $\left[\frac{\left(1 + \text{purchase yield} \times \frac{\text{days from purchase to maturity}}{\text{year}}\right)}{\left(1 + \text{sale yield} \times \frac{\text{days from sale to maturity}}{\text{year}}\right)} - 1\right] \times \frac{\text{year}}{\text{days held}}$

DISCOUNT INSTRUMENTS

Some instruments are known as “discount” instruments. This means that only the face value of the instrument is paid on maturity, without a coupon, in return for a smaller amount paid originally (instead of the face value paid originally in return for a larger amount on maturity). Treasury bills, for example, are discount instruments.

Consider, for example, a Belgian treasury bill for BEF 10 million issued for 91 days. On maturity, the investor receives only the face value of BEF 10 million. If the yield on the bill is 10 percent, the price the investor will be willing to pay now for the bill is its present value calculated at 10 percent:

Price = $\frac{\text{BEF 10 million}}{\left(1 + 0.10 \times \frac{91}{365}\right)}$ = BEF 9,756,749.53

Example 2.4

A French T-bill with face value FRF 10 million matures in 74 days. It is quoted at 8.4%. What is the price of the bill?

$\frac{\text{FRF 10 million}}{\left(1 + 0.084 \times \frac{74}{360}\right)}$ = FRF 9,830,264.11

.084 ENTER 74 × 360 ÷ 1 +
10,000,000 □ x ≥ y ÷

Key Point

Secondary market price = present value
(again!)

For a discount instrument

Maturity proceeds = face value

$$\text{Secondary market price} = \frac{\text{face value}}{\left(1 + \text{market yield} \times \frac{\text{days left to maturity}}{\text{year}}\right)}$$

Discount / true yield

In the USA and UK, a further complication arises in the way the interest rate is quoted on discount instruments – as a “discount rate” instead of a yield. If you invest 98.436 in a sterling time deposit or CD at 10 percent for 58 days, you expect to receive the 98.436 back at the end of 58 days, together with interest calculated as:

$$98.436 \times 0.10 \times \frac{58}{365} = 1.564$$

In this case, the total proceeds at maturity – principal plus interest – are $98.436 + 1.564 = 100.00$.

This means that you invested 98.436 to receive $(98.436 + 1.564) = 100$ at the end of 58 days. If the same investment were made in a discount instrument, the face value of the instrument would be 100, and the amount of discount would be 1.564. In this case, the *discount rate* is the amount of discount expressed as an annualized percentage of the face value, rather than as a percentage of the original amount paid. The discount rate is therefore:

$$(1.564 \div 100) \times \frac{365}{58} = 9.84\%$$

The discount rate is always less than the corresponding yield. If the discount rate on an instrument is D , then the amount of discount is:

$$F \times D \times \frac{\text{days}}{\text{year}}$$

where F is the face value of the instrument.

The price P to be paid is the face value less the discount:

$$P = F \times \left(1 - D \times \frac{\text{days}}{\text{year}}\right)$$

If we expressed the price in terms of the equivalent yield rather than the discount rate, we would still have the same formula as earlier:

$$P = \frac{F}{\left(1 + i \times \frac{\text{days}}{\text{year}}\right)}$$

Combining these two relationships, we get:

$$D = \frac{i}{1 + i \times \frac{\text{days}}{\text{year}}}$$

where i is the equivalent yield (often referred to as the “true yield”). This can perhaps be understood intuitively, by considering that because the discount is received at the beginning of the period whereas the equivalent yield is received at the end, the discount rate should be the “present value of the yield”. Reversing this relationship:

$$i = \frac{D}{1 - D \times \frac{\text{days}}{\text{year}}}$$

Instruments quoted on a discount rate in the USA and UK include domestic currency treasury bills and trade bills, while a yield basis is used for loans, deposits and certificates of deposit (CDs). USA CP is also quoted on a discount rate basis, while ECP and sterling CP are quoted on a yield basis. Note however that while the sterling T-bills issued by the UK government are quoted on a discount rate, the ECU T-bills it issues are quoted on a true yield, as other international ECU instruments are.

Example 2.5

A USA treasury bill of \$1 million is issued for 91 days at a discount rate of 6.5%. What is the amount of the discount and the amount paid?

Amount of discount = \$1 million \times 0.065 \times $\frac{91}{360}$ = \$16,430.56

Price paid = face value – discount = \$983,569.44

1,000,000 ENTER .065 \times 91 \times 360 \div	(Amount of discount)
1,000,000 \square $x \geq y$ –	(Amount paid)

Example 2.6

A UK treasury bill with remaining maturity of 70 days is quoted at a discount rate of 7.1%. What is the equivalent yield?

$\frac{7.1\%}{1 - 0.071 \times \frac{70}{365}} = 7.198\%$

Answer: 7.198%

.071 ENTER 70 \times 365 \div 1 \square $x \geq y$ –
7.1 \square $x \geq y$ \div

**Calculation
summary**

$$\text{Rate of true yield} = \frac{\text{discount rate}}{\left(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}}\right)}$$

$$\text{Discount rate} = \frac{\text{rate of true yield}}{\left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)}$$

$$\text{Amount of discount} = \text{face value} \times \text{discount rate} \times \frac{\text{days}}{\text{year}}$$

$$\text{Amount paid} = \text{face value} \times \left(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}}\right)$$

Instruments quoted on a discount rate

USA: T-bills
BA
CP

UK: T-bills (£)
BA

Key Point**Bond-equivalent yields**

For trading purposes, a government treasury bond which has less than a year left to maturity may be just as acceptable as a treasury bill with the same maturity left. As the method used for quoting yields generally differs between the two instruments, the rate quoted for treasury bills in the USA is often converted to a “bond-equivalent yield” for comparison. This is considered in Chapter 5.

CDS PAYING MORE THAN ONE COUPON

Most CDs are short-term instruments paying interest on maturity only. Some CDs however are issued with a maturity of several years. In this case, interest is paid periodically – generally every six months or every year. The price for a CD paying more than one coupon will therefore depend on all the intervening coupons before maturity, valued at the current yield. Suppose that a CD has three more coupons yet to be paid, one of which will be paid on maturity together with the face value F of the CD. The amount of this last coupon will be:

$$F \times R \times \frac{d_{23}}{\text{year}}$$

where: R = the coupon rate on the CD

d_{23} = the number of days between the second and third (last) coupon

year = the number of days in the conventional year.

The total amount paid on maturity will therefore be:

$$F \times \left(1 + R \times \frac{d_{23}}{\text{year}} \right)$$

The value of this amount discounted to the date of the *second* coupon payment, at the current yield i , is:

$$\frac{F \times \left(1 + R \times \frac{d_{23}}{\text{year}} \right)}{\left(1 + i \times \frac{d_{23}}{\text{year}} \right)}$$

To this can be added the actual cashflow received on the same date – that is, the second coupon, which is:

$$F \times R \times \frac{d_{12}}{\text{year}}$$

where d_{12} = the number of days between the first and second coupons.

The total of these two amounts is:

$$F \times \left[\frac{\left(1 + R \times \frac{d_{23}}{\text{year}} \right)}{\left(1 + i \times \frac{d_{23}}{\text{year}} \right)} + R \times \frac{d_{12}}{\text{year}} \right]$$

Again, this amount can be discounted to the date of the first coupon payment at the current yield i and added to the actual cashflow received then, to give:

$$F \times \left[\frac{\left(1 + R \times \frac{d_{23}}{\text{year}} \right)}{\left(1 + i \times \frac{d_{23}}{\text{year}} \right) \left(1 + i \times \frac{d_{12}}{\text{year}} \right)} + \frac{R \times \frac{d_{12}}{\text{year}}}{\left(1 + i \times \frac{d_{12}}{\text{year}} \right)} + \left(R \times \frac{d_{01}}{\text{year}} \right) \right]$$

where d_{01} = the number of days up to the first coupon since the previous coupon was paid (or since issue if there has been no previous coupon)

Finally, this entire amount can be discounted to the purchase date, again at the current yield of i , by dividing by:

$$\left(1 + i \times \frac{d_{p1}}{\text{year}} \right)$$

where d_{p1} = the number of days between purchase and the first coupon date.

The result will be the present value of all the cashflows, which should be the price P to be paid. This can be written as:

$$\begin{aligned}
 P = & \frac{F}{\left(1 + i \times \frac{d_{p1}}{\text{year}}\right) \left(1 + i \times \frac{d_{12}}{\text{year}}\right) \left(1 + i \times \frac{d_{23}}{\text{year}}\right)} \\
 & + \frac{F \times R \times \frac{d_{23}}{\text{year}}}{\left(1 + i \times \frac{d_{p1}}{\text{year}}\right) \left(1 + i \times \frac{d_{12}}{\text{year}}\right) \left(1 + i \times \frac{d_{23}}{\text{year}}\right)} \\
 & + \frac{F \times R \times \frac{d_{12}}{\text{year}}}{\left(1 + i \times \frac{d_{p1}}{\text{year}}\right) \left(1 + i \times \frac{d_{12}}{\text{year}}\right)} \\
 & + \frac{F \times R \times \frac{d_{01}}{\text{year}}}{\left(1 + i \times \frac{d_{p1}}{\text{year}}\right)}
 \end{aligned}$$

In general, for a CD with N coupon payments yet to be made:

$$P = F \times \left[\frac{1}{A_N} + \left(\frac{R}{\text{year}} \times \sum_{k=1}^N \left[\frac{d_{k-1;k}}{A_k} \right] \right) \right]$$

**Calculation
summary**

where: $A_k = \left(1 + i \times \frac{d_{p1}}{\text{year}}\right) \left(1 + i \times \frac{d_{12}}{\text{year}}\right) \left(1 + i \times \frac{d_{23}}{\text{year}}\right) \dots \left(1 + i \times \frac{d_{k-1;k}}{\text{year}}\right)$

F = face value of the CD

R = coupon rate of the CD

year = number of days in the conventional year

$d_{k-1;k}$ = number of days between (k-1)th coupon and kth coupon

d_{p1} = number of days between purchase and first coupon.

Example 2.7

What is the amount paid for the following CD?

Face value:	\$1 million
Coupon:	8.0% semi-annual
Maturity date:	13 September 1999
Settlement date:	15 January 1998
Yield:	7%

The last coupon date was 15 September 1997 (13 September was a Saturday). Future coupons will be paid on 13 March 1998, 14 September 1998 (13 September is a Sunday), 15 March 1999 (13 March is a Saturday) and 13 September 1999.

With the previous notation:

$d_{01} = 179, d_{p1} = 57, d_{12} = 185, d_{23} = 182, d_{34} = 182$

Price =
$$\frac{\$1 \text{ million} \times \left(1 + 0.08 \times \frac{182}{360}\right)}{\left(1 + 0.07 \times \frac{57}{360}\right)\left(1 + 0.07 \times \frac{185}{360}\right)\left(1 + 0.07 \times \frac{182}{360}\right)\left(1 + 0.07 \times \frac{182}{360}\right)}$$

+
$$\frac{\$1 \text{ million} \times 0.08 \times \frac{182}{360}}{\left(1 + 0.07 \times \frac{57}{360}\right)\left(1 + 0.07 \times \frac{185}{360}\right)\left(1 + 0.07 \times \frac{182}{360}\right)}$$

+
$$\frac{\$1 \text{ million} \times 0.08 \times \frac{185}{360}}{\left(1 + 0.07 \times \frac{57}{360}\right)\left(1 + 0.07 \times \frac{185}{360}\right)}$$

+
$$\frac{\$1 \text{ million} \times 0.08 \times \frac{179}{360}}{\left(1 + 0.07 \times \frac{57}{360}\right)}$$

= \$1,042,449.75

TIME CALC	
13.091999 DATE2 15.031999 DATE1 DAYS	(d ₃₄)
360 ÷ .08 × 1 +	
RCL DAYS 360 ÷ .07 × 1 + ÷	
14.091998 DATE2 DAYS +/-	(d ₂₃)
<input type="checkbox"/> x ≥ y ↓ 360 ÷ .08 × +	
RCL DAYS +/- 360 ÷ .07 × 1 + ÷	
13.031998 DATE1 DAYS	(d ₁₂)
<input type="checkbox"/> x ≥ y ↓ 360 ÷ .08 × +	
RCL DAYS 360 ÷ .07 × 1 + ÷	
15.091997 DATE2 DAYS +/-	(d ₀₁)
<input type="checkbox"/> x ≥ y ↓ 360 ÷ .08 × +	
15.011998 DATE2 DAYS +/-	(d _{p1})
<input type="checkbox"/> x ≥ y ↓ 360 ÷ .07 × 1 + ÷	
1,000,000 ×	

EXERCISES

21. You invest in a 181-day sterling CD with a face value of £1,000,000 and a coupon of 11%. What are the total maturity proceeds?



22. You buy the CD in the previous question 47 days after issue, for a yield of 10%. What amount do you pay for the CD?



You then sell the CD again after holding it for only 63 days (between purchase and sale), at a yield to the new purchaser of 9.5%. What yield have you earned on your whole investment on a simple basis? What is your effective yield?

23. At what different yield to the purchaser would you need to have sold in the previous question, in order to achieve an overall yield on the investment to you of 10% (on a simple basis)?

24. You place a deposit for 91 days at 11.5% on an ACT/360 basis. What would the rate have been if it had been quoted on an ACT/365 basis? What is the effective yield on an ACT/365 basis and on an ACT/360 basis?



25. You purchase some sterling Eurocommercial paper as follows:

Purchase value date:	2 July 1996
Maturity value date:	2 September 1996
Yield:	8.2%
Amount:	£2,000,000.00

What do you pay for the paper?

26. An investor seeks a yield of 9.5% on a sterling 1 million 60-day banker's acceptance. What is the discount rate and the amount of this discount?

27. If the discount rate were in fact 9.5%, what would the yield and the amount paid for the banker's acceptance be?

28. If the amount paid in the previous question is in fact £975,000.00, what is the discount rate?

29. The rate quoted for a 91-day US treasury bill is 6.5%.

- What is the amount paid for US\$1,000,000.00 of this T-bill?
- What is the equivalent true yield on a 365-day basis?



30. You buy a US treasury bill 176 days before it matures at a discount rate of 7% and sell it again at a discount rate of 6.7% after holding it for 64 days. What yield have you achieved on a 365-day year basis?

31. What would be the yield in the previous question if you sold it after only 4 days at a discount rate of 7.5%?

- 32.** The market rate quoted for a 91-day T-bill is 5% in the USA, UK, Belgium and France. Each T-bill has a face value of 1 million of the local currency. What is the amount paid for the bill in each country?
- 33.** Place the following instruments in descending order of yield, working from the rates quoted:
- | | |
|------------------------------|-------------------|
| 30-day UK T-bill (£) | $8\frac{1}{4}\%$ |
| 30-day UK CP (£) | $8\frac{3}{16}\%$ |
| 30-day ECP (£) | $8\frac{1}{8}\%$ |
| 30-day US T-bill | $8\frac{5}{16}\%$ |
| 30-day interbank deposit (£) | $8\frac{1}{4}\%$ |
| 30-day US CP | $8\frac{1}{2}\%$ |
| 30-day US\$ CD | $8\frac{5}{8}\%$ |
| 30-day French T-bill | $8\frac{1}{2}\%$ |
- 34.** A US \$1 million CD with semi-annual coupons of 7.5% per annum is issued on 26 March 1996 with a maturity of 5 years. You purchase the CD on 20 May 1999 at a yield of 8.0%. What is the amount paid?

■ ■ ■

“An important point is to consider which comes first. Are forward-forward rates (and hence futures prices and FRA rates) the mathematical result of the yield curve? Or are the market’s expectations of future rates the starting point?”

Forward-Forwards and Forward Rate Agreements (FRAs)

Forward-forwards, FRAs and futures

Applications of FRAs

Exercises

FORWARD-FORWARDS, FRAs AND FUTURES

Overview

Forward-forwards, forward rate agreements (FRAs) and futures are very similar and closely linked instruments, all relating to an interest rate applied to some period which starts in the future. We shall first define them here and then examine each more closely. Futures in particular will be considered in the next chapter.

Terminology

A **forward-forward** is a cash borrowing or deposit which starts on one forward date and ends on another forward date. The term, amount and interest rate are all fixed in advance. Someone who expects to borrow or deposit cash in the future can use this to remove any uncertainty relating to what interest rates will be when the time arrives.

An **FRA** is an off-balance sheet instrument which can achieve the same economic effect as a forward-forward. Someone who expects to borrow cash in the future can buy an FRA to fix in advance the interest rate on the borrowing. When the time to borrow arrives, he borrows the cash in the usual way. Under the FRA, which remains quite separate, he receives or pays the difference between the cash borrowing rate and the FRA rate, so that he achieves the same net effect as with a forward-forward borrowing.

A **futures** contract is similar to an FRA – an off-balance sheet contract for the difference between the cash interest rate and the agreed futures rate. Futures are however traded only on an exchange, and differ from FRAs in a variety of technical ways.

Pricing a forward-forward

Suppose that the 3-month sterling interest rate is 13.0 percent and the 6-month rate is 13.1 percent. If I borrow £100 for 6 months and simultaneously deposit it for 3 months, I have created a net borrowing which begins in 3 months and ends in 6 months. The position over the first 3 months is a net zero one. If I undertake these two transactions at the same time, I have created a forward-forward borrowing – that is, a borrowing which starts on one forward date and ends on another.

Example 3.1

If I deposit £1 for 91 days at 13%, then at the end of the 91 days, I receive:

$$£1 + 0.13 \times \frac{91}{365} = £1.03241$$

If I borrow £1 for 183 days at 13.1%, then at the end of the 183 days, I must repay:

$$£1 + 0.131 \times \frac{183}{365} = £1.06568$$

My cashflows are: an inflow of £1.03241 after 91 days and an outflow of £1.06568 after 183 days. What is the cost of this forward-forward borrowing? The calculation is similar to working out a yield earlier in the book:

$$\begin{aligned} \text{cost} &= \left(\frac{\text{cash outflow at the end}}{\text{cash inflow at the start}} - 1 \right) \times \frac{\text{year}}{\text{days}} = \left(\frac{1.06568}{1.03241} - 1 \right) \times \frac{365}{92} \\ &= 12.79\% \end{aligned}$$

.131 ENTER 183 × 365 ÷ 1 +	(value after 183 days)
.13 ENTER 91 × 365 ÷ 1 +	(value after 91 days)
÷	
1 -	
365 × 92 ÷	(forward-forward interest rate)

In general:

$$\text{Forward-forward rate} = \left[\frac{\left(1 + i_L \times \frac{d_L}{\text{year}} \right)}{\left(1 + i_S \times \frac{d_S}{\text{year}} \right)} - 1 \right] \times \left(\frac{\text{year}}{d_L - d_S} \right)$$

**Calculation
summary**

where: i_L = interest rate for longer period
 i_S = interest rate for shorter period
 d_L = number of days in longer period
 d_S = number of days in shorter period
year = number of days in conventional year

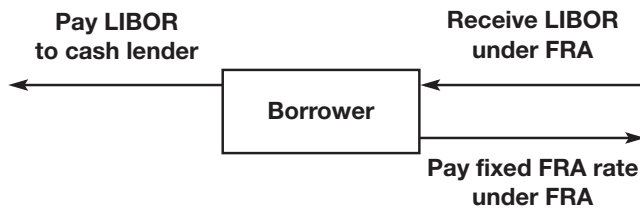
Note that this construction of a theoretical forward-forward rate only applies for periods up to one year. A money market deposit for longer than one year typically pays interim interest after one year (or each six months). This extra cashflow must be taken into account in the forward-forward structure. This is explained in Chapter 6.

Forward rate agreements (FRAs)

An FRA is an off-balance sheet agreement to make a settlement in the future with the same economic effect as a forward-forward. It is an agreement to pay or receive, on an agreed future date, the difference between an agreed interest rate and the interest rate actually prevailing on that future date, calculated on an agreed notional principal amount. It is settled against the actual interest rate prevailing at the beginning of the period to which it relates, rather than paid as a gross amount.

Example 3.2

A borrower intends to borrow cash at LIBOR from 91 days forward to 183 days forward, and he fixes the cost with an FRA. His costs will be as follows:



His flows will therefore be:

– LIBOR
+ LIBOR
– FRA rate
net cost: – FRA rate

If the cash is actually borrowed at a different rate – say $\text{LIBOR} + \frac{1}{4}\%$ – then the net cost will be $(\text{FRA rate} + \frac{1}{4}\%)$, but the all-in cost is still fixed.

Pricing

In calculating a theoretical FRA price, we can apply exactly the same ideas as in the forward-forward calculation above. As we are not actually borrowing cash with the FRA however, we might calculate using middle rates for both the 6-month and the 3-month period – rather than separate bid and offered rates. Conventionally, however, FRAs are always settled against LIBOR rather than LIMEAN. As a calculation using middle rates produces a rate which is comparable to LIMEAN, we would therefore need to add the difference between LIMEAN and LIBOR – generally around $\frac{1}{16}$ percent – to this theoretical “middle price” forward-forward. An alternative approach is to base the initial theoretical price calculation on LIBOR, rather than LIMEAN, for both periods. The result is different, but generally only very slightly.

Quotation

Having established a theoretical FRA price, a dealer would then build a spread round this price.

In Example 3.1, the theoretical FRA rate could be calculated as: $12.785\% + 0.0625\% = 12.85\%$. Putting a spread of, say, six basis points around this would give a price of $12.82\% / 12.8\%$. A customer wishing to hedge a future borrowing (“buying” the FRA) would therefore deal at 12.88 percent. A customer wishing to hedge a future deposit (“selling” the FRA) would deal at 12.82 percent.

An FRA is referred to by the beginning and end dates of the period covered. Thus a “5 v 8” FRA is one to cover a 3-month period, beginning in 5 months and ending in 8 months. Our example above would be a “3 v 6” FRA.

Settlement

Suppose that the actual 3-month LIBOR, in 3 months' time, is 11.5 percent. Settlement then takes place between these two rates. On a principal of £100, the effective interest settlement in our example would be:

$$£100 \times (0.1288 - 0.1150) \times \frac{92}{365} = £0.3478$$

As the settlement rate of 11.50 percent is lower than the agreed FRA rate, this amount is due from the FRA buyer to the other party. Conventionally however, this settlement takes place at the *beginning* of the 3-month borrowing period. It is therefore discounted to a present value at the current 3-month rate to calculate the actual settlement amount paid:

$$\frac{£100 \times (0.1285 - 0.1150) \times \frac{92}{365}}{1 + 0.1150 \times \frac{92}{365}} = £0.3307$$

In general:

$$\text{The FRA settlement amount} = \text{principal} \times \frac{(f - L) \times \frac{\text{days}}{\text{year}}}{\left(1 + L \times \frac{\text{days}}{\text{year}}\right)}$$

**Calculation
summary**

where: f = FRA rate
 L = interest rate (LIBOR) prevailing at the beginning of the period to which the FRA relates
 days = number of days in the FRA period
 year = number of days in the conventional year.

If the period of the FRA is longer than one year, the corresponding LIBOR rate used for settlement relates to a period where interest is conventionally paid at the end of each year as well as at maturity. A 6 v 24 FRA, for example, covers a period from 6 months to 24 months and will be settled against an 18-month LIBOR rate at the beginning of the FRA period.

An 18-month deposit would, however, typically pay interest at the end of one year and after 18 months. The agreed FRA rate and the settlement LIBOR would therefore be based on this. As FRA settlements are paid at the beginning of the period on a discounted basis, each of these payments therefore needs to be discounted separately. Strictly, the net settlement payment calculated for the end of 18 months should be discounted at an appropriate compounded 18-month rate and the net settlement amount calculated for the end of the first year should be discounted at 1-year LIBOR. In practice, the FRA settlement LIBOR is generally used for both. In this case, the final discounted settlement amount would be:

**Calculation
summary**

$$\text{principal} \times \frac{(f - L) \times \frac{d_1}{\text{year}}}{\left(1 + L \times \frac{d_1}{\text{year}}\right)} + \frac{(f - L) \times \frac{d_2}{\text{year}}}{\left(1 + L \times \frac{d_1}{\text{year}}\right) \times \left(1 + L \times \frac{d_2}{\text{year}}\right)}$$

where: d_1 = number of days in the first year of the FRA period
 d_2 = number of days from d_1 until the end of the FRA period.

This same principle can be applied to the settlement of FRAs covering any period.

The short-term yield curve

A “yield curve” shows how interest rates vary with term to maturity. For example, a Reuters screen might show the following rates:

1 month	9.5%
2 months	9.7%
3 months	10.0%
6 months	10.0%
12 months	10.2%
2 years	10.5%

In a free market, these rates show where the market on average believes rates “should” be, as supply and demand would otherwise tend to move them up or down. Clearly the rates at some maturity level or levels are influenced by central bank policy. If the market believes that the central bank is about to change official 3-month rates, for example, this expectation will already have been factored into the market 3-month rate.

If the 3-month maturity is indeed the rate manipulated by the central bank for this particular currency, a more logical curve to look at might be one that shows what the market expects 3-month rates to be at certain times in the future. For example, what is the 3-month rate now, what will it be after 1 month, after 2 months, after 3 months, etc? Given enough such rates, it is possible to work “backwards” to construct the yield curve shown above.

Suppose, for example, that the 3-month rate now is 10.0 percent and the market expects that there will be a 0.25 percent cut in rates during the next 3 months – so that at the end of 3 months, the 3-month rate will be 9.75 percent. Given these data, what should the 6-month rate be now?

The answer must be the rate achieved by taking the 3-month rate now, compounded with the expected 3-month rate in 3 months’ time; otherwise there would be an expected profit in going long for 3 months and short for 6 months or vice versa, and the market would tend to move the rates. In this way, the 6-month rate now can be calculated as:

$$\left[\left(1 + 0.10 \times \frac{91}{360} \right) \times \left(1 + 0.0975 \times \frac{92}{360} \right) - 1 \right] \times \frac{360}{183} = 10.00\%$$

This rate is in fact the 6-month rate shown above. If we now work in the other direction, we would find that the forward-forward rate from 3 months to 6 months (“3 v 6”) would be 9.75 percent as expected:

$$\left[\frac{\left(1 + 0.10 \times \frac{183}{360}\right)}{\left(1 + 0.10 \times \frac{91}{360}\right)} - 1 \right] \times \frac{360}{92} = 9.75\%$$

This shows that a “flat” short-term yield curve – in our example, the 3-month and 6-month rates are the same at 10.0 percent – does not imply that the market expects interest rates to remain stable. Rather, it expects them to fall.

An important point here is to consider the question of which comes first. Are forward-forward rates (and hence futures prices and FRA rates) the mathematical result of the yield curve? Or are the market’s expectations of future rates (i.e. forward-forwards, futures and FRAs) the starting point, and from these it is possible to create the yield curve? The question is a circular one to some extent, but market traders increasingly look at constructing a yield curve from expected future rates for various periods and maturities.

Constructing a strip

In the previous section, when we compounded the 3-month rate now with the expected 3-month rate in 3 months’ time, we were effectively creating a “strip” – that is, a series of consecutive legs which, compounded together, build up to a longer overall period.

Example 3.3

Suppose that it is now January and we have the following rates available. At what cost can we construct a fixed-rate borrowing for 9 months? All rates are ACT/360.

3-month LIBOR:	8.5% (92 days)
3 v 6 FRA:	8.6% (91 days)
6 v 9 FRA:	8.7% (91 days)

We construct the strip as follows:

- borrow cash now for 3 months;
- buy a 3 v 6 FRA now based on the total repayment amount in April (principal plus interest);
- refinance this total amount in April at the 3-month LIBOR in April;
- buy a 6 v 9 FRA now based on the total repayment amount in July (principal plus interest calculated at the 3 v 6 FRA rate now);
- refinance this amount in July at the 3-month LIBOR in July.

Suppose that in April when the first FRA settles, 3-month LIBOR is 9.0% and that in July when the second FRA settles, 3-month LIBOR is 9.5%. Although market practice is for the FRA settlements to be made at the beginning of the relevant period discounted at LIBOR, the economic effect is the same as if the settlement were at the end of the period but not discounted – assuming that the discounted settlement amount could simply be invested or borrowed at LIBOR for the period. For clarity, we will therefore assume that the FRA settlements are at the ends of the periods and not discounted.

This gives us the following cashflows based on a borrowing of 1:

January: + 1

April: $-\left(1 + 0.085 \times \frac{92}{360}\right)$ (repayment)

+ $\left(1 + 0.085 \times \frac{92}{360}\right)$ (refinancing)

July: $-\left(1 + 0.085 \times \frac{92}{360}\right) \times \left(1 + 0.09 \times \frac{91}{360}\right)$ (repayment)

+ $\left(1 + 0.085 \times \frac{92}{360}\right) \times \left(0.09 - 0.086\right) \times \frac{91}{360}$ (FRA settlement)

+ $\left(1 + 0.085 \times \frac{92}{360}\right) \times \left(1 + 0.086 \times \frac{91}{360}\right)$ (refinancing)

October: $-\left(1 + 0.085 \times \frac{92}{360}\right) \times \left(1 + 0.086 \times \frac{91}{360}\right) \times \left(1 + 0.095 \times \frac{91}{360}\right)$ (repayment)

+ $\left(1 + 0.085 \times \frac{92}{360}\right) \times \left(1 + 0.086 \times \frac{91}{360}\right) \times \left(0.095 - 0.087\right) \times \frac{91}{360}$ (FRA settlement)

Most of these flows offset each other, leaving the following net flows:

January: + 1

October: $-\left(1 + 0.085 \times \frac{92}{360}\right) \times \left(1 + 0.086 \times \frac{91}{360}\right) \times \left(1 + 0.087 \times \frac{91}{360}\right) = -1.066891$

The cost of funding for 9 months thus depends only on the original cash interest rate for 3 months and the FRA rates, compounded together. The cost per annum is:

$$(1.066891 - 1) \times \frac{360}{274} = 8.79\%$$

In general:

Calculation summary

$$\text{The interest rate for a longer period up to one year} = \left[\left(1 + i_1 \times \frac{d_1}{\text{year}}\right) \times \left(1 + i_2 \times \frac{d_2}{\text{year}}\right) \times \left(1 + i_3 \times \frac{d_3}{\text{year}}\right) \times \dots - 1 \right] \times \frac{\text{year}}{\text{total days}}$$

where: i_1, i_2, i_3, \dots are the cash interest rate and forward-forward rates for a series of consecutive periods lasting d_1, d_2, d_3, \dots days.

APPLICATIONS OF FRAs

As with any instrument, FRAs may be used for hedging, speculating or arbitrage, depending on whether they are taken to offset an existing underlying position or taken as new positions themselves.

Hedging

Example 3.4

A company has a 5-year borrowing with 3-monthly rollovers – that is, every three months, the interest rate is refixed at the prevailing 3-month LIBOR. The company expects interest rates to rise before the next rollover date. It therefore buys an FRA to

start on the next rollover date and finish 3 months later. If the next rollover date is 2 months away, this would be a “2 v 5” FRA. If the company is correct and interest rates do rise, the next rollover will cost more, but the company will make a profit on the FRA settlement to offset this. If rates fall however, the next rollover will be cheaper but the company will make an offsetting loss on the FRA settlement. The FRA settlement profit or loss will of course depend on how the 3-month rate stands after two months compared with the FRA rate now, not compared with the cash rate now. Either way, the net effect will be that the company’s borrowing cost will be locked in at the FRA rate (plus the normal margin which it pays on its credit facility):

Company pays	LIBOR + margin	to lending bank
Company pays	FRA rate	to FRA counterparty
Company receives	LIBOR	from FRA counterparty
Net cost	FRA rate + margin	

The FRA payments are in practice netted and also settled at the beginning of the borrowing period after discounting. The economic effect is still as shown.

Example 3.5

A company expects to make a 6-month deposit in two weeks’ time and fears that interest rates may fall. The company therefore sells a 2-week v $6\frac{1}{2}$ -month FRA. Exactly as above, but in reverse, the company will thereby lock in the deposit rate. Although the company may expect to receive LIBID on its actual deposit, the FRA will always be settled against LIBOR:

Company receives	LIBID	from deposit
Company receives	FRA rate	from FRA counterparty
Company pays	LIBOR	to FRA counterparty
Net return	FRA rate – (LIBOR – LIBID)	

As the FRA rate is theoretically calculated to be comparable to LIBOR, it is reasonable to expect the net return to be correspondingly lower than the FRA rate by the (LIBID – LIBOR) spread.

Speculation

The most basic trading strategy is to use an FRA to speculate on whether the cash interest rate when the FRA period begins is higher or lower than the FRA rate. If the trader expects interest rates to rise, he buys an FRA; if he expects rates to fall, he sells an FRA.

Example 3.6

A bank with no position expects interest rates to rise. The bank therefore buys an FRA. If rates rise above the FRA rate it will make a profit; otherwise it will make a loss.

Arbitrage

Arbitrage between FRAs and futures is considered later.

EXERCISES



35. Current market rates are as follows for SEK:

3 months (91 days)	9.87 / 10.00%
6 months (182 days)	10.12 / 10.25%
9 months (273 days)	10.00 / 10.12%

What is the theoretical FRA 3 v 9 for SEK now?

36. You borrow DEM 5 million at 7.00% for 6 months (183 days) and deposit DEM 5 million for 3 months (91 days) at 6.75%. You wish to hedge the mismatched position, based on the following FRA prices quoted to you:

3 v 6: 7.10% / 7.15%
6 v 9: 7.20% / 7.25%

- Do you buy or sell the FRA?
- At what price?
- For a complete hedge, what amount do you deal?

When it comes to fixing the FRA, the 3-month rate is 6.85% / 6.90%:

- What is the settlement amount? Who pays whom?
- What is the overall profit or loss of the book at the end of six months, assuming that your borrowings are always at LIBOR and your deposits at LIBID?



“In general, a futures contract in any market is a contract in which the commodity being bought and sold is considered as being delivered (even though this may not physically occur) at some future date rather than immediately.”

Interest Rate Futures

Overview

Exchange structure and margins

Futures compared with FRAs

Pricing and hedging FRAs with futures

Trading with interest rate futures

Exercises

OVERVIEW

In general, a futures contract in any market is a contract in which the commodity being bought and sold is considered as being delivered (even though this may not physically occur) at some future date rather than immediately – hence the name. The significant differences between a “futures contract” and a “forward” arise in two ways. First, a futures contract is traded on a particular exchange (although two or more exchanges might trade identically specified contracts). A forward however, which is also a deal for delivery on a future date, is dealt “over the counter” (OTC) – a deal made between any two parties, not on an exchange. Second, futures contracts are generally standardized, while forwards are not. The specifications of each futures contract are laid down precisely by the relevant exchange and vary from commodity to commodity and from exchange to exchange. Some contracts, for example, specifically do not allow for the commodity to be delivered; although their prices are calculated as if future delivery takes place, the contracts must be reversed before the notional delivery date, thereby capturing a profit or a loss. Interest rate futures, for example, cannot be delivered, whereas most bond futures can.

The theory underlying the pricing of a futures contract depends on the underlying “commodity” on which the contract is based. For a futures contract based on 3-month interest rates, for example, the pricing is based on forward-forward pricing theory, explained earlier. Similarly, currency futures pricing theory is the same as currency forward outright pricing theory and bond futures pricing theory is based on bond pricing.

The principle – and the different characteristics of interest rate and currency futures – are most easily understood by examples.

Example 4.1

A 3-month EuroDEM interest rate futures contract traded on LIFFE:

Exchange LIFFE (London International Financial Futures and Options Exchange), where a variety of futures and options contracts on interest rates, bonds, equities and commodities is traded.

Commodity The basis of the contract is a deposit of DEM 1 million (EuroDEM) lasting for 90 days based on an ACT/360 year.

Delivery It is not permitted for this contract to be delivered; if a trader buys such a contract, he cannot insist that, on the future delivery date, his counterparty makes arrangements for him to have a deposit for 90 days from then onwards at the interest rate agreed. Rather, the trader must reverse his futures contract before delivery, thereby taking a profit or loss.

Delivery date The contract must be based on a notional delivery date. In this case, the delivery date must be the first business day before the third Wednesday of the delivery month (March, June, September, December and the next two months following dealing).

Trading It is possible to trade the contract until 11:00 am on the last business day before the delivery day. Trading hours are from 07:30 to 16:10 each business day in London for “open outcry” (physical trading, face to face on the exchange), and from 16:25 to 17:59 for “APT” (automated pit trading – computerized trading outside the exchange).

Price The price is determined by the free market and is quoted as an index rather than an interest rate. The index is expressed as 100 minus the implied interest rate. Thus a price of 93.52 implies an interest rate of 6.48% ($100 - 93.52 = 6.48$).

Price movement Prices are quoted to two decimal places and can move by as little as 0.01. This minimum movement is called a “tick” and is equivalent to a profit or loss of DEM 25. This is calculated as:

$$\begin{aligned} & \text{Amount of contract} \times \text{price movement} \times \frac{\text{days}}{\text{year}} \\ &= \text{DEM 1 million} \times 0.01\% \times \frac{90}{360} = \text{DEM 25} \end{aligned}$$

Settlement price At the close of trading, LIFFE declares an Exchange Delivery Settlement Price (EDSP) which is the closing price at which any contracts still outstanding will be automatically reversed. The EDSP is 100 minus the British Bankers’ Association 3-month LIBOR.

Example 4.2

Yen/dollar futures contract traded on the IMM:

Exchange IMM (International Monetary Market; a division of CME, Chicago Mercantile Exchange, where futures and options on currencies, interest rates equities and commodities are traded).

Commodity The basis of the contract is JPY 12.5 million.

Delivery It is possible for the JPY 12.5 million to be delivered, if the contract is not reversed before maturity, against an equivalent value in USD.

Delivery date The third Wednesday in January, March, April, June, July, September, October and December, as well as the month in which the current spot date falls.

Trading It is possible to trade the contract until 09:16 two business days before the delivery date. Trading hours are from 07:20 to 14:00 each business day in Chicago for open outcry, and from 14:30 to 07:05 Monday to Thursday, 17:30 to 07:05 Sundays on GLOBEX (computerized trading outside the exchange – only the next four March / June / September / December months traded on GLOBEX).

Price The price is expressed as the dollar value of JPY 1.

Price movement Prices are quoted to six decimal places, with a minimum movement (one tick) of \$0.000001. A one tick movement is equivalent to a profit or loss of USD 12.50.

Settlement price At the close of trading, the EDSP is the closing spot JPY/USD rate as determined by the IMM.

The typical contract specification for short-term interest rate futures is for a “3-month” interest rate, although, for example, 1-month contracts also exist

in some currencies on some exchanges. The precise specification can vary from exchange to exchange but is in practice for one-quarter of a year. Thus the sterling 3-month interest rate futures contract traded on LIFFE, for example, technically calculates $91\frac{1}{4}$ days' settlement.

Suppose, for example, that the sterling futures price moves from 92.40 to 92.90 (an implied interest rate change of 0.5 percent). The settlement amount would be:

$$\text{Contract amount} \times 0.0050 \times \frac{1}{4}$$

As sterling interest rates are conventionally calculated on an ACT/365 basis, this is equivalent to:

$$\text{Contract amount} \times 0.0050 \times \frac{91\frac{1}{4}}{365}$$

Calculation summary

Short-term interest rate futures:

$$\text{Price} = 100 - (\text{implied forward-forward interest rate} \times 100)$$

$$\text{Profit / loss on long position in a 3-month contract} =$$

$$\text{notional contract size} \times \frac{(\text{sale price} - \text{purchase price})}{100} \times \frac{1}{4}$$

EXCHANGE STRUCTURE AND MARGINS

Market participants

The users of an exchange are its members and their customers. An exchange also has “locals” – private traders dealing for their own account only.

Dealing

There are two methods of dealing. The first, traditional, method is “open outcry”, whereby the buyer and seller deal face to face in public in the exchange’s “trading pit”. This should ensure that a customer’s order will always be transacted at the best possible rate. The second is screen-trading, designed to simulate the transparency of open outcry. In some cases (such as LIFFE’s “automated pit trading”), this is used for trading outside normal business hours. Some exchanges however use only screen-based trading.

Clearing

The futures exchange is responsible for administering the market, but all transactions are cleared through a clearing house, which is usually separate. On LIFFE, for example, this function is performed by the London Clearing House (LCH). Following confirmation of a transaction, the clearing house substitutes itself as a counterparty to each user and becomes the seller to every buyer and the buyer to every seller.

Margin requirements

In order to protect the clearing house, clearing members are required to place collateral with it for each deal transacted. This collateral is called “initial margin”.

Members are then debited or credited each day with “variation margin” which reflects the day’s loss or profit on contracts held. Customers, in turn, are required to pay initial margin and variation margin to the member through whom they have dealt. The initial margin is intended to protect the clearing house for the short period until a position can be revalued and variation margin called for if necessary. As variation margin is paid each day, the initial margin is relatively small in relation to the potential price movements over a longer period.

The calculation of variation margin is equivalent to “marking to market” – that is, revaluing a futures contract each day at the current price. The variation margin required is the tick value multiplied by the number of ticks price movement. For example, if the tick value is DEM 25 on each contract, and the price moves from 94.73 to 94.21 (a fall of 52 ticks), the loss on a long futures contract is DEM $(25 \times 52) = \text{DEM } 1,300$. Depending on the size of initial margin already placed with the exchange, and the exchange’s current rules, a variation margin may not be called for below a certain level.

Delivery

A futures position can be closed out by means of an exactly offsetting transaction. Depending on the specification of the particular futures contract, contracts which are not settled before maturity are required to be either “cash settled” – that is, reversed at maturity and the price difference paid – or (if delivery is permitted) delivered. The mechanics of the delivery process differ for each type of contract.

Limit up / down

Some markets impose limits on trading movements in an attempt to prevent wild price fluctuations and hence limit risk to some extent. For example, when the IMM opens in the morning, it is not possible for the opening price in the yen/dollar futures contract to differ from the previous day’s closing price by more than 200 ticks.

FUTURES COMPARED WITH FRAs

An FRA is an OTC equivalent to an interest rate futures contract. Exactly the same forward-forward pricing mechanism is therefore used to calculate a futures price – although the futures price is then expressed as an index (100 – rate). In practice, an FRA trader will often take his price from the futures market (which may not be precisely in line with the theoretical calculations),

rather than directly from the forward-forward calculation. This is because the FRA trader would use the futures market rather than a forward-forward to hedge his FRA book – because of both the balance sheet implications and the transaction costs. In practice therefore, the FRA rate for a period coinciding with a futures contract would be $(100 - \text{futures price})$.

An important practical difference between FRAs and futures is in the settlement mechanics. An FRA settlement is paid at the beginning of the notional borrowing period, and is discounted. The futures “settlement” – the profit or loss on the contract – is also all settled by the same date, via the margin payments made during the period from transaction until the futures delivery date. However, in most futures markets, this settlement is *not discounted*. A 90-day FRA is not therefore exactly matched by an offsetting futures contract even if the amounts and dates are the same.

It should also be noted that FRAs and futures are “in opposite directions.” A *buyer* of an FRA will profit if interest rates rise. A *buyer* of a futures contract will profit if interest rates fall. If a trader sells an FRA to a counterparty, he must therefore also *sell* a futures contract to cover his position.

OTC vs. exchange-traded

It is worthwhile summarizing the differences between OTC contracts such as an FRA, and futures contracts.

Key Point

Amount The amount of a futures contract is standardized. The amount of an OTC deal is entirely flexible.

Delivery date The delivery date of a futures contract is standardized. The delivery date of an OTC deal is entirely flexible.

Margin Dealing in futures requires the payment of collateral (called “initial margin”) as security. In addition, “variation margin” is paid or received each day to reflect the day’s loss or profit on futures contracts held. When trading OTC, professional traders usually deal on the basis of credit lines, with no security required.

Settlement Settlement on an instrument such as an FRA is discounted to a present value. Settlement on a futures contract, because it is paid through variation margin, is not discounted.

Credit risk The margin system ensures that the exchange clearing house is generally fully protected against the risk of default. As the counterparty to each futures contract is the clearing house, there is therefore usually virtually no credit risk in dealing futures. OTC counterparties are generally not of the same creditworthiness.

Delivery Some futures contracts are not deliverable and must be cash settled. It is usually possible to arrange an OTC deal to include delivery.

Liquidity and spread Standardization and transparency generally ensure a liquid market in futures contracts, together with narrower spreads between bid and offer than in OTC deals. For delivery dates far in the future on the other hand, there may be insufficient liquidity in the futures market, where an OTC price may be available.

Underlying commodity Futures contracts are not available in all underlying commodities. For example, there is no ringgit/peseta futures contract but a ringgit/peseta forward is easily available.

PRICING AND HEDGING FRAs WITH FUTURES

In Chapter 3 on forward-forwards and FRAs, we calculated FRA rates from cash market interest rates – for example, a 3 v 6 FRA from a 3-month interest rate and a 6-month interest rate. In practice, however, a trader may well generate an FRA price from futures prices and also hedge the resulting position by buying or selling futures.

Example 4.3

Suppose we have the following prices on 17 March for DEM and wish to sell a 3 v 6 FRA for DEM 10 million. How should the FRA be priced (ignoring any buy / sell spread) and hedged, based on these prices?

June futures price (delivery 18 June):	91.75 (implied interest rate: 8.25%)
Sept futures price (delivery 17 Sept):	91.50 (implied interest rate: 8.50%)
Dec futures price (delivery 17 Dec):	91.25 (implied interest rate: 8.75%)

The FRA will be for the period 19 June to 19 September (92 days) and will settle against LIBOR fixed on 17 June. The June futures contract EDSP will also be LIBOR on 17 June. The FRA rate should therefore be the implied June futures rate of 8.25%.

The settlement amount for the FRA will be:

$$\frac{\text{DEM 10 million} \times (0.0825 - \text{LIBOR}) \times \frac{92}{360}}{1 + \text{LIBOR} \times \frac{92}{360}}$$

The profit or loss on the futures contract (which is not discounted) is:

$$\text{number of contracts} \times \text{DEM 1 million} \times (0.0825 - \text{LIBOR}) \times \frac{90}{360}$$

In order for these to be equal, we need:

$$\text{number of contracts} = 10 \times \frac{\frac{92}{90}}{\left(1 + \text{LIBOR} \times \frac{92}{360}\right)}$$

We do not know what LIBOR will be. Taking 8.25% as our best guess however, we have:

$$\text{number of contracts} = 10 \times \frac{\frac{92}{90}}{\left(1 + 0.0825 \times \frac{92}{360}\right)} = 10.01$$

As futures contracts can only be traded in whole numbers, we hedge by selling 10 futures contracts.

In Chapter 3, we considered a strip of FRAs to create a rate for a longer period. The same theory applies just as well here.

Example 4.4

With the same prices as in Example 4.3, we wish to sell a 3 v 9 FRA and a 6 v 12 FRA. How should these be priced and hedged?

A 3 v 6 FRA can be priced at 8.25% and hedged exactly as in Example 4.3. A 6 v 9 FRA (91 days from 19 September to 19 December) can similarly be priced at the implied September futures rate of 8.50% and hedged by selling 10 September futures (although there is a slight discrepancy in the dates as the futures contract delivery is 17 September). The 3 v 9 FRA should be equivalent to a strip combining the 3 v 6 FRA and 6 v 9 FRA (because, if not, there would be an arbitrage opportunity). This gives the 3 v 9 FRA rate as:

$$\left[\left(1 + 0.0825 \times \frac{92}{360}\right) \times \left(1 + 0.085 \times \frac{91}{360}\right) - 1\right] \times \frac{360}{183} = 8.463\%$$

The hedge required is the combination of the hedges for each leg: sell 10 June futures and 10 September futures.

In the same way, we can build up the 6 v 12 FRA from a strip of the 6 v 9 FRA and 9 v 12 FRA (90 days, from 19 December to 19 March), and hedge it by selling 10 September futures and 10 December futures.

$$\left[\left(1 + 0.085 \times \frac{91}{360}\right) \times \left(1 + 0.875 \times \frac{90}{360}\right) - 1\right] \times \frac{360}{181} = 8.718\%$$

In Example 4.4, when we reach 17 June, we need to close out the June and September futures hedges against the 3 v 9 FRA. The June contract will be closed at the current 3-month LIBOR and the September contract at the current futures price which will approximate the current 3 v 6 FRA; combined in a strip, these two rates approximate to the current 6-month LIBOR against which the 3 v 9 FRA will settle at the same time. The hedge therefore works. The result of the hedge is not perfect however, for various reasons:

- In creating a strip of FRAs, we compounded, by increasing the notional amount for each leg to match the previous leg's maturing amount. With futures, we cannot do this – futures contracts are for standardized notional amounts only.
- Similarly, the futures profit / loss is based on a 90-day period rather than 91 or 92 days etc. as the FRA period.
- FRA settlements are discounted but futures settlements are not.
- The futures price when the September contract is closed out in June may not exactly match the theoretical forward-forward rate at that time.

- Even if the September futures price does exactly match the theoretical forward-forward rate, there is in fact a slight discrepancy in dates. On 17 June, the 3 v 6 FRA period is 19 September to 19 December (LIBOR is therefore fixed on 17 September) but the September futures delivery is 17 September (EDSP is therefore fixed on 16 September).

The result of these effects is considered later, in Example 4.9.

Pricing FRAs from futures in this way is not as theoretically straightforward as pricing FRAs from the cash market because in practice, the FRA period is unlikely to coincide exactly with a futures date. If we only have 3-month futures prices available, we only know what the market expects 3-month interest rates to be at certain times in the future. We do not know what 3-month rates are expected to be at any other time, or, for example, what 4-month or 5-month rates are expected to be at any time. We therefore need to interpolate between the prices we do have, to build up the rates we need.

Example 4.5

With the same prices as in Example 4.3, we wish to sell a 3 v 8 FRA (153 days from 19 June to 19 November) and a 6 v 11 FRA (153 days, from 19 September to 19 February). How do we price and hedge these?

For the 3 v 8 FRA, we are effectively asking what the market expects the 5-month rate to be in 3 months' time. The information available is what the market expects the 3-month rate to be in 3 months' time (8.25%, the 3 v 6 rate) and what it expects the 6-month rate to be at the same time (8.463%, the 3 v 9 rate from Example 4.4). An approach is therefore to interpolate, to give the 3 v 8 FRA as:

$$\begin{aligned}
 & 3 \text{ v } 6 + (3 \text{ v } 9 - 3 \text{ v } 6) \times \frac{(\text{days in } 3 \text{ v } 8 - \text{days in } 3 \text{ v } 6)}{(\text{days in } 3 \text{ v } 9 - \text{days in } 3 \text{ v } 6)} \\
 & = 8.25\% + (8.463\% - 8.25\%) \times \frac{153 - 92}{183 - 92} = 8.393\%
 \end{aligned}$$

The hedge can similarly be considered as the sale of the following futures contracts:

$$\begin{aligned}
 & 10 \text{ June} + (10 \text{ June} + 10 \text{ September} - 10 \text{ June}) \times \frac{153 - 92}{183 - 92} \\
 & = 10 \text{ June} + 6.7 \text{ September}
 \end{aligned}$$

We therefore sell 10 June futures and 7 September futures.

In the same way, we can interpolate between the 6 v 9 rate and the 6 v 12 rate for the 6 v 11 rate:

$$\begin{aligned}
 & 6 \text{ v } 9 + (6 \text{ v } 12 - 6 \text{ v } 9) \times \frac{(\text{days in } 6 \text{ v } 11 - \text{days in } 6 \text{ v } 9)}{(\text{days in } 6 \text{ v } 12 - \text{days in } 6 \text{ v } 9)} \\
 & = 8.50\% + (8.718\% - 8.50\%) \times \frac{153 - 91}{181 - 91} = 8.650\%
 \end{aligned}$$

This is hedged by selling 10 September futures and 7 December futures.

So far, we have considered FRAs where the start of the FRA period coincides with a futures contract. In practice, we need to be able to price an FRA which starts between two futures dates. Again, interpolation is necessary.

Example 4.6

With the same prices as before, we wish to sell and hedge a 5 v 10 FRA.

From the previous example, we have the following prices:

3 v 8 FRA (153 days from 19 June to 19 November): 8.393%, hedged by selling 10 June futures and 7 September futures

6 v 11 FRA (153 days from 19 September to 19 February): 8.650%, hedged by selling 10 September futures and 7 December futures

We are now asking what the market expects the 5-month rate to be in 5 months' time. The information available is what the market expects this rate to be in 3 months' time and in 6 months' time. An approach is therefore to interpolate again, to give the 5 v 10 FRA as:

$$\begin{aligned} & 3 \text{ v } 8 + (6 \text{ v } 11 - 3 \text{ v } 8) \times \frac{(\text{days to fixing } 5 \text{ v } 10 - \text{days to fixing } 3 \text{ v } 8)}{(\text{days to fixing } 6 \text{ v } 11 - \text{days to fixing } 3 \text{ v } 8)} \\ & = 8.393\% + (8.650\% - 8.393\%) \times \frac{153 - 92}{184 - 92} = 8.563\% \end{aligned}$$

The hedge for this would follow the same approach, as a sale of the following futures contracts:

$$\begin{aligned} & (10 \text{ June} + 6.7 \text{ Sept}) + (10 \text{ Sept} + 6.7 \text{ Dec} - 10 \text{ June} - 6.7 \text{ Sept}) \times \frac{153 - 92}{184 - 92} \\ & = 3.4 \text{ June} + 8.9 \text{ September} + 4.4 \text{ December} \end{aligned}$$

We would really like to sell 16.7 contracts. As before, we need to approximate by selling 17 contracts. We could therefore for example sell 3 June futures, 10 September futures and 4 December futures. (The extra September contract is an approximation for 0.4 June and 0.4 December.)

In these examples we have built up a 5 v 10 FRA by interpolating in two stages:

- (i) interpolate between known rates for different standard forward periods but starting from the same time:

$$\begin{aligned} & 3 \text{ v } 8 \text{ from } 3 \text{ v } 6 \text{ and } 3 \text{ v } 9 \\ & 6 \text{ v } 11 \text{ from } 6 \text{ v } 9 \text{ and } 6 \text{ v } 12 \end{aligned}$$

- (ii) interpolate between rates for the same non-standard period but starting from different times:

$$5 \text{ v } 10 \text{ from } 3 \text{ v } 8 \text{ and } 6 \text{ v } 11$$

An alternative would be to approach these operations in reverse:

- (i) interpolate between known rates for the same standard forward periods but starting from different times:

$$\begin{aligned} & 5 \text{ v } 8 \text{ from } 3 \text{ v } 6 \text{ and } 6 \text{ v } 9 \\ & 5 \text{ v } 11 \text{ from } 3 \text{ v } 9 \text{ and } 6 \text{ v } 12 \end{aligned}$$

- (ii) interpolate between rates for different standard periods but starting from the same non-standard time:

$$5 \text{ v } 10 \text{ from } 5 \text{ v } 8 \text{ and } 5 \text{ v } 11$$

The result of this approach would be slightly different but generally not significantly. Neither approach is perfect. However the 5 v 10 rate is calculated, it is an estimate for a particular forward period, from a particular time, neither of which can actually be known from the current futures prices.

Hedging the basis risk

In Example 4.6, the hedge put in place in March for selling the 5 v 10 FRA was to sell 3 June futures, 10 September futures and 4 December futures. When the June futures contract closes on 17 June, there are still two months before the FRA settles. The hedge needs to be maintained but if we replace the sale of 3 now non-existent June contracts by the nearest available – that is, the sale of 3 September contracts – there is a risk. Suppose that interest rates rise. We will make a loss on the 5 v 10 FRA we have sold and a corresponding gain on the futures hedge. If, however, the yield curve flattens somewhat at the same time (shorter-term rates rise more than longer-term rates), we will make less of a profit on the 3 new September contracts than we would have made on the 3 June contracts. The hedge will not therefore be as effective. This is a form of basis risk – that the movement in the instrument used as a hedge does not match the movement in the instrument being hedged.

An attempt to hedge against this basis risk is as follows. If there is a flattening or steepening of the yield curve while the June contract does still exist, any change in the September futures price is assumed to be approximately equal to the average of the change in the June futures price and the December futures price. For example, if the June price falls 10 ticks and the December price rises 10 ticks (yield curve flattening), the September price is assumed not to change. On this assumption, the following two positions would give the same profit or loss as each other:

sell 6 September futures
or
sell 3 June futures and sell 3 December futures.

It follows that the following two positions would also give the same profit or loss as each other:

sell 6 September futures and buy 3 December futures
or
sell 3 June futures.

In this way, if we must replace the sale of 3 June futures contracts (because they no longer exist) by the sale of 3 September contracts, we should then additionally sell a further 3 September contracts and buy 3 December contracts in an attempt to hedge the basis risk.

In Example 4.6 therefore, when the 3 June contracts expire, we replace them by a further sale of 6 September futures and a purchase of 3 December futures, to leave a net hedge then of short 16 September futures and 1 December futures.

TRADING WITH INTEREST RATE FUTURES

Basis

In practice, the actual futures price trading in the market is unlikely to be exactly the same as the “fair” theoretical value which can be calculated according to the underlying cash market prices. The difference between the actual price and the theoretical “fair” price is termed the “basis”.

Suppose the following prices for a 3-month interest rate futures contract:

Actual futures price	94.40
Fair futures price	94.31 (based on a 5.69% forward-forward rate)
Implied cash price	93.90 (based on cash 3-month LIBOR of 6.10%)

“Basis” or “simple basis” is the difference between the price based on the current cash rate and the actual price ($93.90 - 94.40 = -0.50$). This difference will tend towards zero on the last trading day for the futures contract.

“Theoretical basis” is the difference between the price based on the current cash rate and the fair price ($93.90 - 94.31 = -0.41$). This difference depends on the calculation of the fair price and will also tend towards zero on the last trading day for the futures contract.

“Value basis” is the difference between the fair and actual prices ($94.31 - 94.40 = -0.09$). If the value basis is temporarily large, arbitrageurs will trade in such a way as to reduce it.

Clearly:

$$\text{basis} = \text{theoretical basis} + \text{value basis}$$

“Basis risk” is the risk arising from the basis on a futures position. Suppose for example that on 1 April a futures trader sells a 1 v 4 FRA to a customer which will settle on 4 May and that he hedges this by selling futures contracts for the nearest futures contract – say for delivery on 18 June. The trader cannot be perfectly hedged because on 4 May the cash market 3-month LIBOR against which the FRA will be settled will not necessarily have moved since 1 May to the same extent as the futures price. He thus has a basis risk.

Calculation summary

$$\text{Basis} = \text{implied cash price} - \text{actual futures price}$$

$$\text{Theoretical basis} = \text{implied cash price} - \text{theoretical futures price}$$

$$\text{Value basis} = \text{theoretical futures price} - \text{actual futures price}$$

Volume and open interest

“Open interest” in a particular futures contract represents the number of purchases of that contract which have not yet been reversed or “closed out”. It is thus a measure of the extent to which traders are maintaining their positions. The daily volume in a particular contract represents the total number of contracts traded during the day. In both cases, contracts

are not double-counted; either all the long positions or all the short positions are counted, but not both.

The development of volume and open interest are useful in assessing how a futures price might move. If the futures price is rising, for example, and volume and open interest are also rising, this may suggest that the rising price trend will continue for the immediate future, as there is currently enthusiasm for opening new contracts and maintaining them. If the open interest is falling, however, this may suggest that traders are taking profits and not maintaining their positions. Analogous interpretations are possible with a falling futures price.

Speculation

As with an FRA, the most basic trading strategy is to use a futures contract to speculate on the cash interest rate on maturity of the futures contract being higher or lower than the interest rate implied in the futures price now. If the trader expects interest rates to rise, he sells the futures contract; if he expects rates to fall, he buys the futures contract.

As noted earlier, the profit or loss for a futures buyer is:

$$\text{contract size} \times \frac{\text{increase in price}}{100} \times \frac{\text{length of contract in months}}{12}$$

Arbitrage

Example 4.7

Market prices are currently as follows. The futures delivery date is in 2 months' time:

2 v 5 FRA	7.22 / 7.27%
3 month futures	92.67 / 92.68

A trader can arbitrage between these two prices by dealing at 7.27% in the FRA and at 92.68 in the futures market. He buys the FRA and is therefore effectively paying an agreed 7.27% in two months' time. He also buys the futures contract and is therefore effectively receiving $(100 - 92.68)\% = 7.32\%$. He has therefore locked in a profit of 5 basis points.

In practice, Example 4.7 will be complicated by several factors in the same way as in the previous section on hedging FRAs.

First, there is the problem that the FRA settlement is discounted but the futures settlement generally is not. The trader needs to decrease slightly the number of futures contracts traded to adjust for this. As we do not know the discount factor in advance, we need to estimate it – for example, by using the FRA rate itself.

Second, the period of the 2 v 5 FRA might be, for example, 92 days, while the futures contract specification is effectively 90 days (for a currency where the money market convention is ACT/360) and the two settlements will reflect this. The trader needs to increase slightly the number of futures contracts traded to adjust for this.

Combining these last two points, we could make an adjustment as follows:

$$\text{notional amount of FRA} \times \frac{1}{\left(1 + \text{FRA rate} \times \frac{92}{360}\right)} \times \frac{92}{90}$$

Third, the futures delivery date is unlikely to coincide with the start date of the 2 v 5 FRA period, which gives rise to a basis risk. The trader therefore also needs to adjust for this. If the nearest futures date is earlier than two months, the trader could buy some futures for the nearest futures date and some for the following date, in a ratio dependent on the time between the two futures dates and the two-month date. If the nearest futures date is later than two months, the trader could buy all the futures for that date and then superimpose another futures trade as an approximate hedge against the basis risk. As before, this hedge involves buying more futures for the nearest date and selling futures for the following date.

Calendar spread

A spread is a strategy whereby the trader buys one futures contract and sells another, because he expects the difference between them to change but does not necessarily have any expectation about the whole yield curve moving in one direction or the other. A calendar spread, for example, is used when the trader expects the yield curve to change shape – that is, become more or less positive – but does not have a view on rates overall rising or falling. This is similar to the basis risk hedge already described, but using the spread to speculate rather than to hedge.

Example 4.8

On 19 June, USD rates are as follows and a trader expects that the yield curve will become even more negative:

3-month LIBOR:	6.375%
6-month LIBOR:	6.0625%
9-month LIBOR:	5.75%
September futures price:	94.34
December futures price:	95.03

If longer-term rates fall relative to shorter-term ones as expected, the December futures price will rise relative to the September futures price. The trader therefore sells September and buys December futures. In this case, he is “selling” the spread. Suppose after one month, the prices are as follows:

2-month LIBOR:	6.75%
5-month LIBOR:	6.35%
8-month LIBOR:	5.87%
September futures price:	93.98
December futures price:	95.05

The trader can now reverse his position, having made the following profit and loss on the two trades:

September contract:	+ 36 ticks (94.34 – 93.98)
December contract:	+ 2 ticks (95.05 – 95.03)
Total profit:	+ 38 ticks × USD 25 = USD 950

The spread was successful because there was a shift in the yield curve as expected.

A longer-term spread can be taken if a trader has a view on short-term yields compared with bond yields – for example, a spread between a 3-month Eurodollar futures and long-term USD bond futures. In this case an adjustment has to be made for the difference in maturity of the underlying instrument. Settlement on the short-term futures relates to a 90-day instrument, while settlement on the bond futures relates to a notional 15-year bond. For a given change in yield therefore, there will be a far greater profit or loss on the bond futures than on the short-term futures. To balance this, the trader would buy or sell a much smaller notional amount of the bond futures than of the short-term futures.

Cross-market spread

A spread can similarly be taken on the difference between two markets. For example, if ECU interest rates are above DEM rates and a trader believes that the spread between them will narrow, he could buy ECU futures and sell DEM futures.

Strip trading

Just as we used a strip of futures to hedge an FRA in the last section, we could use a strip to hedge an interest rate risk directly. If a trader, for example, buys a June futures and a September futures at the same time, he has hedged against interest rates for six months rather than just for three months.

Example 4.9

A dealer expects to borrow DEM 10 million for six months from 19 June and wishes to lock in a future borrowing rate.

Date:	17 March
Amount:	DEM 10 million
June futures price:	91.75 (implied interest rate: 8.25%)
September futures price:	91.50 (implied interest rate: 8.50%)

To hedge the borrowing, the dealer sells 10 June and 10 September DEM futures. Three months later, the rates are as follows:

Date:	17 June
3-month LIBOR:	9.00%
6-month LIBOR:	9.50%
June futures EDSP:	91.00
September futures price:	90.22

The dealer now reverses the futures contracts and has the following profits:

June contract:	75 ticks (91.75 – 91.00)
September contract:	128 ticks (91.50 – 90.22)
Total profit:	203 ticks

The total profit on the June / September strip is 203 ticks, in *3-month interest rate terms*. This is equivalent to 1.015% in *6-month interest rate terms*. This profit is received in June, but could be invested (say at LIBOR) until December when the borrowing matures. This would give a profit of:

$$1.015\% \times \left(1 + 0.095 \times \frac{183}{360}\right) = 1.064\%$$

Therefore:

$$\begin{aligned} \text{Effective borrowing rate} &= \text{6-month LIBOR} - \text{futures profit} \\ &= 9.50\% - 1.064\% \\ &= 8.436\% \end{aligned}$$

The rate of 8.436% achieved in Example 4.9 comes from the same prices as we used in Example 4.4. In that example, however, we calculated the FRA rate slightly differently, as:

$$\left[\left(1 + 0.0825 \times \frac{92}{360}\right) \times \left(1 + 0.085 \times \frac{91}{360}\right) - 1\right] \times \frac{360}{183} = 8.463\%$$

This demonstrates the effect of the various discrepancies mentioned after Example 4.4.

The strip in Example 4.9 is a rather short strip. It is possible to buy or sell a longer series of contracts to make a longer strip, which can be used, for example, to hedge or arbitrage against a longer-term instrument. Another strategy known as a “boomerang” involves buying a strip and simultaneously selling the same number of contracts all in the nearest date. This is a type of spread, and, traded this way round, will make a profit, for example, if a negative yield curve becomes more negative.

EXERCISES

37. You sell a June futures contract (3-month US dollars) at 94.20. You subsequently close out the position at 94.35. In which direction did you expect US interest rates to move? What is your profit or loss?
38. Given the following 3-month Deutschemark futures prices, what is the implied 3 v 12 Deutschemark FRA rate? Assume that the June futures contract settlement date is exactly 3 months from spot settlement.
- | | |
|-----------|-------|
| June | 95.45 |
| September | 95.20 |
| December | 95.05 |
39. Given the futures prices in the previous question, what is the implied 4 v 8 FRA rate? If you sell a 4 v 8 FRA to a customer, how would you hedge the position using futures?
40. It is now June. You are overlent by DEM 10 million over the six-month period from September to March (i.e. you have lent out DEM 10 million more over that period than you have borrowed, and will need to borrow in due course to cover this mismatch). Given the rates quoted to you, what is the cheapest way to hedge your risk, assuming that all the dates match (i.e. the September futures settlement is in exactly three months' time from now, etc.)?
- | | | |
|-----------------------|--------------------|-------------|
| a. Cash market | 3 mths (92 days): | 4.50%/4.65% |
| | 6 mths (183 days): | 4.60%/4.75% |
| | 9 mths (273 days): | 4.75%/4.90% |
| b. FRAs | 3 v 6: | 4.86%/4.96% |
| | 6 v 9: | 4.90%/5.00% |
| | 3 v 9: | 4.97%/5.07% |
| c. Futures | Sep: | 95.03 |
| | Dec: | 95.05 |



“The different conventions used in different markets to relate price and yield should not affect the economics of an instrument. The economics are determined by what the price actually is and what the future cashflows are. From these, the investor can use a consistent approach of his/her choice to calculate yields for comparison.”

Bond Market Calculations

Overview of capital market instruments

Features and variations

Introduction to bond pricing

Different yield measures and price calculations

A summary of the various approaches to price/yield

Duration, modified duration and convexity

Bond futures

Cash-and-carry arbitrage

Exercises

OVERVIEW OF CAPITAL MARKET INSTRUMENTS

We have already considered short-term securities issued by borrowers in the money market – treasury bills, CDs, commercial paper and bills of exchange. Longer-term securities are considered as “capital market” instruments rather than money market ones. Like commercial paper, capital market borrowing normally involves lending directly from the investor to the borrower without the intermediation of a bank. As a result of this disintermediation, only companies of high creditworthiness – or governments, quasi-governmental bodies and supranational bodies – can usually borrow from the capital markets. Those that do so, however, may raise finance more cheaply than they could on the same terms from a bank.

A straightforward security issued as a medium-term borrowing, with a fixed coupon paid at regular intervals, is generally called a bond – although a medium-term CD issued by a bank is very similar. There is, however, a wide range of variations on this basic theme.

Domestic, foreign and Eurobonds

As with money market instruments, there is a distinction between domestic bonds (where the issuer sells the bond in its own country) and Eurobonds (where the issuer sells the bond internationally). A further distinction is drawn for foreign bonds, where a company issues a bond in a foreign country. For example, a US company might issue a bond in the domestic Japanese market – a “Samurai” bond. Other examples of this are “Yankee” bonds (foreign issues in the US domestic market), “Bulldog” bonds (in the UK), “Matador” bonds (in Spain), and “Alpine” bonds (in Switzerland).

Eurobonds are issued in bearer form – investors do not have to be registered as in the domestic US bond markets, for example. Ownership is evidenced by physical possession and the interest is payable to the holder presenting the coupon, which is detachable from the bearer bond. This is clearly an advantage to an investor who wishes to remain anonymous. Interest on Eurobonds is quoted on a 30(E)/360 basis (see later for an explanation of the various day/year conventions) and paid gross of tax.

As in other markets, there is an important distinction between a “Eurobond” (a bond issued internationally) and a “euro bond” (a bond denominated in euros, whether issued “domestically” in the euro zone, or internationally).

Government bond markets

Domestic debt markets are usually dominated by government debt for two reasons. First, government debt outstanding is usually large in relation to the market as a whole and therefore offers good liquidity. Second, government debt issued in the domestic currency is usually considered virtually riskless,

because the government can always print its own money to redeem the issue. Its debt therefore provides a good benchmark for yields. The following are a few of the important markets:

USA

US government issues with an original maturity between one year and ten years are called “treasury notes”. Issues with an original maturity longer than ten years (in practice up to 30 years) are called “treasury bonds”. The difference in name does not affect calculations for these bonds. All pay semi-annual coupons and have both the accrued coupon and price calculated on an ACT/ACT basis (see later in this chapter for an explanation of the different calculation bases).

In addition to US treasury bonds, there is a large market in bonds issued by US federal agencies (generally supported by the US government), such as the Federal National Mortgage Association (FNMA or “Fanny Mae”), Student Loan Marketing Association (SLMA or “Sally Mae”), Federal Home Loan Mortgage Corporation (FHMC or “Freddie Mac”) and Government National Mortgage Association (GNMA or “Ginny Mae”). Coupons and price are calculated on a 30(A)/360 basis.

UK

UK government securities are called gilt-edged securities or “gilts”. Conventional gilts are generally called “shorts” if they have a remaining maturity up to five years, “mediums” between five and fifteen years, and “longs” over fifteen years. Almost all gilts pay coupons semi-annually, although there are a few with quarterly coupons. Some are index-linked, where both the coupon and redemption amount paid depend on the inflation rate. Accrued coupon and price have both been calculated on an ACT/365 basis prior to 1998, with the price / yield calculation changing to ACT/ACT in December 1997 when strip trading began and the accrued coupon calculation changing to ACT/ACT in late 1998.

Most gilts are conventional bonds with a fixed maturity. Some may be redeemed by the government early, and some (“convertibles”) may be converted by the holder into other specific issues. There are a few irredeemable gilts, although these are no longer issued.

Germany

German government bonds pay annual coupons, with accrued coupon and price both calculated on a 30(E)/360 basis. Important bonds are Bunds (Bundesanleihen), normally with an original maturity of ten years, BOBLs or OBLs (Bundessobligationen), with maturities up to five years, and Schätze (federal treasury notes) with two-year maturities.

France

French government bonds pay annual coupons, with accrued coupon and price both calculated on an ACT/ACT basis. The most important bonds are BTANs (bons du trésor à taux fixe et intérêt annuel), with original maturities of between two and five years, and OATs (obligations assimilables du trésor) with original maturities of between seven and thirty years.

FEATURES AND VARIATIONS

What we have described so far are essentially straightforward bonds with a fixed coupon and a fixed maturity date when all the bond is redeemed (that is, repaid). There are several variations available.

Floating rate note (FRN)

An FRN is a bond whose coupon is not fixed in advance, but rather is refixed periodically on a “refix date” by reference to some floating interest rate index. In the Euromarkets, this is often some fixed margin over six-month LIBOR.

Essentially, investing in an FRN is equivalent to investing in a short-term money market instrument such as a six-month CD, and then reinvesting the principal in a new CD on a rolling basis as the CD matures.

Index-linked bonds

With an index-linked bond, the coupon, and possibly the redemption amount, are linked to a particular index rather than fixed. For example, some governments issue index-linked bonds where the coupon paid is a certain margin above the domestic inflation index. Index-linked bonds issued by companies are often linked to stock indexes or commodity price indexes.

Zero-coupon bonds

Zero-coupon bonds are bonds that make no interest payments. Similar to commercial paper or treasury bills in the money market, the price must be less than the face value, so that they are sold at a discount to their nominal value.

The only cashflows on a zero-coupon bond are the price paid and the principal amount received at maturity, so that the investor’s return is represented by the relationship between these two amounts. With a “normal” coupon-bearing bond, the investor is vulnerable to the risk that, by the time he/she receives the coupons, interest rates have fallen so that he/she can only reinvest the coupons received at a lower rate. Whether the reinvestment rate has in fact fallen or risen, the final outcome for the investor is not known

exactly at the beginning. With a zero-coupon bond however, the outcome is known exactly because there are no coupons to reinvest. Because of this certainty, investors may accept a slightly lower overall yield for a zero-coupon bond. Differences in tax treatment between capital gains / losses and coupon income may also affect the attractiveness of a zero-coupon bond compared with a coupon-bearing bond.

Strips

The process of stripping a bond is separating all the cashflows – generally a series of coupon payments and a redemption payment – and then trading each cashflow as a separate zero-coupon item. Various governments (US, UK, German and French, for example) facilitate the trading of their securities as strips in this way. Before government securities were officially strippable however, strips were created by investment banks. The bank sets up a special purpose vehicle to purchase the government security, holds it in custody, and issues a new stripped security in its own name on the back of this collateral.

Amortisation

A straightforward bond has a “bullet” maturity, whereby all the bond is redeemed at maturity. An alternative is for the principal amount to be repaid in stages – “amortised” – over the bond’s life.

Perpetual bonds

A perpetual bond is one with no redemption date. Most such bonds have been FRNs issued by banks, although there are some government perpetuals, such as the “War Loan” UK gilt.

Calls and puts

Bonds are often issued with options for the issuer to “call” (i.e. redeem) the bond prior to maturity. This often requires that the redemption amount is higher than it would be at maturity. This option is particularly helpful with fixed rate bonds, to protect the issuer from paying too much if market interest rates fall after the bond has been issued. This is because he can then redeem the bond early and issue a new one with a lower coupon.

Bonds can also be issued with an option for the investor to “put” (i.e. require redemption on) the bond before maturity.

Bond warrants

A warrant attached to a bond is an option to buy something – generally more of the same bond or a different bond.

Medium-term note (MTN)

An MTN is a hybrid between commercial paper and a bond. Like commercial paper, an MTN is issued continually rather than as a one-off. Like a bond, it has a maturity over one year and pays a coupon, either fixed- or floating-rate. The interest calculation may be either on a money market basis – for example ACT/360 for US domestic MTNs – or on a bond basis – 30(E)/360 for Euro-MTNs.

Asset-backed securities

In a straightforward bond, the investor is relying on the issuer's overall creditworthiness for payment of the bond's coupons and repayment of the principal. With an asset-backed security however, the bond is collateralised by a specific asset or pool of assets which generate the necessary cashflows.

In a mortgage-backed security, for example, a collection of property mortgages may be pooled together. The investor buys a security which is issued by a special-purpose company. The security is however collateralised specifically by the pool of mortgages so that the investor's risk is that of the original underlying mortgage transactions. Each of these in turn is collateralised by the property in question. Ultimately therefore, the bond market investor is buying a bond with property values collateralising the risk. As each individual mortgage is repaid, the mortgage-backed security is amortised to the extent of the principal amount of that individual mortgage. Whether the amortisation is applied to each individual investor's bond holding pro rata, or to particular holdings at random, depends on the security's structure. Either way, it is impossible for the investor to predict his/her cashflows precisely, regardless of whether the security is fixed-rate or floating-rate. In assessing the yield on such a security, the investor must therefore make assumptions about the likely pattern of amortisation, based on historic comparisons and interest rate expectations.

INTRODUCTION TO BOND PRICING

We have already seen that for a given yield, any future cashflow C has a present value equal to:

$$\frac{C}{(1+i)^N}$$

where i is the annual yield and N is the number of years ahead that the cashflow occurs.

Given a series of cashflows – some of which could be negative and some positive – the net present value of the whole series is the sum of all the present values.

The principles of pricing in the bond market are the same as the principles of pricing in other financial markets – the total price paid for a bond now is

the net present value now of all the future cashflows which arise from holding the bond. The price is expressed as the price for 100 units of the bond.

In general therefore, the theoretical all-in price P of a straightforward bond should be given by:

$$P = \sum_k \frac{C_k}{\left(1 + \frac{i}{n}\right)^{\frac{d_k \times n}{\text{year}}}}$$

**Calculation
summary**

where: C_k = the k^{th} cashflow arising
 d_k = number of days until C_k
 i = yield on the basis of n payments per year
 year = number of days in the conventional year.

The important thing to note here is the concept that the all-in price of a bond equals the NPV of its future cashflows.

All-in price of a bond = NPV of the bond's future cashflows

Key Point

Because the price is the net present value, the greater the yield used to discount the cashflows, the lower the price.

A bond's price falls as the yield rises and vice versa

Key Point

There are four small but significant differences in practice between calculations for a bond price and the price of a money market instrument such as a medium-term CD. These differences are helpful in understanding the ideas behind bond pricing:

1. The coupon actually paid on a CD is calculated on the basis of the exact number of days between issue and maturity. With a bond, the coupons paid each year or half-year (or occasionally each quarter) are paid as fixed "round" amounts. For example, if a 10 percent coupon bond pays semi-annual coupons, exactly 5 percent will be paid each half-year regardless of the exact number of days involved (which will change according to week-ends and leap years, for example).

Bond coupons are paid in round amounts, unlike CD coupons which are calculated to the exact day

Key Point

2. When discounting to a present value, it is again assumed that the time periods between coupons are “round” amounts – 0.5 years, 1 year, etc., rather than an odd number of days. For this purpose, the first outstanding coupon payment is usually assumed to be made on the regular scheduled date, regardless of whether this is a non-working day.

Key Point

For straightforward bonds, pricing conventionally assumes that periods between coupons are regular

3. When discounting back to a present value from the first outstanding coupon payment date, the price calculation is made on the basis of compound interest rather than simple interest. Suppose, for example, there is a cashflow of 105 occurring 78 days in the future, the yield is 6 percent and the year-count basis is 360. The present value calculation for a CD would be $\frac{105}{(1 + 0.06 \times \frac{78}{360})}$, which uses the 6 percent yield for 78 days on a simple basis. The corresponding calculation for a bond would be $\frac{105}{(1 + 0.06)^{\frac{78}{360}}}$ which compounds the 6 percent yield for $\frac{78}{360}$ of a year.

Key Point

Bond pricing conventionally assumes compound discounting to NPV

4. The day/year count basis for money market instruments and bonds is generally different. The first have been described earlier. The day/year counts for bonds are described later in this chapter.

Given these differences, it is possible to express the equation for a bond price given earlier as follows:

Calculation summary

$$P = \frac{100}{(1 + \frac{i}{n})^W} \left[\frac{R}{n} \times \frac{\left(1 - \frac{1}{(1 + \frac{i}{n})^N}\right)}{\left(1 - \frac{1}{(1 + \frac{i}{n})}\right)} + \frac{1}{(1 + \frac{i}{n})^{N-1}} \right]$$

where: R = the annual coupon rate paid n times per year
W = the fraction of a coupon period between purchase and the next coupon to be received
N = the number of coupon payments not yet paid
i = yield per annum based on n payments per year.

Despite the simplifying assumptions behind this formula, it is important because it is the conventional approach used in the markets. Adjustments to

the formula are of course necessary if there are different cashflows to be discounted – for example, an unusual first coupon period which gives an odd first coupon payment, early partial redemptions of the bond or changes in the coupon rate during the bond's life. Even with these adjustments, the market approach is still generally to calculate the total price as the NPV assuming precisely regular coupon periods and certain day/year conventions.

It is clearly possible to calculate a price without making these assumptions – that is, to use the formula:

$$P = \sum_k \frac{C_k}{\left(1 + \frac{i}{n}\right)^{\frac{d_k \times n}{\text{year}}}}$$

with the exact time periods between cashflows and a consistent day/year basis for all types of bonds. For bonds of short maturity, the result can be significantly different from the conventional formula and this approach is clearly more satisfactory when comparing different bonds. In the UK gilt market, for example, some market participants quote yields taking into account the exact number of days between the actual cashflows. Some also use a year basis of ACT/365 $\frac{1}{4}$ to allow for the average effect of leap years.

Accrued interest

The price we have calculated so far is in fact known as the “dirty” price of the bond and represents the total amount of cash paid by the buyer. From the seller's point of view, however, he expects to receive “accrued” coupon on the bond. The accrued coupon is the coupon which the seller of a bond has “earned” so far by holding the bond since the last coupon date. He feels he is entitled to this portion of the coupon and therefore insists on the bond buyer paying it to him. The buyer, however, will pay no more than the NPV of all the future cashflows. Therefore the total price paid is the dirty price but this is effectively considered as two separate amounts – the “clean” price and the accrued coupon. The price quoted in the market is the “clean” price, which is equal to dirty price minus the accrued coupon. In the market generally, accrued coupon is often called accrued interest.

Example 5.1

A bond pays a 9% coupon annually. Maturity is on 15 August 2003. The current market yield for the bond is 8%. Interest is calculated on a 30(E)/360 basis (see later in this section for an explanation of this convention). What are the accrued coupon, dirty price and clean price for settlement on 12 June 1998?

Time from 15 August 1997 (the last coupon date) to 12 June 1998 is 297 days on a 30(E) basis:

$$\text{Accrued coupon} = 9 \times \frac{297}{360} = 7.4250$$

Time from 12 June 1998 to 15 August 1998 is 63 days on a 30(E) basis:

$$\text{Dirty price} = \frac{100}{1.08^{\frac{63}{360}}} \times \left(0.09 \times \frac{\left(1 - \frac{1}{1.08^5}\right)}{\left(1 - \frac{1}{1.08}\right)} + \frac{1}{1.08^5} \right) = 111.4811$$

$$\text{Clean price} = 111.4811 - 7.4250 = 104.06$$

1.08 ENTER 6 $\square \wedge \square \frac{1}{x}$ 1 $\square x \geq y -$
1.08 $\square \frac{1}{x}$ 1 $\square x \geq y - \div$
.09 \times
1.08 ENTER 5 $\square \wedge \square \frac{1}{x} +$
100 \times
1.08 ENTER 63 ENTER 360 $\div \square \wedge \div$
9 ENTER 297 \times 360 \div
-

(Dirty price)
(Accrued interest)
(Clean price)

Calculation
summary

Dirty price = NPV of cashflows

Accrued coupon = proportion of coupon since last coupon payment

Clean price = dirty price – accrued coupon

Clean bond prices are generally quoted in terms of the price per 100 units of the bond, often to two decimal places. US bonds are, however, generally quoted in units of $\frac{1}{32}$. Thus a price of 95–17 for a US treasury bond means $\$95\frac{17}{32}$ (= 95.53125) per \$100 face value, rather than \$95.17 per \$100 face value. This is sometimes refined further to units of $\frac{1}{64}$ by use of “+” and “–”. Thus a price of 95–17+ means $95\frac{35}{64}$ = 95.546875. Option prices on US government bonds are quoted in units of $\frac{1}{64}$.

Coupon dates

Apart from the first coupon period or the last coupon period, which may be irregular, coupons are generally paid on regular dates – usually annual or semi-annual and sometimes quarterly. Thus semi-annual coupons on a bond maturing on 17 February 2015 would typically be paid on 17 August and 17 February each year. If a semi-annual bond pays one coupon on 30 April for example (that is, at month-end), the other coupon might be on 31 October (also month-end) as with a US treasury bond, or 30 October as with a UK gilt.

Even though the previous coupon may have been delayed – for example, the coupon date was a Sunday so the coupon was paid the following day – the accrual calculation is taken from the regular scheduled date, not the actual payment date. Also, the accrued interest is calculated up to a value date which in some markets can sometimes be slightly different from the settlement date for the transaction. This does not affect the total dirty price paid. Note that in some markets, if the scheduled coupon payment date is a Saturday, the payment is actually made on the previous working day, rather than on the next working day as is the more usual convention.

Ex-dividend

When a bond is bought or sold shortly before a coupon date, the issuer of the bond generally needs some days to change the records in order to make a note of the new owner. If there is not enough time to make this administrative change, the coupon will still be paid to the previous owner.

The length of time taken varies widely between different issuers. The issuer pays the coupon to the holder registered on a date known as the record date. This is therefore the last date on which a bond transaction can be settled in order for the new owner to be recorded in time as entitled to the coupon, and a bond sold up to this date is said to be “cum-dividend”. A bond sold for settlement after this date is said to be sold “ex-dividend” or “ex-coupon”. In some cases it is possible to sell a bond ex-dividend before the normal ex-dividend period.

If a bond sale is ex-dividend, the seller, rather than needing to receive accrued interest from the buyer, will need to pay it to the buyer – because the final days of the coupon period which “belong” to the buyer will in fact be paid to the seller. At the ex-dividend point therefore, the accrued interest becomes negative. In this case, the accrued interest is calculated from value date to the next scheduled coupon date (rather than the next actual coupon payment date if that is different because of a non-working day).

Example 5.2

Consider a UK gilt with a coupon of 7.3% and maturity 17 August 2005, purchased for settlement on 13 August 1998 at a price of 98.45. Assume the record date is 5 working days before the coupon date.

$$\text{The accrued interest is: } \frac{-4}{365} \times 7.3 = -0.08$$

$$\text{The all-in price is therefore } 98.45 - 0.08 = 98.37$$

$$\text{Accrued coupon} = 100 \times \text{coupon rate} \times \frac{\text{days since last coupon}}{\text{year}}$$

For ex-dividend prices, accrued coupon is negative:

$$\text{Accrued coupon} = -100 \times \text{coupon rate} \times \frac{\text{days to next coupon}}{\text{year}}$$

**Calculation
summary**

Day/year conventions

Fractional periods of a year for price / yield calculations and accrued coupon are calculated according to different conventions in different markets. As in the money market, these conventions are generally expressed in the form “days/year” where “days” is the conventional number of days in a particular period and “year” is the conventional number of days in a year.

Calculation of days in the period

ACT The actual number of calendar days in the period.

30(E) To calculate the number of days between two dates $d_1 / m_1 / y_1$ and $d_2 / m_2 / y_2$, first make the following adjustments:

If d_1 is 31, change it to 30.

If d_2 is 31, change it to 30.

The number of days in the period between the two dates is given by:

$$(y_2 - y_1) \times 360 + (m_2 - m_1) \times 30 + (d_2 - d_1)$$

For example there are 5 days between 27 February and 2 March and 33 days between 27 February and 31 March.

30(A) This is similar to 30(E), but with the following difference:

If d_2 is 31 but d_1 is not 30 or 31, do not change d_1 to 30.

The number of days in the period between the two dates is again given by:

$$(y_2 - y_1) \times 360 + (m_2 - m_1) \times 30 + (d_2 - d_1)$$

For example, there are 5 days between 27 February and 2 March and 34 days between 27 February and 31 March.

Calculation of year basis

365 Assume that there are 365 days in a year.

360 Assume that there are 360 days in a year.

ACT Assume that the number of days in a year is the actual number of days in the current coupon period multiplied by the number of coupon payments per year.

The four combinations used in the bond market (in addition to ACT/360 used in many money markets as discussed earlier) are:

ACT/365 Sometimes called “ACT/365 fixed”. Used for Japanese government bonds and, prior to late 1998, for accrued interest on UK gilts.

ACT/ACT Used for US treasury notes and bonds, which always pay coupons semi-annually, and results in a value for the year of 362, 364, 366 or 368 (twice the coupon period of 181, 182, 183 or 184). It is also used for French government bonds, which pay coupons annually, resulting in a value for the year of 365 or 366. It is also the method for accrued interest on UK gilts after late 1998.

30(E)/360 Sometimes called “360/360”. Used for Eurobonds and some European domestic markets such as German government bonds.

30(A)/360 Used for US federal agency and corporate bonds.

In general, the convention for accrued interest in a particular market is the same as the convention used in that market for calculating the fraction of a year for the price / yield formula. This is not always so, however. In Italy, for example, accrued interest is calculated on a 30(E)/360 basis, but the price is calculated on an ACT/365 basis. In Spain, on the other hand, accrued interest is ACT/ACT and the price is ACT/365.

Note that, because coupons are paid in “round” amounts, it is possible for the accrued coupon to be greater than the actual coupon payable for the period. For example, if coupons are paid semi-annually on an ACT/365 basis, the interest accrued from a 15 March coupon date to 14 September would be $\text{coupon} \times \frac{183}{365}$, which is greater than the coupon $\times \frac{1}{2}$ payable. This possibility depends on the ex-dividend period for the bond.

Example 5.3

Consider a bond whose previous coupon was due on 15 January 1999 and whose next coupon is due on 15 July 1999. The number of calendar days in the current coupon period is 181. The day/year calculation under the various conventions is shown below for accrued interest from 15 January up to 30 March, 31 March and 1 April respectively:

	30 March	31 March	1 April
ACT/365	74/365	75/365	76/365
ACT/360	74/360	75/360	76/360
30(E)/360	75/360	75/360	76/360
30(A)/360	75/360	76/360	76/360
ACT/ACT	74/362	75/362	76/362

We have given a list of the conventions used in some important markets in Appendix 1.

Using an HP calculator

The HP17 and HP19 calculators have inbuilt functions specifically for calculating clean bond prices and yields. They allow for coupons to be paid annually or semi-annually and also for the day/year counts to be 30(A)/360 or ACT/ACT. Provided that neither the settlement date nor the next coupon date is the 31st day of a month, 30(A)/360 is the same as 30(E)/360. Also, provided that the current coupon period does not include 29 February, ACT/ACT annual is the same as ACT/365 annual. The calculator cannot however calculate on an ACT/365 semi-annual basis.

HP calculator example

We can repeat Example 5.1 using the HP bond function as follows:

HP19BII	
FIN BOND	
TYPE 360 ANN EXIT	(Set for 30/360, annual coupons)
12.061998 SETT	(Settlement date)
15.082003 MAT	(Maturity date)
9 CPN%	(Coupon)
MORE 8 YLD%	(Yield)
PRICE	(Clean price)
ACCRU	(Accrued interest)
+	(Dirty price)

Similarly to the TVM function of the calculator, it is possible to calculate the clean price from the yield or the yield from the clean price. It is also possible to allow for a redemption amount different from the normal 100 by entering the appropriate value as a “call” amount – i.e. the amount which the issuer must pay on redemption if he wishes to call the bond on the date entered as the maturity date.

HP calculator example

A bond pays a 6.5% coupon semi-annually. Maturity is on 22 July 2015. The bond will be redeemed at a rate of 105 per 100. The current (clean) price for the bond for settlement on 27 March 1998 is 97.45. Interest and price / yield calculations are both on a semi-annual ACT/ACT basis. What is the yield of the bond?

Answer: 6.91%

HP19BII	
FIN BOND	
TYPE A/A SEMI EXIT	(Set for ACT/ACT, semi-annual)
27.031998 SETT	(Settlement date)
22.072015 MAT	(Maturity date)
6.5 CPN%	(Coupon)
105 CALL	(Redemption amount per 100)
MORE 97.45 PRICE	(Clean price)
YLD%	(Yield)

Note that it is probably useful to reset the call amount to 100 after such a calculation, to avoid forgetting this next time!

As already mentioned, some bonds, such as Spanish and Italian bonds, for example, calculate accrued interest on one day/year basis but calculate the dirty price from the future cashflows on a different day/year basis. The HP bond function is not able to calculate with a mixture in this way, and the numbers must be manipulated to get round the problem. The exercises at the end of the chapter include some examples of this. The procedures necessary are as follows:

To calculate the clean price from the yield

- Set the calculator for the day/year basis used for the price / yield calculation.

- Calculate the clean price and accrued interest as usual. Add together to give the true dirty price.
- Reset the calculator for the day/year basis used for the accrued interest.
- Calculate the conventional accrued interest and subtract from the dirty price to give the clean price.

To calculate the yield from the clean price

- Set the calculator for the day/year basis used for the accrued interest calculation.
- Calculate the accrued interest as usual. Add to the known clean price to give the true dirty price.
- Reset the calculator for the basis used for the price / yield calculation.
- Recalculate an adjusted accrued interest and subtract from the true dirty price to give an adjusted clean price.
- Use this adjusted clean price to calculate the yield.

DIFFERENT YIELD MEASURES AND PRICE CALCULATIONS

Yield to maturity

We have so far considered how to calculate a bond's price if we know its yield. This is the same as calculating an NPV given a rate of discount. Calculating the yield if we know the price is the same as calculating the internal rate of return of all the cashflows including the price paid. As noted in Chapter 1, there is no formula for this calculation. Instead, it is necessary to use the price formula and calculate the yield by iteration – estimate a yield, calculate the price based on this estimate, compare with the actual price, adjust the yield estimate, recalculate the price, etc.

The yield we have used is known as the yield to maturity (sometimes YTM). It is also known simply as “yield,” “redemption yield” or “gross redemption yield” (GRY) because it is the yield to the redemption date assuming all cashflows are paid gross (without deduction of tax).

Some bonds make partial redemptions – that is, the principal is repaid in stages rather than all at maturity. Using the same approach as we have so far requires that all the cashflows, including the partial redemptions, are discounted at the yield to give the dirty price. In this case, the yield is sometimes known as “yield to equivalent life.”

The yield to maturity has the same disadvantage that we considered in Chapter 1; it assumes that all cashflows can be reinvested at the same rate. Consider, for example, a 7-year bond with annual coupons of 10 percent, a price of 95.00 and an annual yield of 11.063 percent. This means that if all the cashflows are discounted at 11.063 percent, they have an NPV of 95. Equivalently, if all the cashflows received over the bond's life can be reinvested to maturity at 11.063 percent, the final internal rate of return will also be 11.063 percent.

In practice, reinvestment rates will be different. However, given that a bond does have a market price, an investor wishes to be told, in summary, what rate of return this implies. There are as many answers to the question as there are assumptions about reinvestment rates. One possibility would be to calculate the current market forward-forward rates at which all the coupon cashflows could be reinvested. In practice, the conventional summary answer given to the question is the internal rate of return – i.e. the yield to maturity.

Yield vs. coupon

It is intuitively reasonable that when a bond's yield is the same as its coupon, the bond's price should be par (that is, 100) – because the bond's future cashflows are created by applying an interest rate to the face value of 100 and discounting back again to an NPV at the same rate should arrive back at 100.

Similarly, as the yield rises above the coupon rate, the NPV will fall and vice versa. Therefore:

Key Point

Generally, if a bond's price is greater than par, its yield is less than the coupon rate, and vice versa

Although this intuitive result is almost true, the price is not in fact exactly 100 when the yield is equal to the coupon, except on a coupon date. This is because accrued interest is calculated on a simple interest basis and price on a compound interest basis.

Example 5.4

Consider the same bond as in Example 5.1, but assume the yield is now 9%. What is the clean price?

$$\text{As before, accrued coupon} = 9 \times \frac{297}{360} = 7.4250$$

$$\text{Dirty price} = \frac{100}{1.09^{\frac{63}{360}}} \times \left(0.09 \times \frac{\left(1 - \frac{1}{1.09^{\frac{63}{360}}}\right)}{\left(1 - \frac{1}{1.09}\right)} + \frac{1}{1.09^5} \right) = 107.3685$$

$$\text{Clean price} = 107.3685 - 7.4250 = 99.94$$

```
FIN BOND TYPE 360 ANN EXIT
12.061998 SETT
15.082003 MAT
9 CPN%
MORE 9 YLD%
PRICE
```

In Example 5.4, if the clean price were exactly 100, the yield would in fact be 8.986%.

This calculation is complicated further, of course, when the day/year basis for the price / yield calculation is different from the day/year convention for the accrued interest. In general, however:

On a non-coupon date, if a bond is priced at par, the yield is slightly lower than the coupon rate

Key Point

The effect increases as the settlement date moves away from the coupon dates and is greater for bonds of short maturity.

Current yield

A much more simple measure of yield, which can be calculated easily, is the current yield. This ignores any capital gain or loss arising from the difference between the price paid and the principal amount received at redemption. It also ignores the time value of money. Instead it considers only the coupon income as a proportion of the price paid for the bond – essentially considering the investment as an indefinite deposit:

$$\text{Current yield} = \frac{\text{coupon rate}}{\frac{\text{clean price}}{100}}$$

Calculation summary

Simple yield to maturity

Simple yield to maturity does take the capital gain or loss into account as well as the coupon but, like current yield, ignores the time value of money. The capital gain or loss is amortized equally over the time left to maturity: The simple yield to maturity is used in the Japanese bond market.

$$\text{Simple yield to maturity} = \frac{\text{coupon rate} + \left(\frac{\text{redemption amount} - \text{clean price}}{\text{years to maturity}} \right)}{\frac{\text{clean price}}{100}}$$

Calculation summary

Example 5.5

For a 7-year bond paying annual 10% coupons and with a price of 95.00, what are the yield to maturity, current yield and simple yield to maturity?

Yield to maturity = 11.06%

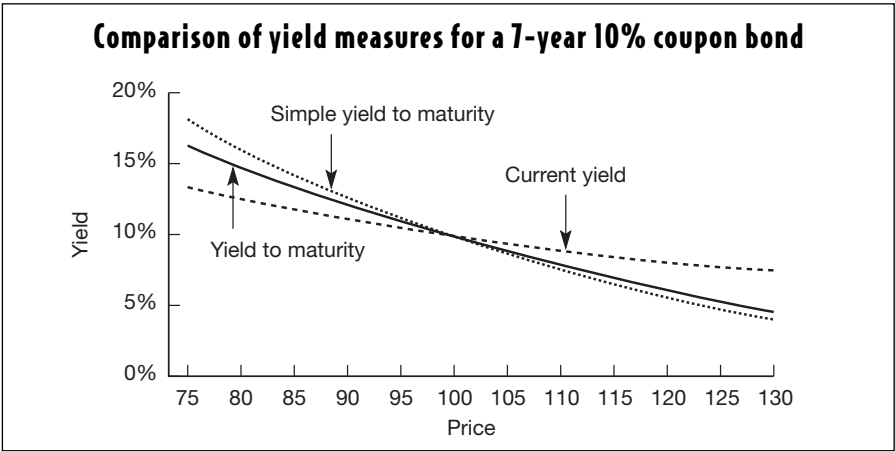
$$\text{Current yield} = \frac{10\%}{\frac{95.00}{100}} = 10.53\%$$

$$\text{Simple yield to maturity} = \left(\frac{10 + \left(\frac{100 - 95}{7} \right)}{\frac{95.00}{100}} \right) \% = 11.28\%$$

FIN TVM	
7 N	
95 +/- PV	
10 PMT	
100 FV	
I%YR	(Yield to maturity)
10 ENTER .95 ÷	(Current yield)
100 ENTER 95 - 7 ÷ 10 + .95 ÷	(Simple yield to maturity)

Figure 5.1 compares the different measures for the same bond at different prices.

Fig 5.1



Yield in a final coupon period

When a bond has only one remaining coupon to be paid (at maturity), it looks very similar, in cashflow terms, to a short-term, money-market instrument. For this reason, yields for such bonds are sometimes quoted on a basis comparable to money market yields. This means that the future cashflow is discounted to a present value (the price) at a simple rate rather than a compound rate – but still using the day/year basis conventional for that particular bond market.

Example 5.6

Consider the following bond:

Coupon:	8% semi-annual
Maturity date:	20 October 2000
Settlement date:	14 August 2000
Clean price:	100.26
Price / yield calculation basis:	ACT/ACT
Accrued interest calculation basis:	ACT/ACT

The current coupon period from 20 April to 20 October is 183 days

Therefore the accrued interest is $8 \times \frac{116}{366} = 2.5355$

Therefore the dirty price is $100.26 + 2.5355 = 102.7955$

(a) Following the logic of usual bond pricing, the price / yield equation would be:

$$\text{dirty price} = \frac{\text{final cashflow}}{(1 + \text{yield})^{\frac{\text{days}}{\text{year}}}}$$

This can be manipulated to become:

$$\begin{aligned} \text{yield} &= \left(\frac{\text{final cashflow}}{\text{dirty price}} \right)^{\frac{\text{year}}{\text{days}}} - 1 \\ &= \left(\frac{104}{102.7955} \right)^{\frac{366}{67}} - 1 = 6.57\% \end{aligned}$$

(b) Using simple interest, however, the result would be:

$$\begin{aligned} \text{yield} &= \left(\frac{\text{final cashflow}}{\text{dirty price}} - 1 \right) \times \frac{\text{year}}{\text{days}} \\ &= \left(\frac{104}{102.7955} - 1 \right) \times \frac{366}{67} = 6.40\% \end{aligned}$$

Method (b) is assumed by the HP calculator. It is used in the market, for example, for US treasury bonds (on an ACT/ACT basis), but not generally for Eurobonds.

104 ENTER 102.7955 ÷ 366 ENTER 67 ÷ □ ∧ 1 -

(a)

104 ENTER 102.7955 ÷ 1 - 366 × 67 ÷

(b)

Alternative yield calculation in a final coupon period

$$i = \left[\frac{\text{total final cashflow including coupon}}{\text{dirty price}} - 1 \right] \times \frac{\text{year}}{\text{days to maturity}}$$

where days and year are measured on the relevant bond basis

**Calculation
summary**

Bond-equivalent yields for treasury bills

In the previous section we considered what is the yield of a bond in its last coupon period, calculated by a method which enables comparison with a money market instrument. In the US market, the reverse is also considered: what is the yield of a treasury bill calculated by a method which enables comparison with a bond which will shortly mature? The bond-equivalent yield of a treasury bill is therefore the coupon of a theoretical US treasury bond, trading at par, with the same maturity date, which would give the same return as the bill.

If the bill has 182 days or less until maturity, the calculation for this is the usual conversion from discount to yield, except that it is then quoted on a 365-day basis:

$$i = \frac{D}{\left(1 - D \times \frac{\text{days}}{360} \right)} \times \frac{365}{360}$$

where: D = discount rate (on a 360-day basis)
 days = number of days until maturity
 i = the bond-equivalent yield we are trying to calculate

If 29 February falls during the 12-month period starting on the purchase date, 365 is conventionally replaced by 366.

If the bill has more than 182 days until maturity, however, the calculation must take account of the fact that the equivalent bond would pay a coupon during the period as well as at the end. The amount of the first coupon is taken to be the coupon accrued on the bond between purchase and coupon date (which is half a year before maturity). This coupon is:

$$P \times i \times \left(\frac{\text{days} - \frac{365}{2}}{365} \right)$$

$$= P \times i \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right)$$

where: P = the price paid for the bond (= its par value)

If this coupon is reinvested at the same yield i , then at maturity it is worth:

$$P \times i \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right) \times \left(1 + \frac{i}{2} \right)$$

The bond also returns at maturity the face value P and the final coupon – a total of:

$$P \times \left(1 + \frac{i}{2} \right)$$

Adding together these amounts, the total proceeds at maturity are:

$$P \times \left(1 + \frac{i}{2} \right) \times \left(1 + i \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right) \right)$$

Since the return on this theoretical bond is the same as the return on the treasury bill, these total proceeds must be the face value F of the bill paid on maturity, and the price paid P must be the same as the discounted value paid for the bill. Thus:

$$F = P \times \left(1 + \frac{i}{2} \right) \times \left(1 + i \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right) \right)$$

$$\text{and } P = F \times \left(1 - D \times \frac{\text{days}}{360} \right)$$

Therefore:

$$i^2 \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right) + i \times \frac{2 \times \text{days}}{365} + 2 \times \left(1 - \frac{1}{\left(1 - D \times \frac{\text{days}}{360} \right)} \right) = 0$$

Solving for a quadratic equation, this gives the formula shown below. Again, if 29 February falls in the 12-month period starting on the purchase date, 365 is conventionally replaced by 366.

Bond-equivalent yield for US treasury bill

If 182 days or less to maturity:

$$i = \frac{D}{1 - D \times \frac{\text{days}}{360}} \times \frac{365}{360}$$

If more than 182 days to maturity:

$$i = \frac{-\frac{\text{days}}{365} + \left(\left(\frac{\text{days}}{365} \right)^2 + 2 \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right) \times \left(\frac{1}{1 - D \times \frac{\text{days}}{360}} - 1 \right) \right)^{\frac{1}{2}}}{\left(\frac{\text{days}}{360} - \frac{1}{2} \right)}$$

If 29 February falls in the 12-month period starting on the purchase date, replace 365 by 366.

**Calculation
summary**

It should be noted that if there happen to be a treasury bill and a treasury bond maturing on the same day in less than 182 days, the bond-equivalent yield for the bill is not exactly the same as the yield quoted for a bond in its final coupon period, even though this is intended to take the same approach as money market instruments. Example 5.7 demonstrates this.

Example 5.7

Consider a US treasury bill maturing on 20 October 2000 and quoted at a discount rate of 6.2229% for settlement 14 August 2000. (The discount rate would not normally be quoted to that level of precision, but we need this for a comparison with the previous example.)

The bond-equivalent yield for this bill is:

$$\frac{6.2229\%}{1 - 0.062229 \times \frac{67}{360}} \times \frac{365}{360} = 6.38\%$$

Suppose that we purchase face value \$10,400 of the bill. The price paid would be:

$$\$10,400 \times \left(1 - 0.062229 \times \frac{67}{360} \right) = \$10,279.55$$

The cashflows for this bill are exactly the same as if we buy face value \$10,000 of the bond in the previous example; the final proceeds are principal plus a semi-annual coupon and the dirty price paid is 102.7955. The yield quoted for that bond however was 6.40%, because the year basis was 2×181 days.

0.062229 ENTER 67 × 360 ÷ 1 - +/- 6.2229 □ x ≥ y ÷ 365 × 360 ÷

Money market yield

The difference in price / yield conventions between instruments such as a long-term CD and a bond makes comparison between them difficult. An investor may therefore wish to bring them into line by calculating a money market yield for a bond.

From earlier, we saw that the differences between the two approaches (apart from the fact that coupons on bonds are paid in round amounts and those on CDs are not) are:

1. CD price calculations use exact day counts rather than assume regular time intervals between coupon payments.
2. CDs calculate the final discounting from the nearest coupon date back to the settlement date using simple interest rather than compound interest.
3. CDs and bonds generally also have different day/year count bases.

It is possible to calculate a money market yield for a bond, exactly as we did for a medium-term CD in Chapter 2. An alternative is to compromise between the two approaches by using the bond approach for (1) above and the CD approach for (2) and (3). The result is to adjust the basic bond dirty price formula to the following:

Calculation summary

Money market yield for a bond

$$P = \frac{100}{\left(1 + \frac{i}{n} \times W\right)} \left[\frac{R}{n} \times \frac{\left(1 - \frac{1}{\left(1 + \frac{i}{n} \times \frac{365}{360}\right)^N}\right)}{\left(1 - \frac{1}{\left(1 + \frac{i}{n} \times \frac{365}{360}\right)}\right)} + \frac{1}{\left(1 + \frac{i}{n} \times \frac{365}{360}\right)^{N-1}} \right]$$

where i and W are the yield and fraction of a coupon period on a money market basis rather than a bond basis.

Example 5.8

What is the money market yield for the same bond as used in Example 5.1, assuming that the appropriate money market convention is ACT/360?

We know that the dirty price is 111.4811. On an ACT/360 basis, the fraction of a coupon period from settlement to the next coupon is $\frac{64}{360}$ rather than $\frac{63}{360}$. We therefore have:

$$111.4811 = \frac{100}{\left(1 + i \times \frac{64}{360}\right)} \times \left(0.09 \times \frac{\left(1 - \frac{1}{\left(1 + i \times \frac{365}{360}\right)^6}\right)}{\left(1 - \frac{1}{\left(1 + i \times \frac{365}{360}\right)}\right)} + \frac{1}{\left(1 + i \times \frac{365}{360}\right)^5} \right)$$

The solution to this is that the money market yield is 7.878%.

This cannot be solved by the HP bond calculator. The HP does have an equation solver however (available only in algebraic mode rather than RPN

mode) which can be used to solve the formula as follows. Note that the equation solver respects the normal mathematical conventions for the order of operations, whereas the HP used normally in algebraic mode does not (operating instead in the order in which operations are entered, thus requiring the use of extra parentheses).

SOLVE □ ↓

PRICE = $100 \times (\text{CPN} \times (1 - 1 \div (1 + \text{YLD} \times 365 \div 360) \square \wedge N) \div (1 - 1 \div (1 + \text{YLD} \times 365 \div 360)) + 1 \div (1 + \text{YLD} \times 365 \div 360) \square \wedge (N - 1)) \div (1 + \text{YLD} \times \text{DAYS} \div 360)$

CALC

111.4811 PRICE

.09 CPN

6 N

64 DAYS

YLD

Moosmüller yield

The Moosmüller method of yield calculation (Figure 5.1) is used in some German markets. It is also used by the US Treasury (rather than the market) for US bonds. It is similar to the money market yield in the previous section, in that it calculates the final discounting for the next coupon date to settlement using simple interest rather than compound interest. The day/year convention however is not changed to a money market basis. The result is the following formula, which can be seen to be a hybrid between the formulas for yield to maturity and money market yield.

$$P = \frac{100}{(1 + \frac{i}{n} \times W)} \left[\frac{R}{n} \times \frac{\left(1 - \frac{1}{(1 + \frac{i}{n})^N}\right)}{\left(1 - \frac{1}{(1 + \frac{i}{n})}\right)} + \frac{1}{(1 + \frac{i}{n})^{N-1}} \right]$$

Calculation
summary

**Comparison of yield to maturity with Moosmüller yield
for a 10% coupon bond priced at par**

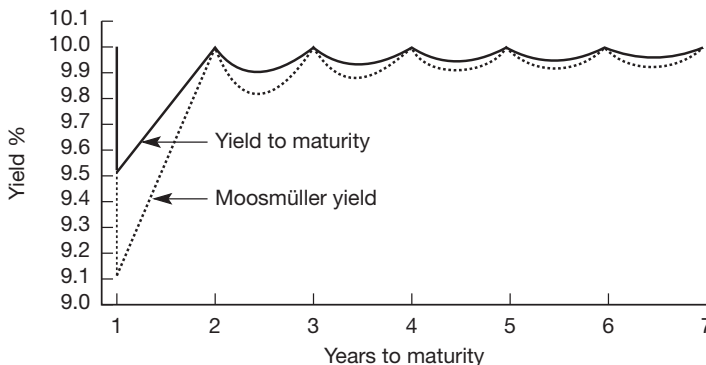


Fig 5.2

Zero-coupon and strip prices

Pricing a zero-coupon bond or a single component of a stripped bond, is similar in concept to pricing a coupon-bearing bond except that the fraction of an interest period used for discounting – generally taken as the time to the next scheduled coupon date for a coupon-bearing bond – must be determined. It is appropriate to use the date which would be the next coupon date if it existed – known as a “quasi-coupon date”. There remain then the same choices as with coupon-bearing bonds: which day/year convention to use, whether to use simple interest for short maturities, and whether to use a mixture of simple and compound interest for longer maturities. As there is no coupon, the clean price and dirty price are the same.

Example 5.9

What is the price of the following zero-coupon bond?

Maturity date:	28 September 2005
Settlement date:	15 January 1998
Yield:	8.520%
Price / yield calculation basis:	ACT/ACT (semi-annual)

As the quasi-coupon periods are semi-annual, the next quasi-coupon date is 28 March 1998 and the previous one was 28 September 1997 (despite the fact that these are Saturday and Sunday respectively). On an ACT/ACT basis therefore, the fraction of a quasi-coupon period from settlement to the next quasi-coupon date is $\frac{72}{181}$. The price is therefore:

$$\frac{100}{\left(1 + \frac{0.0852}{2}\right)^{\frac{72}{181} + 15}} = 52.605014$$

This is exactly the same as the conventional general bond price formula given earlier, with R (coupon rate) = 0, n (coupon frequency) = 2 and ACT/ACT semi-annual as the day/year convention.

```
.0852 ENTER 2 ÷ 1 +
72 ENTER 181 ÷ 15 + □ ^
100 □ x ≥ y ÷
```

OR

```
FIN BOND TYPE A/A SEMI EXIT
15.011998 SETT
28.092005 MAT
0 CPN%
MORE 8.52 YLD%
PRICE
```

Because the quasi-coupon dates are used as reference points for discounting to a present value but do not reflect actual cashflows, two identical zero-coupon bonds stripped from different coupon-bearing bonds could appear to have inconsistent yields.

Example 5.10

Suppose that the bond in Example 5.9 is in fact a coupon stripped from a UK gilt (semi-annual coupons). Consider another zero-coupon bond in the same currency with the same maturity date and face value, but which is in fact a coupon stripped from a French government bond (annual coupons). What is the yield of this bond if the price is the same as the bond in Example 5.9?

The fraction of a period to the next quasi-coupon date (28 September 1998) is now $\frac{256}{365}$. The price is therefore given by:

$$52.605014 = \frac{100}{(1 + \text{yield})^{\frac{256}{365} \times 7}}$$

Solving for the yield gives 8.699%.

Continuing from the previous example:

MORE TYPE ANN EXIT

MORE YLD%

OR

52.605014 ENTER 100 ÷

256 ENTER 365 ÷ 7 + \square $1/x$ \square \wedge

\square $1/x$ 1 –

We would not expect the yields in Examples 5.9 and 5.10 to be the same, because one is semi-annual and the other is annual. However, the usual conversion from semi-annual to annual will not bring them in line:

$$\left(1 + \frac{0.0852}{2}\right)^2 - 1 = 8.701\%$$

In this case, the annual equivalent of the yield quoted for the gilt strip is greater than the yield quoted for the identical French strip. The reason is the uneven division of days between the semi-annual quasi-coupon periods (181 days, then 184 days etc.).

A SUMMARY OF THE VARIOUS APPROACHES TO PRICE / YIELD

The different conventions used in different markets to relate price and yield are just that – different conventions. They should not affect the economics of an instrument. If the market convention is to trade a particular instrument in terms of yield rather than price, the investor must first convert this yield to a price using the appropriate conventions. The economics of the investment are then determined by what the price actually is and what the future cashflows are. From these, to compare two investments, the investor can ignore the yield quoted by the market and use a single consistent approach of his/her choice to calculate yields for comparison. In the next chapter we see how, in reverse, he/she can calculate a price for each investment using zero-coupon yields; again, this can be done consistently, ignoring market conventions.

Summarising the issues we have already seen, the following factors need to be considered.

Day/year convention for accrued coupon

This is ACT/365 (for example, Norway), ACT/ACT (France), 30(E)/360 (Germany) or 30(A)/360 (US federal agency).

Day/year convention for discounting cashflows to the dirty price

This is often the same as for accrued coupon, but may not be (Italy). For a consistent calculation disregarding market convention, ACT/365.25 might be used to compensate on average for the distorting effect of leap years.

Adjustment for non-working days

Conventional bond calculations generally ignore the effect of non-working days, assuming coupons are always paid on the regular scheduled date (even if this is not a working day). This approach is not taken with medium-term CDs. The UK Stock Exchange calculations for gilt prices for example however, have in the past discounted to the dirty price using actual payment dates for the cashflows.

Simple interest v. compound interest

In some markets, the yield for a bond in its final coupon period is calculated on the basis of simple interest (US). For bonds with more than one coupon remaining, compound interest is usual, although it is possible for example to take simple interest for the first fractional coupon period compounded with periodic interest thereafter (the US Treasury's calculation method for new issues). For a medium-term CD, simple interest discount factors for each period are compounded together.

Compounding method

It is usual to discount bond cashflows by compounding in "round" years. Using the same notation as earlier in this chapter, the discount factor is $\left(\frac{1}{1 + \frac{i}{n}}\right)^{W + \text{a number of whole coupon periods}}$. A more precise approach is to use the total exact number of days to each cashflow rather than a round number of years: a factor of $\left(1 + \frac{i}{n}\right)^{\frac{\text{total days to cashflow}}{\text{year}} \times n}$. For medium-term CDs, the approach is to compound each exact time separately.

Other considerations

One basic question is whether a yield is quoted annually, semi-annually or quarterly. The usual convention is to quote an annual yield if the coupons are paid annually, a semi-annual yield if the coupons are paid semi-annually etc. Care would need to be taken for example in comparing a semi-annual

yield for a UK gilt paying semi-annual coupons with a quarterly yield for a gilt paying quarterly coupons.

The concept of a bond yield as an internal rate of return implies that all coupons are also reinvested at the yield. An alternative is to assume a particular reinvestment rate (or series of different reinvestment rates). The coupon cashflows are then all reinvested at this rate until maturity. The yield is then the zero-coupon yield implied by the total future value accumulated in this manner, and the initial bond price.

DURATION, MODIFIED DURATION AND CONVEXITY

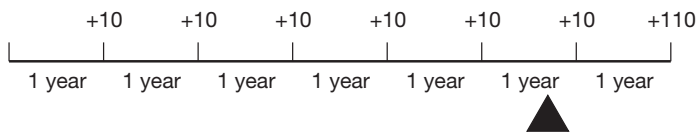
Duration

The maturity of a bond is not generally a good indication of the timing of the cashflows arising from the bond, because a significant proportion of the cashflows may occur before maturity in the form of coupon payments, and also possibly partial redemption payments.

One could calculate an average of the times to each cashflow, weighted by the size of the cashflows. Duration is very similar to such an average. Instead of taking each cashflow as a weighting however, duration takes the present value of each cashflow.

Example 5.11

Consider the same 7-year 10% coupon bond as in Example 5.5, with a price of 95.00 and a yield of 11.063%. The size and timing of the cashflows of the bond can be shown as follows:

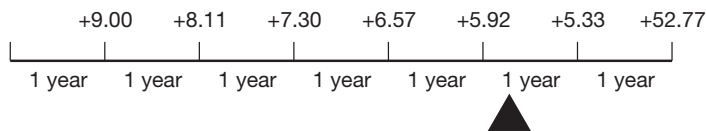


The average time to the cashflows weighted by the cashflows themselves would be calculated as:

$$\begin{aligned}
 & \frac{\sum (\text{cashflow} \times \text{time to cashflow})}{\text{sum of the cashflows}} \\
 &= \frac{(10 \times 1) + (10 \times 2) + (10 \times 3) + (10 \times 4) + (10 \times 5) + (10 \times 6) + (110 \times 7)}{10 + 10 + 10 + 10 + 10 + 10 + 110} \text{ years} \\
 &= \frac{980}{170} \text{ years} = 5.76 \text{ years}
 \end{aligned}$$

If you consider the diagram above as a beam with weights of 10, 10, . . . 110 placed on it, the point 5.76 along the beam is the point at which the beam would be balanced.

Now consider the same averaging process but instead using the present value of each cashflow (discounted at the yield of 11.063%). These present values are as follows:



The weighted average is now:

$$\frac{(9.00 \times 1) + (8.11 \times 2) + (7.30 \times 3) + (6.57 \times 4) + (5.92 \times 5) + (5.33 \times 6) + (52.77 \times 7)}{(9.00 + 8.11 + 7.30 + 6.57 + 5.92 + 5.33 + 52.78)} \text{ yrs}$$

$$= \frac{504.37}{95.00} \text{ years} = 5.31 \text{ years}$$

The duration of the bond is thus 5.31 years.

Note that the lower half of the calculation ($9.00 + 8.11 \dots + 52.77$) is simply the price of the bond, because it is the NPV of the cashflows.

```
10 ENTER 1.11063 ÷
10 ENTER 1.11063 ENTER 2  $\square \wedge \div 2 \times +$ 
10 ENTER 1.11063 ENTER 3  $\square \wedge \div 3 \times +$ 
10 ENTER 1.11063 ENTER 4  $\square \wedge \div 4 \times +$ 
10 ENTER 1.11063 ENTER 5  $\square \wedge \div 5 \times +$ 
10 ENTER 1.11063 ENTER 6  $\square \wedge \div 6 \times +$ 
110 ENTER 1.11063 ENTER 7  $\square \wedge \div 7 \times +$ 
95 ÷
```

Calculation summary

$$\text{Duration} = \frac{\sum (\text{present value of cashflow} \times \text{time to cashflow})}{\sum (\text{present value of cashflow})}$$

It is worth noting that for a zero-coupon bond, there is only one cashflow, at maturity. The duration of a zero-coupon bond is therefore the same as its maturity.

The concept of duration can be extended to any series of cashflows, and hence to a portfolio of investments.

Duration is useful partly because of its relationship with the price sensitivity of a bond (see modified duration below) and partly because of the concept of investment “immunization.”

If I invest in a bond and there is a fall in yields (both short-term and long-term), there are two effects on my investment. First, I will not be able to earn as much as I had expected on reinvesting the coupons I receive. As a result, if I hold the bond to maturity, my total return will be less than anticipated. The price of the bond will however rise immediately (yield down, price up). If I hold the bond for only a very short time therefore, my total return will be more than anticipated because I will not have time to be affected by lower reinvestment rates. There must be some moment between now and the bond’s maturity when these two effects – the capital gain from the higher bond price and the loss on reinvestment – are in balance and the total return is the same yield as originally anticipated. The same would be true if yields rise – there is some point at which the capital loss due to the higher bond price would be balanced by the reinvestment gains.

Suppose that an investor wishes to be sure of the total return on his/her portfolio between now and a particular time in the future, regardless of interest rate movements. It can be shown that, if he/she arranges the portfolio to have a duration equal to that period (rather than have a maturity equal to it), he will then not be vulnerable to yield movements up or down during that period – the portfolio will be “immunized”.

There are practical problems with this concept. First, the idea assumes that short-term reinvestment rates and long-term bond yields move up or down together. Second, the portfolio’s duration will change as its yield changes, because the calculation of duration depends on the yield. In order to keep the portfolio’s duration equal to the time remaining up to his/her particular investment horizon, and so remain immunized, the investor therefore needs to adjust the portfolio continually by changing the investments.

Modified duration

It is useful to know the sensitivity of a bond price to yield changes – that is, if a yield rises by a certain amount, how much will the bond’s price fall? The answer can be seen as depending on how steeply the price / yield curve slopes (see Figure 5.3).

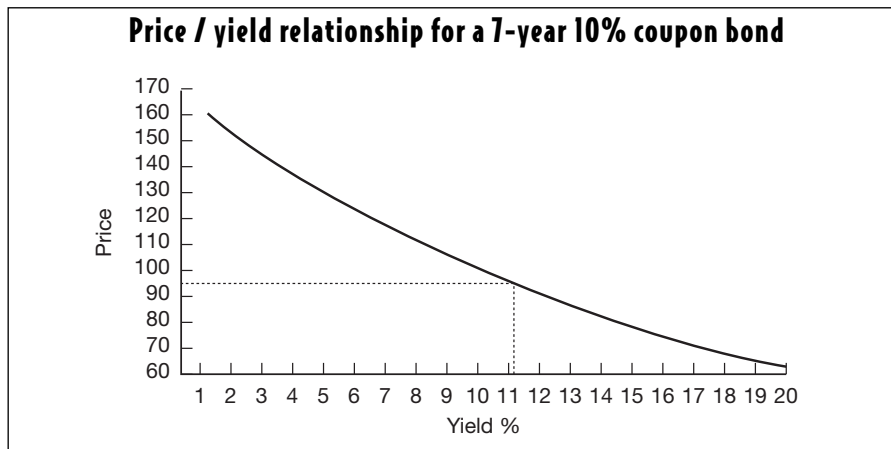


Fig 5.3

If the curve is very steeply sloped, then a given move up in the yield will cause a sharp fall in the price from 95.00. If the curve is not steeply sloped, the price fall will be small. This gives the following approximation:

$$\text{change in price} = \text{change in yield} \times \text{slope of curve}$$

We are probably more interested in how much the value of our investment changes relative to the size of the investment (rather than as an absolute amount) – that is, $\frac{\text{change in price}}{\text{dirty price}}$. This must therefore be equal to:

$$\text{change in yield} \times \frac{\text{slope of curve}}{\text{dirty price}}$$

Mathematically, the slope of the curve is $\frac{dP}{di}$. By taking the general bond price

formula:

$$P = \sum_k \frac{C_k}{\left(1 + \frac{i}{n}\right)^{\frac{d_k \times n}{\text{year}}}}$$

and differentiating, we get:

$$\frac{dP}{di} = - \frac{\sum \frac{C_k}{\left(1 + \frac{i}{n}\right)^{\frac{d_k \times n}{\text{year}}}} \times \frac{d_k}{\text{year}}}{\left(1 + \frac{i}{n}\right)}$$

Comparing this with the formula for duration in the previous section, it can be seen that:

$$\frac{\text{slope of curve}}{\text{dirty price}} = \frac{\frac{dP}{di}}{\text{dirty price}} = - \frac{\text{duration}}{\left(1 + \frac{i}{n}\right)}$$

Therefore it can be seen that the sensitivity factor relating $\frac{\text{change in price}}{\text{dirty price}}$ to $-(\text{change in yield})$ is $\frac{\text{duration}}{\left(1 + \frac{i}{n}\right)}$. Because this factor is so similar to duration, it is known as “modified duration.”

Calculation summary

$$\text{Modified duration} = \frac{-\frac{dP}{di}}{\text{dirty price}} = \frac{\text{duration}}{\left(1 + \frac{i}{n}\right)}$$

So that we have:

Calculation summary

Approximation

$$\text{Change in price} \approx - \text{dirty price} \times \text{change in yield} \times \text{modified duration}$$

In some markets, modified duration is known as “volatility”.

Example 5.12

Consider the same bond as before, and assume a 1% rise in the yield from 11.063% to 12.063%.

$$\frac{5.31}{\left(1 + 0.11063\right)} = 4.78 \text{ years}$$

As the duration is 5.31 years and $n = 1$ (coupons are annual), the modified duration is:

Using the price sensitivity, we know:

$$\begin{aligned} \text{Change in price} &\approx - \text{dirty price} \times \text{change in yield} \times \text{modified duration} \\ &= -95.00 \times 0.01 \times 4.78 = -4.54 \end{aligned}$$

We should therefore expect the price to fall from 95.00 to $95.00 - 4.54 = 90.46$.

If now we check this by repricing at 12.063% (for example, by using the HP's TVM function), we find that the bond's price has actually fallen to 90.60.

Dollar value of an 01

A measure very closely related to modified duration is the “dollar value of an 01” (DV01) – or the “present value of an 01” (PV01), or “the value of a basis point”. This is simply the change in price due to a 1 basis point change in yield – usually expressed as a positive number. From the above, it can be seen that:

$$\text{DV01} = \text{modified duration} \times \text{dirty price} \times 0.0001$$

**Calculation
summary**

Convexity

For small yield changes, the approximation above is fairly accurate. For larger changes, it becomes less accurate. The reason that the modified duration did not produce exactly the correct price change in the last example is that the slope of the curve changes as you move along the curve. The equation:

$$\text{change in price} = \text{change in yield} \times \frac{dP}{di}$$

in fact calculates the change along the straight line shown in Figure 5.4 rather than along the curve.

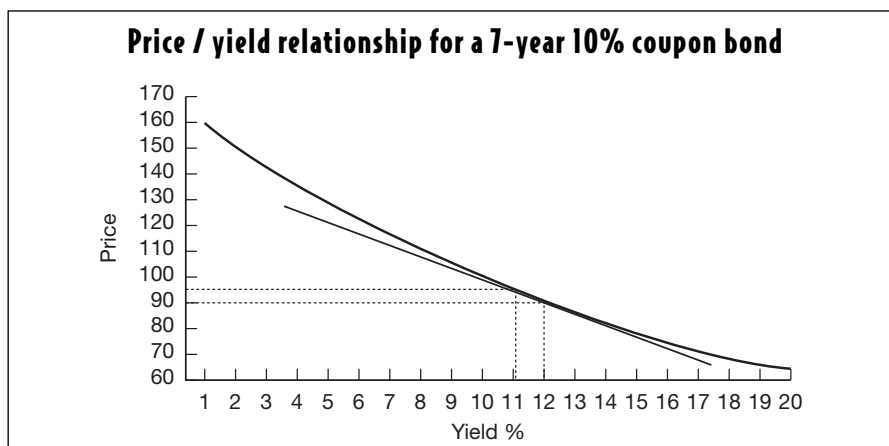


Fig 5.4

As a result, using modified duration to calculate the change in price due to a particular change in yield will generally underestimate the price. When the yield rises, the price does not actually fall as far as the straight line suggests; when the yield falls, the price rises more than the straight line suggests. The difference between the actual price and the estimate depends on how curved

the curve is. This amount of curvature is known as “convexity” – the more curved it is, the higher the convexity.

Mathematically, convexity is defined as:

Key Point

$$\text{Convexity} = \frac{d^2P}{di^2} \bigg/ \text{dirty price}$$

By applying this to the bond price formula, we get:

Calculation summary

$$\text{Convexity} = \frac{\sum \left[\frac{C_k}{\left(1 + \frac{i}{n}\right)^{\frac{d_k}{\text{year}} + 2}} \times \frac{d_k}{\text{year}} \times \left(\frac{d_k}{\text{year}} + \frac{1}{n} \right) \right]}{\text{dirty price}}$$

Using convexity, it is possible to make a better approximation of the change in price due to a change in yield:

Calculation summary

Better approximation

$$\text{Change in price} \approx - \text{dirty price} \times \text{modified duration} \times \text{change in yield} + \frac{1}{2} \text{dirty price} \times \text{convexity} \times (\text{change in yield})^2$$

Example 5.13

Using the same details again as in the last example, the bond's convexity can be calculated as:

$$\left(\frac{10}{(1.11063)^3} \times 2 + \frac{10}{(1.11063)^4} \times 6 + \frac{10}{(1.11063)^5} \times 12 + \frac{10}{(1.11063)^6} \times 20 + \frac{10}{(1.11063)^7} \times 30 + \frac{10}{(1.11063)^8} \times 42 + \frac{110}{(1.11063)^9} \times 56 \right) \bigg/ 95.00$$

$$= 31.08$$

A 1% rise in the yield would then give the following approximate change in price:

$$-95 \times 4.78 \times .01 + \frac{1}{2} \times 95 \times 31.08 \times (.01)^2 = -4.39$$

We should therefore now expect the price to fall from 95.00 to $95.00 - 4.39 = 90.61$ – which is very close to the actual price of 90.60.

For bonds without calls, it can be shown in general that:

- for a given yield and maturity, a higher coupon rate implies lower duration and convexity
- for a given coupon rate and maturity, a higher yield implies lower duration and convexity.

In Example 5.11 convexity is a positive number. This is always true for a straightforward bond – that is, the shape of the price / yield curve is roughly as shown in Figure 5.4. It is possible, however, for convexity to become negative at some point along the curve. Suppose, for example, that the bond issuer has the choice of redeeming the bond early if market yields fall below a

certain level. In that case, the price will cease rising beyond a certain point as yields fall. The result is a reversal of the curvature at low yields. Mortgage-backed securities, where the home-owners are more likely to repay mortgages and refinance as yields fall, can provide a similar situation.

In general, high positive convexity is good from an investor's point of view. If two bonds have equal price, yield and duration but different convexities, the bond with higher convexity will perform relatively better if the yield changes. In practice, therefore, the two bonds should not be priced the same. In the same way, when hedging a portfolio, an investor should try to ensure higher convexity in his/her long positions and lower convexity in short positions.

Portfolio duration

As mentioned above, the concept of duration – and also modified duration and convexity – can be applied to any series of cashflows, and hence to a whole portfolio of investments rather than to a single bond.

To calculate precisely a portfolio's duration, modified duration or convexity, the same concept should be used as for a single bond, using all the portfolio's cashflows and the portfolio's yield (calculated in the same way as for a single bond). In practice, a good approximation is achieved by taking a weighted average of the duration etc. of each investment:

Approximations for a portfolio

$$\text{Duration} = \frac{\sum (\text{duration of each investment} \times \text{value of each investment})}{\text{value of portfolio}}$$

$$\text{Modified duration} = \frac{\sum (\text{mod. dur. of each investment} \times \text{value of each investment})}{\text{value of portfolio}}$$

$$\text{Convexity} = \frac{\sum (\text{convexity of each investment} \times \text{value of each investment})}{\text{value of portfolio}}$$

**Calculation
summary**

A portfolio's modified duration, for example, gives the sensitivity of the whole portfolio's value to yield changes. Although this has the limitation that it assumes all yields move up or down in parallel along the yield curve, it can provide a useful quick measure of risk. With the same limitation, an organization wishing to avoid any such risk can match the modified durations of its liabilities and its assets.

Example 5.14

We own the following portfolio:

	<i>Face value</i>	<i>Dirty price</i>	<i>Modified duration</i>
Bond A	10 million	107.50	5.35
Bond B	5 million	98.40	7.20
Bond C	7 million	95.25	3.45

How much of the following bond should we short to make the portfolio immune to small changes in yield, assuming parallel movements in the yield curve?

	<i>Dirty price</i>	<i>Modified duration</i>
Bond D	110.20	9.75

For any small change in yield, the change in value of our portfolio is approximately:

– change in yield ×

$$\left[\left(10 \text{ mln} \times \frac{107.50}{100} \times 5.35 \right) + \left(5 \text{ mln} \times \frac{98.40}{100} \times 7.20 \right) + \left(7 \text{ mln} \times \frac{95.25}{100} \times 3.45 \right) \right]$$

We therefore need to short enough of bond D to have an offsetting effect. The above change in value therefore needs to equal:

$$- \text{change in yield} \times \left(\text{face value of bond D} \times \frac{110.20}{100} \times 9.75 \right)$$

Therefore face value of bond D to be sold =

$$\frac{(10 \text{ mln} \times 1.075 \times 5.35) + (5 \text{ mln} \times 0.984 \times 7.2) + (7 \text{ mln} \times 0.9525 \times 3.45)}{1.102 \times 9.75}$$

= 10.8 million

10 ENTER 1.075 × 5.35 ×
5 ENTER .984 × 7.2 × +
7 ENTER .9525 × 3.45 × +
1.102 ÷ 9.75 ÷

BOND FUTURES

A bond futures contract is generally an agreement whereby the seller must deliver to the buyer an agreed amount of a bond at an agreed time, for an agreed price. In practice, bond futures contracts are generally closed out before maturity in the same way that short-term futures contracts are, and the profit / loss is captured through variation margins. In the case of most bond futures contracts however, a bond futures buyer can in theory insist on delivery of a bond at maturity of the contract. There are several complications which do not arise in the case of short-term interest rate futures.

Bond specification

A bond futures contract must be based on a precisely specified bond, in the same way that a short-term interest rate futures contract is generally based precisely on a 3-month deposit. It may be preferable, however, to allow any one of a range of existing bonds – which can change over time – to be delivered at maturity of the futures contract. Bond futures prices are therefore usually based on a *notional* bond index rather than any one of these particu-

lar bonds. In the case of a US treasury bond futures, for example, the bond specified is a fictional 8 percent bond of at least 15 years' maturity.

Deliverable bonds and conversion factors

The seller of the futures contract cannot of course deliver this fictional bond. Instead, he is usually entitled to deliver any one of a range of bonds which is defined in the specifications for the futures contract. In this case, it is the seller who chooses which bond to deliver to the buyer. In the case of US treasury bond futures, for example, the seller may deliver any bond maturing at least 15 years after the first day of the delivery month if the bond is not callable. If it is callable, the earliest call date must be at least 15 years after the first day of the delivery month.

Because the different deliverable bonds have different coupons and maturities, they need to be put on a common basis. The futures exchange therefore publishes a “price factor” or “conversion factor” for each deliverable bond. The amount paid by the buyer if the bond is delivered then depends on this conversion factor. In the case of a treasury bond, this is the price per \$1 nominal value of the specific bond at which the bond has a gross redemption yield of 8 percent on the first day of the delivery month (i.e. it has the same yield as the coupon of the fictional bond underlying the contract). The maturity of the deliverable bond is found by measuring the time from the first day of the delivery month to the maturity of the bond (first call day if callable) and rounding down to the nearest quarter.

Conversion factor =
clean price at delivery for one unit of the deliverable bond, at which that
bond's yield equals the futures contract notional coupon rate

Key Point

On the delivery day, the specific bond nominated by the seller will be delivered and the seller will receive from the buyer the relevant invoicing amount. The invoicing amount is based on the “Exchange Delivery Settlement Price” (EDSP – the futures price at the close of trading for that futures contract, as determined by the exchange) and the size of the futures contract (\$100,000 in the case of a US treasury bond futures):

$$\text{Invoicing amount} = \text{face value} \times \left(\frac{\text{EDSP}}{100} \times \text{conversion factor} + \text{accrued coupon rate} \right)$$

This choice of deliverable bonds also gives rise to the concept of “cheapest to deliver”: the seller will always choose to deliver whichever bond is the cheapest for him to do so – known as the “cheapest to deliver” or “CTD” bond.

Key Point

A bond futures contract is based on a notional bond. The real bonds deliverable into the contract are made comparable by their conversion factors

Delivery date

Delivery specifications vary between futures contracts and exchanges. In the case of a US treasury bond futures, for example, the seller is entitled to deliver on any business day in the delivery month. Clearly he will deliver later if the coupon he is accruing is higher than the cost of funding the position and earlier if the coupon is lower.

Coupons

If there is an intervening coupon payment on the actual bond which the futures seller expects to deliver, he will take this into account in calculating the futures price.

Pricing

The theoretical futures price depends on the elimination of arbitrage. The seller of a futures contract, if he delivers a bond at maturity of the futures contract, will receive the invoicing amount on delivery. He will also receive any intervening coupon plus interest earned on this coupon between receipt of the coupon and delivery of the bond to the futures buyer.

In order to hedge themselves, the futures sellers must buy the bond in the cash market at the same time as they sell the futures contract. For this they must pay the current bond price plus accrued coupon. This total amount must be funded from the time they buy the bond until the time they deliver it to the futures buyer.

By delivery, the market futures price (which becomes the EDSP at the close of the contract) should converge to the cash market price of the CTD bond divided by the conversion factor. If this were not so, there would be an arbitrage difference between the invoicing amount for the CTD at delivery and the cost of buying the CTD bond in the cash market at the same time. During the period from selling the futures contract to delivery, the futures seller pays or receives variation margin (as with short-term interest rate futures) based on the notional amount of the futures contract. These variation margin cashflows represent the difference between the price at which he sold the futures contract and the EDSP.

It can be seen from the cashflows shown below that, in order to achieve a hedge which balances correctly, the nominal amount of cash bond sold should be $\frac{\text{notional size of futures contract}}{\text{conversion factor}}$. If the buyer requires delivery of the bond, the seller will need to buy or sell a small amount of the cash bond at maturity – the difference between the notional futures amount, which must be delivered, and the amount already bought as the hedge. This will be done at the

price of the cash bond at maturity, which should converge to (EDSP × conversion factor), as noted above.

Assuming that the futures seller makes zero profit / loss, the cashflows received and paid by the futures seller should net to zero. For any given notional face value of futures, these cashflows are as follows:

Receive on delivery of the bond

$$\text{face value} \times \left(\frac{\text{EDSP}}{100} \times \text{conversion factor} + \text{accrued coupon rate at delivery} \right)$$

Receive as variation margin

$$\text{face value} \times \left(\frac{\text{futures price} - \text{EDSP}}{100} \right)$$

Receive any intervening coupon

$$\frac{\text{face value}}{\text{conversion factor}} \times (\text{coupon rate} + \text{reinvestment})$$

Pay the total cost of funding the cash bond purchase

$$\frac{\text{face value}}{\text{conversion factor}} \times \left(\frac{\text{cash bond price}}{100} + \text{accrued coupon rate at start} \right) \times \left(1 + \text{funding rate} \times \frac{\text{days to delivery}}{\text{year}} \right)$$

Pay (or receive) the cost of the difference between the amount of bond hedged, and the amount of the bond to be delivered to the buyer

$$\left(\text{face value} - \frac{\text{face value}}{\text{conversion factor}} \right) \times \left(\frac{\text{EDSP} \times \text{conversion factor}}{100} + \text{accrued coupon rate at delivery} \right)$$

From this, it follows that:

$$\begin{aligned} &\text{Theoretical bond futures price} = \\ &\frac{\left(\left[\text{bond price} + \text{accrued coupon now} \right] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] - \right.}{\text{conversion factor}} \\ &\quad \left. - (\text{accrued coupon at delivery}) - (\text{intervening coupon reinvested}) \right) \\ &\text{where } i = \text{short-term funding rate} \end{aligned}$$

**Calculation
summary**

Example 5.15

What is the theoretical September T-bond futures price on 18 June if the cheapest-to-deliver bond is the Treasury 12 $\frac{3}{4}$ % 2020, trading at 142-09 (i.e. 142 $\frac{9}{32}$)? The conversion factor for the 2020 is 1.4546. Coupon dates are 15 May and 15 November. Short-term funds can be borrowed at 6.45%.

Answer:

Payment for the bond purchased by the futures seller to hedge himself is made on

$$\text{Accrued coupon now} = 12.75 \times \frac{35}{368} = 1.212636$$

19 June. Coupon on the purchase of the bond is accrued for 35 days. The current coupon period is 184 days. Therefore:

Assume that delivery of the bond to the futures buyer requires payment to the futures seller on 30 September. The futures seller must then fund his position from 19 June to 30 September (103 actual days) and coupon on the sale of the bond will then be accrued for 138 days.

Therefore:

$$\text{Accrued coupon at delivery} = 12.75 \times \frac{138}{368} = 4.781250$$

$$\begin{aligned} \text{Theoretical futures price} &= \frac{(142.281250 \times 1.212636) \times (1 + 0.0645 \times \frac{103}{360}) - 4.781250}{1.4546} \\ &= 97.1818 \\ &= 97-06 \text{ (i.e. } 97 \frac{6}{32} \text{)} \end{aligned}$$

12.75 ENTER 35 × 368 ÷	(Accrued interest at start)
142.28125 +	(Clean bond price at start)
.0645 ENTER 103 × 360 ÷ 1 + ×	
12.75 ENTER 138 × 368 ÷	(Accrued interest at end)
–	
1.4546 ÷	(Theoretical futures price)

The construction above of a theoretical futures price does not take account of the fact that the seller of a bond futures contract has a choice of which bond to deliver. This effectively gives the seller an option built into the futures contract. This optionality has a value, which should in general be reflected in a slightly lower theoretical futures price than the formula above suggests.

Forward bond prices

The arbitrage method used above to calculate a theoretical bond futures price could equally well be used to calculate a forward market price for a bond which is to be delivered on any date later than the normal convention – that is, a forward bond price. In this case, there is no conversion factor involved.

**Calculation
summary**

$$\begin{aligned} \text{Forward bond price} &= \\ &= \left(\left[\text{bond price} + \text{accrued coupon now} \right] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] \right) \\ &\quad - (\text{accrued coupon at delivery}) - (\text{intervening coupon reinvested}) \end{aligned}$$

Ignoring differences between the day/year conventions for the bond and the short-term financing, the formula above can be rewritten as:

$$\begin{aligned}
 &(\text{forward price} - \text{cash price}) = \\
 &\text{cash price} \times \left(i - \frac{\text{coupon rate}}{\text{cash price} / 100} \right) \times \frac{\text{days to delivery}}{\text{year}} \\
 &+ \left[\text{accrued coupon now} \times i \times \frac{\text{days}}{\text{year}} - \text{interest earned on intervening coupon} \right]
 \end{aligned}$$

The expression in square brackets is rather small, so that in general, the amount (forward price – cash price) is positive or negative – that is, the forward price is at a premium or a discount to the cash price – if $\left(i - \frac{\text{coupon rate}}{\text{cash price} / 100} \right)$ is positive or negative. This is reasonable, because it reflects whether the cost of funding a long bond position is greater or less than the coupon accruing on the position.

Generally, a forward bond price is at a premium (discount) to the cash price if the short-term funding cost is greater than (less than) $\frac{\text{coupon rate}}{\text{cash price} / 100}$.

Key Point

Using futures to hedge a cash position

If a dealer takes a cash position in bonds, he can use bond futures to hedge his position in the same way that a cash position in short-term interest rates can be hedged by interest rate futures. The commodity traded in a bond futures contract – the notional bond – will however behave differently from any particular cash bond. As a result, the notional amount of the futures hedge needs to be different from the face value of the cash position.

Suppose that a dealer takes a position in the CTD bond. We know that the theoretical futures price is given by:

$$\begin{aligned}
 &\text{futures price} = \\
 &\frac{\left([\text{bond price} + \text{accrued coupon now}] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] \right) - (\text{accrued coupon at delivery}) - (\text{intervening coupon reinvested})}{\text{conversion factor}}
 \end{aligned}$$

If the price of the CTD changes therefore, the instantaneous change in the futures price is found by differentiating the above formula:

$$\begin{aligned}
 \text{change in futures price} &= \frac{\text{change in CTD price} \times \left(1 + i \times \frac{\text{days}}{\text{year}} \right)}{\text{conversion factor}} \\
 \text{or, change in CTD price} &= \text{change in futures price} \times \left(\frac{\text{conversion factor}}{1 + i \times \frac{\text{days}}{\text{year}}} \right)
 \end{aligned}$$

In order to hedge a position in \$100 face value of the CTD bond therefore, the dealer should take an opposite position in $\$100 \times \left(\frac{\text{conversion factor}}{1 + i \times \frac{\text{days}}{\text{year}}} \right)$ nominal of the futures contract. This will still leave the risks that the actual futures price will not move exactly in line with the CTD bond, and that the CTD bond will change, but it will provide an approximate hedge.

The factor $\left(\frac{\text{conversion factor}}{1 + i \times \frac{\text{days}}{\text{year}}} \right)$ is known as a hedge ratio – the necessary ratio of the size of the futures hedge to the size of the underlying position.

We could reverse this concept to say that a dealer could hedge a futures position by an opposite position in the CTD bond. The amount of the CTD bond required is then:

$$\text{notional amount of futures contract} \times \frac{\left(1 + i \times \frac{\text{days}}{\text{year}} \right)}{\text{conversion factor}}$$

Note that this is not exactly the same as the amount of CTD bond we used in establishing the theoretical futures price in the previous section: the amount there was simply $\frac{\text{notional amount of futures contract}}{\text{conversion factor}}$.

The difference arises because in establishing the theoretical futures price, we needed an arbitrage which we could hold to delivery – even though, in practice, futures contracts are rarely delivered. In this section, on the other hand, we are concerned about a hedge for an *instantaneous* change in price. If we held the hedge to delivery, the $\left(1 + i \times \frac{\text{days}}{\text{year}} \right)$ factor would converge to 1.

Example 5.16

A dealer wishes to hedge his short position in \$15 million face value of the bond which is currently the CTD for the US treasury bond futures contract. The conversion factor for the CTD is 1.1482. Assume that delivery of the futures contract is in 73 days and that the cost of short-term funding is 5.2%. The size of a US treasury bond futures contract is \$100,000 nominal. How should the dealer hedge his position?

$$\text{Hedge ratio} = \left(\frac{\text{conversion factor}}{1 + i \times \frac{\text{days}}{\text{year}}} \right) = \left(\frac{1.1482}{1 + 0.052 \times \frac{73}{360}} \right) = 1.1362$$

The dealer should therefore buy \$15 million \times 1.1362 = \$17,043,000 nominal of futures. As the nominal size of each futures contract is \$100,000, this involves buying $\frac{17,043,000}{100,000} = 170$ futures contracts.

If the dealer wishes to hedge a different bond, he will need to assess how that bond's price is likely to move compared with the CTD's price. For small yield changes, one approach is to use modified duration. Assuming that the yield on the actual bond purchased and the yield on the CTD move in parallel, we have:

$$\begin{aligned} \text{change in bond price} = \\ - \text{dirty bond price} \times \text{modified duration of bond} \times \text{yield change} \end{aligned}$$

and:

$$\begin{aligned} \text{change in price of CTD} = \\ - \text{dirty price of CTD} \times \text{modified duration of CTD} \times \text{yield change} \end{aligned}$$

so that:

$$\begin{aligned} \text{change in bond price} = \\ \text{change in price of CTD} \times \frac{\text{dirty bond price}}{\text{dirty price of CTD}} \times \frac{\text{modified duration of bond}}{\text{modified duration of CTD}} \end{aligned}$$

This gives a hedge ratio as follows:

$$\begin{aligned} \frac{\text{notional amount of futures contract required to hedge a position in bond A}}{\text{face value of bond A}} = \\ \frac{\text{dirty price of bond A}}{\text{dirty price of CTD bond}} \times \frac{\text{modified duration of bond A}}{\text{modified duration of CTD bond}} \\ \times \frac{\text{conversion factor for CTD bond}}{\left(1 + i \times \frac{\text{days}}{\text{year}}\right)} \\ \text{where } i = \text{short-term funding rate} \end{aligned}$$

**Calculation
summary**

Note that this hedge assumes, as with all modified duration hedging, that the yield curve shift is the same for all bonds – that is, that the yields on bond A and the CTD bond respond in the same way to market changes. It might be that even for a yield curve shift which is parallel in general, bond A in particular responds more sluggishly or less sluggishly than the market in general. Any expectation of this would need to be taken into account in the size of the hedge.

CASH-AND-CARRY ARBITRAGE

Overview of repos

“Repo” is short for “sale and repurchase agreement” and is essentially a transaction whereby the two parties involved agree to do two deals as a package. The first deal is a purchase or sale of a security – often a government bond – for delivery straight away (the exact settlement date will vary according to the market convention for the security involved). The second deal is a reversal of the first deal, for settlement on some future date.

Key Point

A repo is a purchase or sale of a security now, together with an opposite transaction later

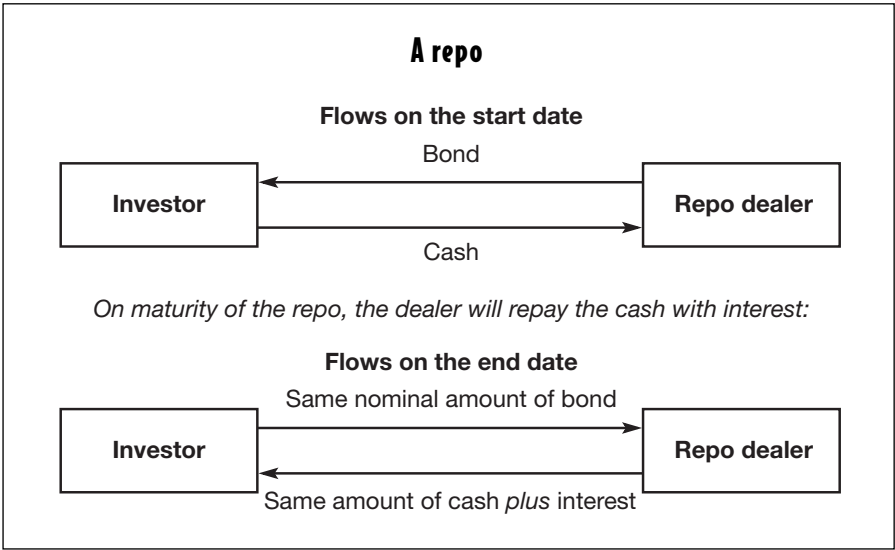
Because it is understood from the outset that the first deal will be reversed, it is clear that both parties intend the transfer of securities (in one direction) and the transfer of cash (in the other direction) to be temporary rather than permanent. The transaction is therefore exactly equivalent to a loan of securities in one direction and a loan of cash in the other. The repo is structured so that the economic benefit of owning the securities – income and capital gains / losses – remains with their original owner. These are in fact the driving forces behind the repo market; all repos are driven by either the need to lend or borrow cash, which is collateralized by securities – or the need to borrow specific securities. The prices for both the original sale and subsequent repurchase are agreed at the outset. The difference between the two prices is calculated to be equivalent to the cost of borrowing secured money.

Key Point

All repos are driven by either the need to borrow cash, or the need to borrow a specific security

A repo is defined as an initial sale of securities followed by a subsequent repurchase. A “reverse repo” is the opposite – an initial purchase of securities followed by a subsequent resale. Because the two parties involved are of course doing opposite transactions, a “repo” to one party is a “reverse” to the other. For this purpose, the deal is generally considered from the repo dealer’s point of view. If he is effectively borrowing cash, the deal is a repo; if he is effectively lending cash, the deal is a reverse. In a repo, the “seller” (or

Fig 5.5



“lender”) is the party selling securities at the outset and repurchasing them later. The “buyer” (or “borrower” or “investor”) is the other party. (See Figure 5.5.) It is important to note that the terminology is taken from the viewpoint of the bond market, not the money market: the party borrowing cash is usually known as the lender in the repo.

Price calculation

The total price at which the first leg of the repo is transacted is the straightforward current market price for the security, plus accrued coupon, taking into account any margin (see below) if agreed. The total price for the second leg, however, reflects only the repo interest rate and not the accrued coupon due on the security at that time. This is because the security is in reality only playing the part of collateral. The repo interest rate is calculated according to the normal convention in the relevant money market. On a DEM repo, for example, this would be calculated on an ACT/360 basis; this is unaffected by the fact that the coupon on the collateral might be calculated on a 30(E)/360 basis.

**The price on the first leg of a classic repo is the market price.
The price on the second leg is the first price plus the repo interest.**

Key Point

Example 5.17

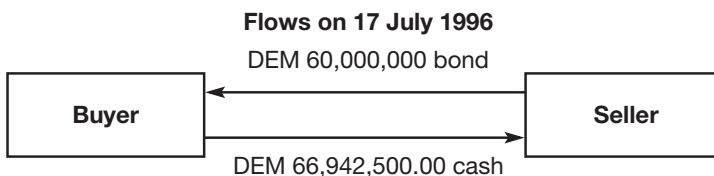
Currency:	DEM
Deal date:	15 July 1996
Settlement date:	17 July 1996
Term:	28 days (14 August 1996)
Repo rate:	4.0% (ACT/360 basis)
Collateral:	DEM 60,000,000 nominal 8.5% bond with maturity 26 March 2004 and annual coupons (30(E)/360 basis)
Clean bond price:	108.95

Clean price of bond for value 17 July 1996 is 108.95

$$\text{Accrued coupon on bond on 17 July 1996} = \frac{111}{360} \times 8.5 = 2.6208333$$

Total purchase price = 111.57083333

$$\text{Purchase amount} = \text{DEM } 60,000,000 \times \frac{111.57083333}{100} = \text{DEM } 66,942,500.00$$

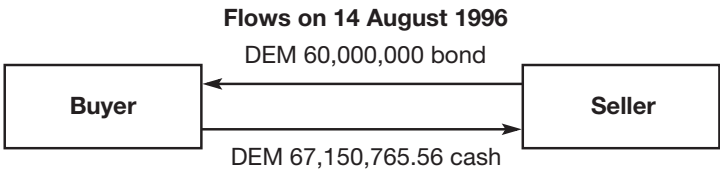


On maturity of the repo, the seller will repay the cash with interest calculated at 4.0%:

Principal = DEM 66,942,500.00

Interest = DEM 66,942,500.00 × 0.04 × $\frac{28}{360}$ = DEM 208,265.56

Total repayment = DEM 67,150,765.56



We can summarize these calculations as follows:

Calculation
summary

The cashflows in a classic repo

Cash paid at the beginning =

$$\text{nominal bond amount} \times (\text{clean price} + \text{accrued coupon}) / 100$$

Cash repaid at the end =

$$\text{cash consideration at the beginning} \times \left(1 + \text{repo rate} \times \frac{\text{repo days}}{\text{repo year}} \right)$$

Implied repo rate

When we constructed a theoretical bond futures price earlier, we did so by considering how the seller of a bond futures contract could hedge himself. This was by borrowing cash, using the cash to buy the bond and holding the bond until the futures contract matures and the bond is delivered against it. If the actual futures price is the same as the theoretical price, this “round trip” should give a zero result – no profit and no loss.

The same calculation can be considered in reverse: assuming that we already know the current futures price and the current bond price, what is the interest rate at which it is necessary to borrow the cash to ensure a zero result?

This interest rate is called the “implied repo rate” and is the break-even rate at which the futures sale can be hedged. The reason for the name “implied repo rate” is that in order to borrow the money to buy the bond, the dealer can repo the bond out. It is thus the “repo borrowing rate implied by the current futures price.” The “cheapest to deliver” bond will generally be the one with the highest implied repo rate (because any other deliverable bond will require a lower repo rate to break even).

Taking our earlier formula and reversing it, we get:

$$\text{Implied repo rate} = \left[\frac{(\text{futures price} \times \text{conversion factor}) + (\text{accrued coupon at delivery of futures}) + (\text{intervening coupon reinvested})}{(\text{bond price} + \text{accrued coupon now})} - 1 \right] \times \frac{\text{year}}{\text{days}}$$

This can also be expressed as:

$$\left[\frac{\text{total cash received at delivery}}{\text{initial cash expenditure}} - 1 \right] \times \frac{\text{year}}{\text{days}}$$

Cheapest-to-deliver bond

The CTD bond will change according to coupon and yield. If the futures seller does hedge his position using one particular deliverable bond, his profit on delivery at maturity will be:

$$\begin{aligned} & \text{nominal value of futures contract} \times \frac{\text{dirty bond price}}{\text{conversion factor}} \\ & \times (\text{implied repo rate} - \text{actual repo rate}) \times \frac{\text{days to delivery}}{\text{year}} \end{aligned}$$

The CTD bond will be the one which maximizes this profit. As yields fall, bonds with lower duration are likely to become cheaper to deliver (because the price of a bond with a low duration rises less, as yields fall, than the price of a bond with a high duration) and *vice versa*.

The arbitrage structure

If in fact the current futures price, bond price and actual repo rate are not all in line, an arbitrage opportunity will be available. Thus, if the actual repo rate is less than the implied repo rate, it will be possible to finance the hedge cheaply – that is, to buy the bond, repo it, sell the futures contract and deliver the bond at maturity of the futures contract, all at a locked-in profit. Such a round-trip is called “cash-and-carry arbitrage.”

If the actual repo rate is higher than the implied repo rate, it is possible to effect a cash-and-carry arbitrage in reverse – that is, borrow a bond through a reverse repo, sell the bond, buy the futures contract and take delivery of a bond at maturity of the futures contract. A problem arises, however, that the buyer of the futures contract has no control over which bond will be delivered. If the “cheapest to deliver” bond at maturity is not the same as the bond he has borrowed, arbitrage will not be complete; he must then sell the bond which has been delivered and buy the bond he has borrowed. In addition to the difference in value of the two bonds, this will also involve extra transaction costs. Where the seller may choose the exact delivery date within the month (as in US treasury bond and UK gilt futures), there is a further uncertainty for the futures buyer.

Example 5.18

CTD Bund 8 7/8% 22/02/2005 price:	105.24
Accrued coupon:	1.922917
Bund futures price:	93.75
Conversion factor for CTD:	1.1181
Repo rate:	3.29%
Days to futures delivery date:	31
Futures contract amount:	DEM 250,000
Accrued coupon on CTD at futures delivery date:	2.6625

Depending on whether the implied repo rate (= the break-even funding rate implied by the current bond futures price and the current cash price of the “cheapest to deliver” bond) is higher or lower than the actual current repo rate, the cash-and-carry arbitrage is:

Either (A)

- Buy the cash CTD bond now.
- Fund this purchase by repoing the bond.
- Sell the bond futures contract.
- Deliver the bond at maturity of the futures contract.

or (B) the opposite:

- Sell the cash CTD bond now.
- Borrow this bond (to deliver it now) through a reverse repo, using the cash raised by the bond sale.
- Buy the futures contract.
- Take delivery of the futures contract at maturity and use the bond to deliver on the second leg of the reverse repo.

(In practice, rather than deliver or take delivery of the bond at maturity of the futures contract, the cash bond purchase or sale and the futures contract can both be reversed at maturity. In (B) particularly, there would be no certainty that the bond delivered to us by the futures seller would match the bond we are obliged to return under the reverse repo.)

Assume for the moment that the profitable arbitrage is (A). (If in fact the result is negative, the profitable arbitrage is (B) instead):

Sell the bond futures contract (notional DEM 250,000)

Buy the cash CTD bond with nominal amount $\frac{\text{DEM } 250,000}{1.1181}$

= DEM 223,594

Cost of buying bond is nominal \times (clean price + accrued coupon)

= DEM 223,594 \times (105.24 + 1.922917) \div 100 = DEM 239,609.85

Total borrowing (principal + interest) to be repaid at the end

= DEM 239,609.85 \times $\left(1 + 0.0329 \times \frac{31}{360}\right)$ = DEM 240,288.68

Anticipated receipt from delivering bond = notional amount of bond \times (futures price \times conversion factor + accrued coupon) \div 100

$$= \text{DEM } 223,594 \times (93.75 \times 1.1181 + 2.6625) / 100 = \text{DEM } 240,328.61$$

In fact, the futures contract requires that DEM 250,000 nominal of the bond be delivered by the seller, rather than the DEM 223,594 which has been purchased as the hedge. The balance of DEM 26,406 would need to be purchased at the time of delivery for onward delivery to the counterparty. Apart from transaction costs, this should involve no significant profit or loss, as the futures exchange delivery settlement price should converge by delivery to (CTD cash price \times conversion factor).

$$\text{Profit} = \text{DEM } (240,328.61 - 240,288.68) = \text{DEM } 39.93$$

Therefore profit per futures contract = DEM 39.93

In practice, the profit in Example 5.16 cannot be calculated precisely for several reasons:

- The CTD bond may not be the same at maturity of the futures contract as it is when the arbitrage is established. This provides an advantage to the futures seller, who can profit by switching his hedge from the original CTD bond to a new one during the life of the arbitrage.
- The futures price and the CTD bond cash price may not converge exactly by maturity of the futures contract (that is, the basis may not move exactly to zero).
- The profit or loss on the futures contract is realized through variation margin payments; because the timing of these payments is unknown in advance, it is impossible to calculate their exact value.

Cash-and-carry arbitrage

Assume the arbitrage is achieved by buying the cash bond and selling the futures:

$$\text{Cash cost at start} = \text{nominal bond amount} \times (\text{cash bond price} + \text{accrued coupon at start}) / 100$$

$$\text{Total payments} = (\text{cash cost at start}) \times \left(1 + \text{repo rate} \times \frac{\text{days to futures delivery}}{\text{year}} \right)$$

$$\text{Total receipts} = \text{nominal bond amount} \times (\text{futures price} \times \text{conversion factor} + \text{accrued coupon at delivery of futures}) / 100$$

$$\text{Profit} = \text{total receipts} - \text{total payments}$$

For each futures contract sold, the nominal bond amount above is:

$$\frac{\text{notional size of contract}}{\text{conversion factor}}$$

**Calculation
summary**

Basis, net cost of carry, and net basis

The concept of “basis” is similar to that for a short-term interest rate futures contract, but using the conversion factor to adjust for the difference

between the actual bond and the notional futures bond. Therefore for any deliverable bond:

**Calculation
summary**

$$\text{basis} = \text{bond price} - \text{futures price} \times \text{conversion factor}$$

Using the formula we derived earlier for the implied repo rate, this can also be expressed as:

$$\begin{aligned} \text{basis} = & \text{coupon} \times \frac{\text{days to delivery}}{\text{year}} \\ & - \text{dirty bond price} \times \text{implied repo rate} \times \frac{\text{days to delivery}}{\text{year}} \end{aligned}$$

where “days to delivery” and “year” are calculated by the appropriate method in each case.

Buying a bond and selling a futures contract is known as “buying the basis”. Selling a bond and buying a futures contract is “selling the basis”.

The “net cost of carry” in holding any position is the difference between the financing cost of holding it and the interest income from the position. If we define this as negative when there is a net cost and positive when there is a net income, we have:

**Calculation
summary**

$$\text{net cost of carry} = \text{coupon income} - \text{financing cost}$$

In the case of 100 units of a bond purchased to hedge a futures sale, this can be expressed as:

$$\begin{aligned} \text{net cost of carry} = & \text{coupon} \times \frac{\text{days to delivery}}{\text{year}} \\ & - \text{dirty bond price} \times \text{actual repo rate} \times \frac{\text{days to delivery}}{\text{year}} \end{aligned}$$

The concept of “net basis” is similar to value basis for a short-term interest rate futures contract and is defined as the difference between basis and cost of carry:

**Calculation
summary**

$$\text{net basis} = \text{basis} - \text{net cost of carry}$$

From the above, it can be seen that:

$$\begin{aligned} \text{net basis} = & \text{dirty bond price} \times (\text{actual repo rate} - \text{implied repo rate}) \\ & \times \frac{\text{days to delivery}}{\text{year}}. \end{aligned}$$

If the actual repo rate is equal to the implied repo rate, the basis and net cost of carry are equal and the net basis is zero. The net basis shows whether there is potentially a profit in a cash-and-carry arbitrage (net basis is negative) or a reverse cash-and-carry arbitrage (net basis is positive).

“Basis risk” in general is the risk that the prices of any two instruments will not move exactly in line. If a long position in bond A is being hedged by a short position in bond B for example, there is a risk that any loss on the long position will not be fully offset by a gain on the short position. Similarly, there is a basis risk involved in hedging a bond position by an off-setting futures position because the two positions again may not move exactly in line. This point arose earlier in “Using futures to hedge a cash position” and also in the chapter on Interest Rate Futures.

EXERCISES



41. What is the yield of a 15-year bond paying an annual coupon of 7.5% and priced at 102.45?



42. You buy the following bond for settlement on a coupon-payment date. What is the cost of the bond? Make the calculation *without* using the built-in bond function or time value of money function on a calculator.

Amount:	FRF 100,000,000.00
Remaining maturity:	3 years
Coupon:	8.0%
Yield:	7.0%

What are the current yield and simple yield to maturity of the bond? What is the duration of the bond?



43. What is the clean price and accrued coupon of the following bond? Show the structure of a formula you would use to calculate the clean price as an NPV from first principles, but then use a calculator bond function for the answer.

Nominal amount:	5 million
Coupon:	6.8% (semi-annual)
Maturity date:	28 March 2005
Settlement date:	14 November 1997
Yield:	7.4% (semi-annual)
Price / yield calculation basis:	ACT/ACT (semi-annual)
Accrued interest calculation basis:	30/360



44. What is the yield of the following bond?

Coupon:	8.3% (annual)
Maturity date:	13 June 2003
Settlement date:	20 October 1997
Price:	102.48
Price / yield calculation basis:	30/360 (annual)
Accrued interest calculation basis:	ACT/365



45. What is the yield of the following zero-coupon bond? Try calculating this first without using the HP bond functions.

Maturity date:	27 March 2006
Settlement date:	20 July 1998
Price:	65.48
Price / yield calculation basis:	ACT/ACT (semi-annual)

46. You buy a US treasury bill at a discount rate of 8% with 97 days left to maturity. What is the bond-equivalent yield?

47. What would the bond-equivalent yield be in the previous question if the T-bill had 205 days left to maturity?
48. If the bond-equivalent yield which you want to achieve in the previous question is 9%, at what discount rate must you buy?
49. What is the price of the following bond?

Coupon:	4.5% in the first year of issue, increasing by 0.25% each year to 5.75% in the final year. All coupons paid annually.
Issue date:	3 March 1997
Maturity date:	3 March 2003
Settlement date:	11 November 1997
Yield:	5.24%
Price / yield calculation basis:	30/360 (annual)
Accrued interest calculation basis:	30/360



50. What is the yield of the following bond?

Coupon:	3.3% (annual)
Maturity date:	20 September 2007
Settlement date:	8 December 1997
Redemption amount:	110 per 100 face value
Price:	98.00 per 100 face value
Price / yield calculation basis:	30/360 (annual)
Accrued interest calculation basis:	30/360



You buy the above bond on 8 December 1997 and then sell it on 15 December at 98.50. What is your simple rate of return over the week on an ACT/360 basis? What is your effective rate of return (ACT/365)?

51. What was the accrued coupon on 28 July 1997 on the following bonds?

a. 7.5% gilt (ACT/365)	Maturity 7 December 2005
b. 5.625% US treasury bond	Maturity 15 August 2005
c. 6.25% Bund	Maturity 26 October 2005
d. 7.25% OAT	Maturity 25 October 2005
e. 3.00% JGB	Maturity 20 September 2005
f. 7.00% OLO	Maturity 15 November 2005
g. 8.80% Bono	Maturity 28 October 2005
h. 9.50% BTP	Maturity 1 August 2005

52. Would the following bond futures contract trade at a discount or a premium to 100?

Date:	27 March 1998
Futures maturity:	December 1998
CTD bond:	clean price 100 coupon 7% annual
CTD conversion factor:	1.0000
9-month money market interest rate:	5% p.a.





53. What is the theoretical September Bund futures price on 23 April if the cheapest-to-deliver bond is the 7.375% Bund of 2005, trading at 106.13? The conversion factor for that bond is 1.1247. The last coupon date was 3 January. Short-term funds can be borrowed at 3.35%. Futures delivery would be on 10 September. Assume that spot settlement for a Bund traded on 23 April would be 25 April and the coupon is accrued on a 30/360 basis.
54. With the same details as in the previous question but supposing that the actual futures price is 93.10, what is the implied repo rate?
55. Given the following information, there is a cash-and-carry arbitrage opportunity. What trades are necessary to exploit it and how much profit can be made?

CTD Bund 8 7/8% 20/12/2000 price:	102.71
Accrued coupon:	3.599 per 100
Bund futures price:	85.31
Conversion factor for CTD:	1.2030
Repo rate:	6.80%
Days to futures delivery date:	24
Futures contract amount:	DEM 250,000
Accrued coupon on CTD at futures delivery date:	4.191 per 100



56. You own the following portfolio on 14 August 1998:

	<i>Face value</i>	<i>Price</i>	<i>Coupon</i>	<i>Maturity</i>	<i>Duration</i>
Bond A	10 million	88.50	5.0% (annual)	8/7/2003	4.41 years
Bond B	5 million	111.00	12.0% (annual)	20/3/2001	2.31 years
Bond C	15 million	94.70	6.0% (annual)	14/10/2002	3.61 years

What is the approximate modified duration of the portfolio? How do you expect the value of the portfolio to change if yields all rise by 10 basis points? Assume that all the bond calculations are on a 30/360 basis.



- 57.

	<i>Price</i>	<i>Coupon</i>	<i>Maturity</i>	<i>Duration</i>
Bond A	88.50	5.0% (annual)	8/7/2003	4.41 years
Bond B	107.50	10.0% (annual)	8/4/2010	7.56 years

You own 10 million face value of bond A. You wish to hedge this position by selling bond futures. Bond B is currently the CTD for the futures contract and has a conversion factor of 1.2754. The notional size of the futures contract is 100,000. Short-term interest rates are 10%. Settlement date is 14 August 1998 and the futures contract delivery date is 15 September 1998. How many futures contracts should you sell? Assume that all the bond calculations are on a 30/360 basis.



“A zero-coupon yield is unambiguous: it is simply a measure of the relationship between a single future value and its present value.”

Zero-coupon Rates and Yield Curves

Zero-coupon yields and par yields

Forward-forward yields

Summary

Longer-dated FRAs

Exercises

ZERO-COUPON YIELDS AND PAR YIELDS

Constructing par yields from zero-coupon yields

A zero-coupon instrument is one which pays no coupon. For example, a company might issue a 5-year bond with a face value of 100 but no coupon. Clearly investors would not pay 100 for this; they would pay considerably less to allow for the fact that alternative investments would earn them interest. A zero-coupon yield is the yield earned on such an instrument, taking into account the fact that it is purchased for less than its face value. It is the (annual) interest rate which it is necessary to use to discount the future value of the instrument (i.e. its face value) to the price paid for it now. This interest rate is always given as the compounded rate, not the simple annual rate (i.e. in this example, it is not sufficient just to divide the difference between face value and purchase price by 5).

In many markets and periods, there may be no zero-coupon instrument available. There must nevertheless be a theoretical zero-coupon rate which is consistent with the “usual” interest rates available in the market – that is, the yields on coupon-bearing instruments. A clear advantage of zero-coupon yields over these “usual” yields to maturity is that they avoid the question of reinvestment risk.

A par yield is the yield to maturity of a coupon-bearing bond priced at par. The reason for considering par yields in particular rather than coupon-bearing bond yields in general is that anything else would be arbitrary. We could, for example, consider a range of existing five-year bonds – one with a 3 percent coupon, one with a 7 percent coupon and one with a 12 percent coupon. Even assuming the same issuer for all the bonds and no tax or other “hidden” effects, we would not expect the market yield to be exactly the same for the three bonds. The cashflows are on average rather further in the future in the case of the 3 percent coupon bond, than in the case of the 12 percent coupon bond; if longer term yields are higher than shorter term yields, for example, the later cashflows should be worth relatively more than the early cashflows, so that the 3 percent coupon bond should have a slightly higher yield than the 12 percent coupon bond. The 7 percent coupon bond’s yield should lie between the other two. Rather than choose one of these (or even a different one) arbitrarily, we choose a bond priced at par to be “representative” of coupon-bearing yields for that particular maturity. On a coupon date, this is a bond whose coupon is the same as the yield.

Clearly, however, it is very unlikely at any one time that there will be a bond priced exactly at par for any particular maturity. Therefore we probably need to use actual non-par, coupon-bearing bond yields available in the market to construct the equivalent zero-coupon yields. From there, we can go on to construct what the par yields would be if there were any par bonds. It should be noted of course that there is not only one par yield curve or one zero-coupon yield curve. Rather, there are as many par yield curves as there are issuers. Government bond yields will generally be lower than corporate bond yields or interest rate swap yields, for example. Associated with each par yield curve is a separate zero-coupon yield curve – also known as a spot yield curve.

It is probably clearest to begin constructing these relationships from the zero-coupon yield curve. A zero-coupon yield or spot yield is unambiguous: it is simply a measure of the relationship between a single future value and its present value. If we then know the market's view of what zero-coupon yields are for all periods, we can calculate precisely the NPV of a series of bond cashflows by discounting each cashflow at the zero-coupon rate for that particular period. The result will be the price for that bond exactly consistent with the zero-coupon curve. A single yield – the internal rate of return – can then be calculated which would arrive at this same price; this single yield is the usual “yield to maturity” quoted for the bond. If we can then construct a bond where the NPV calculated in this way using the zero coupon rates is 100, the coupon of this bond is the par yield.

Example 6.1

Suppose that we have the following zero-coupon yield structure:

1-year:	10.000%
2-year:	10.526%
3-year:	11.076%
4-year:	11.655%

What are the zero-coupon discount factors? What would be the prices and yields to maturity of:

- a 4-year, 5% coupon bond?
- a 4-year, 11.5% coupon bond?
- a 4-year, 13% coupon bond?

The zero-coupon discount factors are:

1-year:	$\frac{1}{(1.1)}$	= 0.90909
2-year:	$\frac{1}{(1.10526)^2}$	= 0.81860
3-year:	$\frac{1}{(1.11076)^3}$	= 0.72969
4-year:	$\frac{1}{(1.11655)^4}$	= 0.64341

$$(a) \text{ Price} = (5 \times 0.90909) + (5 \times 0.81860) + (5 \times 0.72969) + (105 \times 0.64341) = 79.84$$

Yield to maturity = 11.58% (using TVM function of HP calculator)

$$(b) \text{ Price} = (11.5 \times 0.90909) + (11.5 \times 0.81860) + (11.5 \times 0.72969) + (111.5 \times 0.64341) = 100.00$$

Yield to maturity = 11.50% (the par yield is the same as the coupon because the bond is priced at par)

$$(c) \text{ Price} = (13 \times 0.90909) + (13 \times 0.81860) + (13 \times 0.72969) + (113 \times 0.64341) = 104.65$$

Yield to maturity = 11.49% (using TVM function of HP calculator)

1.1 $\square \frac{1}{x}$	(1-year discount factor)
1.10526 ENTER 2 $\square \wedge \square \frac{1}{x}$	(2-year discount factor)
1.11076 ENTER 3 $\square \wedge \square \frac{1}{x}$	(3-year discount factor)
1.11655 ENTER 4 $\square \wedge \square \frac{1}{x}$	(4-year discount factor)
.90909 ENTER .8186 + .72969 + 5 \times	
105 ENTER .64341 \times +	(Price of (a))
FIN TVM +/- PV	
4 N 5 PMT 100 FV I%YR	(Yield to maturity of (a))
.90909 ENTER .8186 + .72969 + 11.5 \times	
111.5 ENTER .64341 \times +	(Price of (b))
.90909 ENTER .8186 + .72969 + 13 \times	
113 ENTER .64341 \times +	(Price of (c))
+/- PV 13 PMT I%YR	(Yield to maturity of (c))

It can be seen from (b) above that if the zero-coupon discount factor for k years is df_k , and the par yield for N years is i , it will always be the case that:

$$(i \times df_1) + (i \times df_2) + + (i \times df_N) + (1 \times df_N) = 1$$

This gives: $i \times (df_1 + df_2 + + df_N) = 1 - df_N$

Therefore: $i = \frac{1 - df_N}{\sum_{k=1}^N df_k}$

**Calculation
summary**

$$\text{Par yield for } N \text{ years} = \frac{1 - df_N}{\sum_{k=1}^N df_k}$$

where: df_k = zero-coupon discount factor for k years

Zero-coupon yields from coupon-bearing yields

In Example 6.1, we effectively calculated the 4-year par yield (11.50%) from the series of zero-coupon yields. We can similarly calculate a zero-coupon yield from a series of coupon-bearing yields. A method to do this is to build up synthetic zero-coupon structures by combining a series of instruments such that all the cashflows net to zero except for the first and the last.

Example 6.2

Suppose that the 1-year interest rate is 10% and that a series of bonds are currently priced as follows:

	Price	Coupon	Maturity
Bond A	97.409	9	2 years
Bond B	85.256	5	3 years
Bond C	104.651	13	4 years

Consider a 2-year investment of 97.409 to purchase 100 face value of bond A and a 1-year borrowing of 8.182 at 10.0%.

The cashflows are:

Year				Net cashflows
0	- 97.409	+ 8.182		- 89.227
1	+ 9.000	- 9.000		
2	- 109.000			+ 109.000

In this way, we have constructed what is in effect a synthetic 2-year zero-coupon instrument, because there are no cashflows between now and maturity. The amount of 8.182 was calculated as the amount necessary to achieve 9.00 after 1 year – that is, the present value of 9.00 after 1 year.

The 2-year zero-coupon rate is therefore $\left(\frac{109.00}{89.227}\right)^{\frac{1}{2}} - 1 = 10.526\%$

Next, consider a 3-year investment of 85.256 to purchase 100 face value of bond B, a 1-year borrowing of 4.545 at 10.0% and a 2-year zero-coupon borrowing of 4.093 at 10.526%. The cashflows are:

Year				Net cashflows
0	- 85.256	+ 4.545	+ 4.093	- 76.618
1	+ 5.000	- 5.000		
2	+ 5.000		- 5.000	
3	- 105.000			+ 105.000

The 3-year zero-coupon rate is $\left(\frac{105.00}{76.618}\right)^{\frac{1}{3}} - 1 = 11.076\%$

Again, 4.545 is the present value of 5.00 after 1 year; 4.093 is the present value of 5.00 after 2 years.

Finally, consider a 4-year investment of 104.651 to purchase 100 face value of bond C, a 1-year borrowing of 11.818 at 10%, a 2-year zero-coupon borrowing of 10.642 at 10.526% and a 3-year zero-coupon borrowing of 9.486 at 11.076%. The cashflows are:

Year					Net Cashflows
0	- 104.651	+ 11.818	+ 10.642	+ 9.486	- 72.705
1	+ 13.000	- 13.000			
2	+ 13.000			- 13.000	
3	+ 13.000		- 13.000		
4	+ 113.000				+ 113.000

The 4-year zero-coupon rate is $\left(\frac{113.000}{72.705}\right)^{\frac{1}{4}} - 1 = 11.655\%$

This process of building up a series of zero-coupon yields from coupon-bearing yields is known as “bootstrapping”. In Example 6.2, we used three bonds as our starting point, none of which were priced at par. We could instead have used bonds priced at par, or time deposits (which also return the same original principal amount at maturity). In the case of interest rate swaps, for example, we would indeed expect to have par swap rates as our starting

point. The bootstrapping process is exactly the same, with initial cashflows of 100 each instead of the non-par bond prices (97.409, 85.256 and 104.651) in Example 6.2. As long as we have four different investments, it is possible to build a synthetic zero-coupon structure and hence calculate the four-year, zero-coupon yield.

The zero-coupon rates calculated in Example 6.2 are in fact the same ones as used in Example 6.1. We can see that the rates are consistent, in that in Example 6.1, we have already used these zero-coupon rates to value the 4-year 13% coupon bond and arrived at the same price. The process is circular.

If all market prices and yields are consistent, it will not matter which existing bonds we use to construct the zero-coupon yields. In practice, however, they will not be exactly consistent and it is generally preferable to use bonds priced as closely as possible to par.

There are two more very important practical issues which we shall mention, but which are beyond the scope of this book. Firstly, it is not generally possible in practice to find a series of existing bonds with the most convenient maturities – say 1 year, 2 years, 3 years, 4 years etc. – from which we can construct the zero-coupon rates. Instead, we must use what are actually available. Secondly, we need to interpolate between existing yields in order to establish rates for all possible maturities. To do this, we must somehow fit a curve along all the points. This requires assumptions to be made about the mathematical nature of the curve, and some degree of compromise in order to make the curve smooth.

FORWARD-FORWARD YIELDS

Constructing forward-forward yields from zero-coupon yields

In Chapter 3, we constructed short-term forward-forward rates from a series of “normal” rates for periods beginning now. Exactly the same approach can be used for constructing long-term, forward-forward yields from long-term, zero-coupon yields.

Example 6.3

Suppose the same zero-coupon curve as in Example 6.3. What are the 1-year v 2-year forward-forward yield, the 2-year v 3-year forward-forward yield and the 1-year v 3-year forward-forward zero-coupon yield?

Using the 1-year zero-coupon rate of 10.000%, an amount of 1 now has a future value after 1 year of 1.10000.

Using the 2-year zero coupon rate of 10.526%, an amount of 1 now has a future value after 2 years of $(1.10526)^2$.

It follows that an amount worth 1.1000 after 1 year is worth $(1.10526)^2$ after 2 years. The yield linking these two amounts over that 1-year forward-forward period is:

$$\frac{(1.10526)^2}{(1.10000)} - 1 = 11.055\%$$

This is therefore the 1-year v 2-year forward-forward yield.

Using the 3-year zero-coupon rate of 11.076%, an amount of 1 now has a future value after 3 years of $(1.11076)^3$

The 2-year v 3-year forward-forward rate is therefore:

$$\frac{(1.11076)^3}{(1.10526)^2} - 1 = 12.184\%$$

The relationship between an amount after 1 year and an amount after 3 years is:

$$\frac{(1.11076)^3}{(1.10000)} = 1.2459$$

The 1-year v 3-year zero-coupon forward-forward is therefore:

$$(1.2459)^{1/2} - 1 = 11.618\%$$

In general, if z_k and z_m are the zero-coupon yields for k years and m years respectively, we have the following:

Forward-forward zero-coupon yield from k years to m years is:

$$\left[\frac{(1 + z_m)^m}{(1 + z_k)^k} \right]^{\frac{1}{(m-k)}} - 1$$

In particular, the forward-forward yield from k years to $(k + 1)$ years is:

$$\frac{(1 + z_{k+1})^{k+1}}{(1 + z_k)^k} - 1$$

**Calculation
summary**

Zero-coupon yields from forward-forward yields

In Chapter 3, we first used “normal” interest rates to calculate forward-forward rates. We then said that this approach could be reversed. It may make more sense to begin with the question: what does the market expect interest rates to be in the future? From this, we can construct a series of “normal” rates.

Exactly the same question arises with long-term, forward-forward yields and zero-coupon yields.

Example 6.4

The 1-year interest rate is now 10%. The market expects 1-year rates to rise to 11.05% after one year and to 12.18% after a further year. What would you expect the current 2-year and 3-year zero-coupon yields to be, consistent with these expectations?

An amount of 1 now will be worth $(1 + 10\%) = 1.10$ after one year. This is expected to increase by a further 11.05% and 12.18% in the following two years. We can therefore construct a strip of rates for the expected value at the end of three years:

$$1.10 \times 1.1105 \times 1.1218 = 1.3703$$

The 3-year zero coupon rate should therefore be:

$$\left(\frac{1.3703}{1}\right)^{\frac{1}{3}} - 1 = 11.07\%$$

The result of Example 6.4 is consistent with our earlier examples, demonstrating that we could begin with a forward-forward yield curve (that is, a series of 1-year, forward-forward rates), create from them a consistent zero-coupon yield curve, and then create a consistent par yield curve.

**Calculation
summary**

$$z_k = [(1 + i_1) \times (1 + i_2) \times (1 + i_3) \times \dots \times (1 + i_k)]^{\frac{1}{k}} - 1$$

where: $i_1, i_2, i_3 \dots i_k$ are the 1-year cash interest rate and the 1-year v 2-year, 2-year v 3-year, ... (k - 1)-year v k-year forward-forward rates

SUMMARY

Although all the yields we have considered should in theory be consistent, actual market yields might not be. This is important for bond valuation. A bond trading in the market has an actual known market price at which traders are willing to sell it. An investor can calculate the yield to maturity implied by this price and decide whether he/she considers the bond a good investment. However, he/she can also calculate what the theoretical price should be if all the bond's cashflows are individually valued to an NPV using a series of separate zero-coupon rates. The result may in practice be different from the actual market price. The investor then needs to decide whether, if the theoretical price is lower, he/she considers the bond overpriced.

Generally, there is no exact arbitrage to bring the market price precisely in line with this theoretical price, because there are no corresponding zero-coupon instruments. The zero-coupon yields are themselves often theoretical. It is therefore often more a question of using zero-coupon yields to compare different bonds, to see which is better value at current market prices. In the case of government securities where the bonds themselves can be stripped, however, a direct comparison can be made between the price of the bond and the prices of its stripped components, so that the arbitrage is possible.

**Calculation
summary**

Conversion between yield curves

- To create a zero-coupon yield from coupon-bearing yields: **bootstrap**
- To calculate the yield to maturity on a non-par coupon-bearing bond from zero-coupon yields: **calculate the NPV of the bond using the zero-coupon yields, then calculate the yield to maturity of the bond from this dirty price**
- To create a par yield from zero-coupon yields: **use the formula above**

- To create a forward-forward yield from zero-coupon yields: use the formula above
- To create a zero-coupon yield from forward-forward yields: create a strip of the first cash leg with a series of forward-forwards

A par yield curve tends to be exaggerated by the corresponding zero-coupon curve. That is, a positive par yield curve is reflected by a more positive zero-coupon curve and a negative par yield curve by a more negative zero-coupon curve. The forward-forward curve tends to exaggerate further a change in shape of the zero-coupon curve (see Figure 6.1).

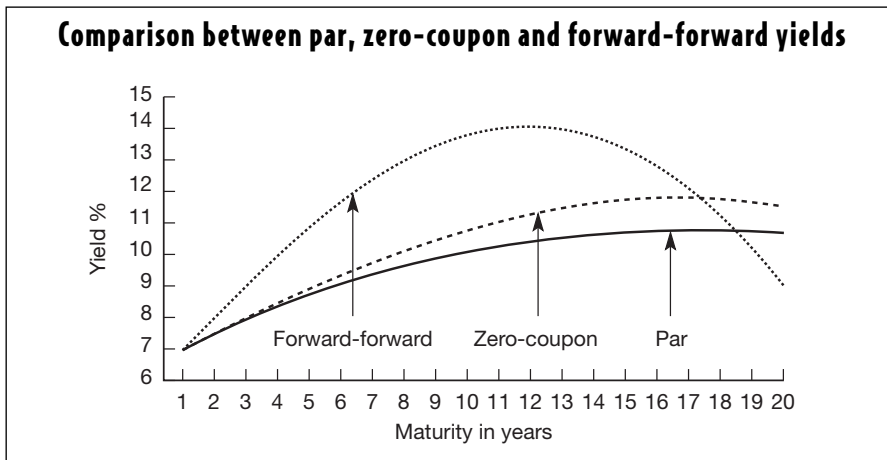


Fig 6.1

LONGER-DATED FRAs

In Chapter 3, we considered FRAs up to one year. For FRAs beyond a year, the calculation of a theoretical FRA price is complicated by the fact that deposits over one year generally pay some interest at the end of the first year or each six months. There are two approaches to this. The first is to consider all the cashflows involved; the second is to calculate zero-coupon discount factors and then use these to calculate an FRA rate.

Example 6.5

Given the following interest rates, what is the theoretical 15 v 18 FRA rate? All rates are on an ACT/360 basis.

- | | |
|-----------------------|--|
| 12 months (365 days): | 8.5% |
| 15 months (456 days): | 8.6% (interest paid after 12 months and 15 months) |
| 18 months (548 days): | 8.7% (interest paid after 12 months and 18 months) |

First approach

Consider an amount of 1 deposited for 12 months, rolled over at the 12 v 18 FRA rate. The result after 18 months will be:

$$\left(1 + 0.085 \times \frac{365}{360}\right) \times \left(1 + [12 \text{ v } 18 \text{ FRA}] \times \frac{183}{360}\right)$$

The same result should be achieved by depositing for 18 months and rolling over the interim interest payment (received after 12 months) at the 12 v 18 FRA rate:

$$1 + \left[0.087 \times \frac{365}{360} \times \left(1 + [12 \text{ v } 18 \text{ FRA}] \times \frac{183}{360}\right)\right] + \left[0.087 \times \frac{183}{360}\right]$$

If these two are equal, we can calculate the 12 v 18 FRA as:

$$12 \text{ v } 18 \text{ FRA} = \left[\frac{\left(1 + 0.087 \times \frac{183}{360}\right)}{\left(1 + 0.085 \times \frac{365}{360} - 0.087 \times \frac{365}{360}\right)} - 1 \right] \times \frac{360}{183} = 9.1174\%$$

The same result should also be achieved by depositing for 15 months and rolling over at the 15 v 18 FRA rate, with the interim interest payment (received after 12 months) rolled over from 12 months to 18 months at the 12 v 18 FRA rate. The result after 18 months will be:

$$\left[0.086 \times \frac{365}{360} \times \left(1 + 0.091174 \times \frac{183}{360}\right)\right] + \left[\left(1 + 0.086 \times \frac{91}{360}\right) + \left(1 + [15 \text{ v } 18 \text{ FRA}] \times \frac{92}{360}\right)\right]$$

If this is equal to the first result, we have:

$$15 \text{ v } 18 \text{ FRA} =$$

$$\left[\frac{\left(1 + 0.085 \times \frac{365}{360}\right) \times \left(1 + 0.091174 \times \frac{183}{360}\right) - \left[0.086 \times \frac{365}{360} \times \left(1 + 0.091174 \times \frac{183}{360}\right)\right]}{\left(1 + 0.086 \times \frac{91}{360}\right)} - 1 \right] \times \frac{360}{92}$$

$$= 9.02\%$$

Second approach

$$\text{The 12-month discount factor is } \frac{1}{\left(1 + 0.085 \times \frac{365}{360}\right)} = 0.92066$$

Consider the following cashflows (a 15-month deposit and a 12-month borrowing):

Months			Net
0	-100	$+(8.6 \times \frac{365}{360} \times 0.92066)$	- 91.9724
12	$+(8.6 \times \frac{365}{360})$	$-(8.6 \times \frac{365}{360})$	
15	$+(100 + 8.6 \times \frac{91}{360})$		+ 102.1739

$$\text{The 15-month discount factor is } \frac{91.9724}{102.1739} = 0.90016$$

Now consider the following cashflows (an 18-month deposit and a 12-month borrowing):

Months			Net
0	-100	$+(8.7 \times \frac{365}{360} \times 0.92066)$	- 91.8790
12	$+(8.7 \times \frac{365}{360})$	$-(8.7 \times \frac{365}{360})$	
18	$+(100 + 8.7 \times \frac{183}{360})$		+ 104.4225

The 18-month discount factor is $\frac{91.8790}{104.4225} = 0.87988$

The 15 v 18 FRA therefore $\left[\frac{\frac{1}{0.87988}}{\frac{1}{0.90016}} - 1 \right] \times \frac{360}{92} = 9.02 \%$

The two approaches are effectively the same; the second is rather more structured.



EXERCISES

58. Calculate the 2-year, 3-year and 4-year zero-coupon yields and discount factors consistent with the following bonds. The 1-year yield is 10.00%.

<i>Maturity</i>	<i>Coupon</i>	<i>Price</i>
2 years	9.0% (annual)	97.70
3 years	7.0% (annual)	90.90
4 years	11.0% (annual)	99.40

What are the 1-year v 2-year, 2-year v 3-year and 3-year v 4-year forward-forward yields?

59. The forward-forward curve is as follows:

1-year yield:	8.00%
1-year v 2-year:	8.24%
2-year v 3-year:	9.00%
3-year v 4-year:	9.50%

- Calculate the 2-year, 3-year and 4-year zero-coupon yields and par yields.
- What is the yield to maturity of a 4-year 12% annual coupon bond, consistent with the rates above?

60. What is the 18 v 24 FRA rate based on the following? All rates are ACT/360.

6 months (182 days):	8.60% / 8.70%
12 months (365 days):	8.70% / 8.80%
18 months (548 days):	8.80% / 8.90% (interest paid 6-monthly)
24 months (730 days):	8.90% / 9.00% (interest paid 6-monthly)

Foreign Exchange



“A forward foreign exchange swap is a temporary purchase or sale of one currency against another. An equivalent effect could be achieved by borrowing one currency, while lending the other.”

Foreign Exchange

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Forward exchange rates

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Arbitraging and creating FRAs

Discounting future foreign exchange risk

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INTRODUCTION

Throughout this chapter, we have generally used ISO codes (also used by the SWIFT system) to abbreviate currency names. You can find a list of the codes in Appendix 5 for reference.

A convention has also been used that, for example, the US dollar/Deutschemark exchange rate is written as USD/DEM if it refers to the number of Deutschemarks equal to 1 US dollar and DEM/USD if it refers to the number of US dollars equal to 1 Deutschemark. The currency code written on the left is the “base” currency; there is always 1 of the base unit. The currency code written on the right is the “variable” currency (or “counter” currency or “quoted” currency). The number of units of this currency equal to 1 of the base currency varies according to the exchange rate. Although some people do use the precisely opposite convention, the one we use here is the more common. The important point to remember is to be consistent.

SPOT EXCHANGE RATES

A “spot” transaction is an outright purchase or sale of one currency for another currency, for delivery two working days after the dealing date (the date on which the contract is made). This allows time for the necessary paperwork and cash transfers to be arranged. Normally, therefore, if a spot deal is contracted on Monday, Tuesday or Wednesday, delivery will be two days after (i.e. Wednesday, Thursday or Friday respectively). If a spot deal is contracted on a Thursday or Friday, the delivery date is on Monday or Tuesday respectively, as neither Saturday nor Sunday are working days in the major markets.

There are, however, some exceptions. For example, a price for USD/CAD without qualification generally implies delivery on the next working day after the dealing day. This is referred to as “funds”. A “spot” price (value two working days after the dealing day, as usual) can generally be requested as an alternative. USD/HKD is also often traded for value the next working day (a “TT” price). Another problem arises in trading Middle East currencies where the relevant markets are closed on Friday but open on Saturday. A USD/SAR spot deal on Wednesday would need to have a split settlement date: the USD would be settled on Friday, but the SAR on Saturday.

If the spot date falls on a public holiday in one or both of the centres of the two currencies involved, the next working day is taken as the value date. For example, if a spot GBP/USD deal is transacted on Thursday 31 August, it would normally be for value Monday 4 September. If this date is a holiday in the UK or the USA, however, all spot transactions on Thursday 31 August are for value Tuesday 5 September. If the intervening day (between today and spot) is a holiday in one only of the two centres, the spot value date is usually also delayed by one day.

How spot rates are quoted

In the spot market, a variable number of units of one currency is quoted per one unit of another currency. When quoting against the US dollar, it is the practice in the interbank market to quote most currencies in terms of a varying number of units of currency per 1 US dollar. This is known as an “indirect” or “European” quotation against the dollar. Rates quoted the opposite way round are known as “direct” quotations against the dollar.

There are some currencies which are conventionally quoted against the USD “direct” rather than “indirect.” The major ones are sterling, Australian dollar, New Zealand dollar, Irish pound and ECU (European currency unit – to be replaced by the proposed euro).

The Canadian dollar is now generally quoted in indirect terms, although a direct quotation is possible. The direct quotation for the Canadian dollar is known as “Canada cross” and the indirect quotation “Canada funds.”

In the currency futures markets, as opposed to the interbank market, all quotations against the US dollar are direct.

Although dealing is possible between any two convertible currencies – for example, Malaysian ringgits against Deutschmarks or French francs against Japanese yen – the interbank market historically quoted mostly against US dollars, so reducing the number of individual rates that needed to be quoted. The exchange rate between any two non-US dollar currencies could then be calculated from the rate for each currency against US dollars. A rate between any two currencies, neither of which is the dollar, is known as a cross-rate. Cross-rates (for example, sterling/Deutschmark, Deutschmark/yen, Deutschmark/Swiss franc) have however increasingly been traded between banks in addition to the dollar-based rates. This reflects the importance of the relationship between the pair of currencies. The economic relationship between the French franc and the Deutschmark, for example, is clearly closer than the relationship between the French franc and the dollar. It is therefore more true nowadays to say that the dollar/franc exchange rate is a function of the dollar/Deutschmark rate and the Deutschmark/franc rate, rather than that the Deutschmark/franc rate is a function of the dollar/Deutschmark rate and the dollar/franc rate. However, the principle of calculating cross-rates remains the same.

As in other markets, a bank normally quotes a two-way price, whereby it indicates at what level it is prepared to buy the base currency against the variable currency (the “bid” for the base currency – a cheaper rate), and at what level it is prepared to sell the base currency against the variable currency (the “offer” of the base currency – a more expensive rate). For example, if a bank is prepared to buy dollars for 1.6375 Deutschmarks, and sell dollars for 1.6385 Deutschmarks, the USD/DEM rate would be quoted as: 1.6375 / 1.6385.

The quoting bank buys the base currency (in this case, dollars) on the left and sells the base currency on the right. If the bank quotes such a rate to a company or other counterparty, the counterparty would sell the base currency on the left, and buy the base currency on the right – the opposite of how the bank sees the deal.

In the money market, the order of quotation is not important and it does differ between markets. From a quotation of either “5.80% / 5.85%” or “5.85% / 5.80%”, it is always clear to the customer that the higher rate is the offered rate and the lower rate is the bid rate. In foreign exchange, however, the market-maker’s bid for the base currency (the lower number in a spot price) is always on the left. This is particularly important in forward prices.

The difference between the two sides of the quotation is known as the “spread.” Historically, a two-way price in a cross-rate would have a wider spread than a two-way price in a dollar-based rate, because the cross-rate constructed from the dollar-based rates would combine both the spreads. Now, however, the spread in, say, a Deutschmark/French franc price would typically be narrower than a dollar/franc spread, because it is now the Deutschmark/franc price that is “driving” the market, as noted above, rather than the dollar/franc price.

Any indirect quotation against the dollar can be converted into a direct quotation by taking its reciprocal. Thus, a USD/DEM quotation of 1.6375 / 1.6385 can be converted to a DEM/USD quotation of $(1 \div 1.6375) / (1 \div 1.6385)$. However, this would still be quoted with a smaller number on the left, so that the two sides of the quotation are reversed: 0.6103 / 0.6107. The important difference is thus that in a direct quotation the bank buys the other currency against the US dollar on the left, sells the other currency against the US dollars on the right – the reverse of an indirect quotation.

Rates are normally quoted to $\frac{1}{100}$ th of a pfennig, cent, etc. (known as a “point” or a “pip”). Thus the US dollar/Deutschmark rate would usually be quoted as 1.6375 / 1.6385, for example. This depends on the size of the number, however, and some typical examples where fewer decimal places are used are:

USD/JPY	105.05 / 15	USD/ITL	1752.00 / 50
USD/BEF	34.095 / 125	USD/ESP	139.80 / 85

In the case of USD/JPY, for example, “15 points” means 0.15 yen.

As the first three digits of the exchange rate (known as the “big figure”) do not change in the short term, dealers generally do not mention them when dealing in the interbank market. In the example above (1.6375 / 1.6385), the quotation would therefore be given as simply 75 / 85. However, when dealers are quoting a rate to a corporate client they will often mention the big figure also. In this case, the quotation would be 1.6375 / 85.

As the market moves very quickly, dealers need to deal with great speed and therefore abbreviate when dealing. For example, if one dealer wishes to buy USD 5 million from another who is quoting him a USD/DEM price, he will say simply “5 mine;” this means “I wish to buy from you 5 million of the base currency and sell the other currency, at your offered price.” Similarly, if he wishes to sell USD 5 million, he will say simply “5 yours,” meaning “I wish to sell you 5 million of the base currency and buy the other currency, at your bid price.”

To earn profit from dealing, the bank’s objective is clearly to sell the US dollars at the highest rate it can against the variable currency and buy US dollars at the lowest rate.

Example 7.1

Deal 1: Bank buys USD 1,000,000 against DEM at 1.6830

Deal 2: Bank sells USD 1,000,000 against DEM at 1.6855

	<i>Inflows</i>	<i>Outflows</i>
Deal 1:	USD 1,000,000	DEM 1,683,000
Deal 2:	<u>DEM 1,685,500</u>	<u>USD 1,000,000</u>
Net result:	DEM 2,500	

Dealers generally operate on the basis of small percentage profits but large turnover. These rates will be good for large, round amounts. For very large amounts, or for smaller amounts, a bank would normally quote a wider spread. The amount for which a quotation is “good” (that is, a valid quote on which the dealer will deal) will vary to some extent with the currency concerned and market conditions.

Cross-rates

In most cases, there is not a universal convention for which way round to quote a cross-rate – that is, which is the base currency and which the variable currency. If a rate between DEM and FRF is requested “in FRF terms”, for example, this would generally mean that FRF is the base and DEM the variable.

Example 7.2

Suppose that we need to quote to a counterparty a spot rate between the Deutschemark and the ringgit, and that our bank does not have a DEM/MYR trading book. The rate must therefore be constructed from the prices quoted by our bank’s USD/DEM dealer and our bank’s USD/MYR dealer as follows:

Spot USD/DEM: 1.6874 / 1.6879
 Spot USD/MYR: 2.4782 / 2.4792

Consider first the left side of the final DEM/MYR price we are constructing. This is the price at which our bank will buy DEM (the base currency) and sell MYR. We must therefore ask: at which price (1.6874 or 1.6879) does our USD/DEM dealer buy DEM against USD, and at which price (2.4782 or 2.4792) does our USD/MYR dealer sell MYR against USD? The answers are 1.6879 (on the right) and 2.4782 (on the left) respectively. Effectively, by dealing at these prices, our bank is both selling USD (against DEM) and buying USD (against MYR) simultaneously, with a net zero effect in USD. If we now consider the right side of the final DEM/MYR price we are constructing, this will come from selling DEM against USD (on the left at 1.6874) and buying MYR against USD (on the right at 2.4792). Finally, since each 1 dollar is worth 1.68 Deutschemarks and also 2.47 ringgits, the DEM/MYR exchange rate must be the ratio between these two:

$2.4782 \div 1.6879 = 1.4682$ is how the bank sells MYR and buys DEM
 $2.4792 \div 1.6874 = 1.4692$ is how the bank buys MYR and sells DEM

Therefore the spot DEM/MYR rate is: 1.4682 / 1.4692.

In summary, therefore, to calculate a spot cross-rate between two indirect dollar rates, divide opposite sides of the exchange rates against the US dollar. Following the same logic shows that to calculate a spot cross-rate between two direct dollar rates, we again need to divide opposite sides of the exchange rates against the US dollar.

Example 7.3

Spot GBP/USD: 1.6166 / 1.6171

Spot AUD/USD: 0.7834 / 0.7839

The GBP/USD dealer buys GBP and sells USD at 1.6166 (on the left).

The AUD/USD dealer sells AUD and buys USD at 0.7839 (on the right).

Therefore:

$$1.6166 \div 0.7839 = 2.0623 \text{ is how the bank sells AUD and buys GBP}$$

Similarly:

$$1.6171 \div 0.7834 = 2.0642 \text{ is how the bank buys AUD and sells GBP}$$

Therefore the spot GBP/AUD rate is: $2.0623 / 2.0642$.

Finally, to calculate a cross-rate between a direct rate and an indirect rate, following the same logic through again shows that we multiply the same sides of the exchange rates against the US dollar.

Example 7.4

Spot USD/DEM: 1.6874 / 1.6879

Spot AUD/USD: 0.7834 / 0.7839

The USD/DEM dealer buys USD and sells DEM at 1.6874 (on the left).

The AUD/USD dealer buys AUD and sells USD at 0.7834 (on the left).

Also, since each 1 Australian dollar is worth 0.78 US dollars, and each of these US dollars is worth 1.68 Deutschemarks, the AUD/DEM exchange rate must be the product of these two numbers. Therefore:

$$1.6874 \times 0.7834 = 1.3219 \text{ is how the bank sells DEM and buys AUD}$$

Similarly:

$$1.6879 \times 0.7839 = 1.3231 \text{ is how the bank buys DEM and sells AUD}$$

Therefore the spot AUD/DEM rate is: $1.3219 / 1.3231$.

Calculation summary

To calculate cross-rates from dollar rates

- Between two indirect rates or two direct rates: divide opposite sides of the dollar exchange rates
- Between one indirect rate and one direct rate: multiply the same sides of the dollar exchange rates

It is just as important to be able to construct rates from non-dollar rates. The same logic, considering the way in which each of the two separate dealers will deal to create the cross-rate, gives the construction:

Example 7.5

Spot USD/DEM: 1.6874 / 1.6879

Spot DEM/FRF: 3.3702 / 3.3707

Spot DEM/SEK: 4.5270 / 4.5300

(i) To construct spot FRF/SEK:

$4.5270 \div 3.3707 = 1.3430$ is how the bank buys FRF and sells SEK

$4.5300 \div 3.3702 = 1.3441$ is how the bank sells FRF and buys SEK

Therefore the spot FRF/SEK rate is: 1.3430 / 1.3441.

(ii) To construct spot SEK/FRF:

$3.3702 \div 4.5300 = 0.7440$ is how the bank buys SEK and sells FRF

$3.3707 \div 4.5270 = 0.7446$ is how the bank sells SEK and buys FRF

Therefore the spot SEK/FRF is 0.7440 / 0.7446.

(iii) To construct spot USD/FRF:

$1.6874 \times 3.3702 = 5.6869$ is how the bank buys USD and sells FRF

$1.6879 \times 3.3707 = 5.6894$ is how the bank sells USD and buys FRF

Therefore the spot USD/FRF rate is: 5.6869 / 5.6894.

(iv) To construct spot FRF/USD, take the reciprocal of the USD/FRF:

$1 \div (1.6879 \times 3.3707) = 0.17577$ is how the bank buys FRF and sells USD

$1 \div (1.6874 \times 3.3702) = 0.17584$ is how the bank sells FRF and buys USD

Therefore the spot FRF/USD rate is 0.17577 / 0.17584.

The construction of one exchange rate from two others in this way can be seen “algebraically”:

Given two exchange rates A/B and A/C, the cross-rates are:

$$B/C = A/C \div A/B$$

$$\text{and } C/B = A/B \div A/C$$

Given two exchange rates B/A and A/C, the cross-rates are:

$$B/C = B/A \times A/C$$

$$\text{and } C/B = 1 \div (B/A \times A/C)$$

When dividing, use opposite sides. When multiplying, use the same sides.

**Calculation
summary**

FORWARD EXCHANGE RATES

Forward outright

Although “spot” is settled two working days in the future, it is not considered in the foreign exchange market as “future” or “forward”, but as the baseline from which all other dates (earlier or later) are considered.

A “forward outright” is an outright purchase or sale of one currency in exchange for another currency for settlement on a fixed date in the future other than the spot value date. Rates are quoted in a similar way to those in the spot market, with the bank buying the base currency “low” (on the left side) and selling it “high” (on the right side). In some emerging markets, forward outright are non-deliverable and are settled in cash against the spot rate at maturity as a contract for differences.

Example 7.6

The spot USD/DEM rate is 1.6874 / 1.6879, but the rate for value one month after the spot value date is 1.6844 / 1.6851.

The “spread” (the difference between the bank’s buying price and the bank’s selling price) is wider in the forward quotation than in the spot quotation. Also, in this example, the US dollar is worth less in the future than at the spot date. USD 1 buys DEM 1.6844 in one month’s time as opposed to 1.6874 at present. In a different example, the US dollar might be worth more in the future than at the spot date.

The forward outright rate may be seen both as the market’s assessment of where the spot rate will be in the future and as a reflection of current interest rates in the two currencies concerned.

Consider, for example, the following “round trip” transactions, all undertaken simultaneously:

- (i) Borrow Deutschmarks for 3 months starting from spot value date.
- (ii) Sell Deutschmarks and buy US dollars for value spot.
- (iii) Deposit the purchased dollars for 3 months starting from spot value date.
- (iv) Sell forward now the dollar principal and interest which mature in 3 months’ time, into Deutschmarks.

In general, the market will adjust the forward price for (iv) so that these simultaneous transactions generate neither a profit nor a loss. When the four rates involved are not in line (DEM interest rate, USD/DEM spot rate, USD interest rate and USD/DEM forward rate), there is in fact opportunity for arbitrage – making a profit by round-tripping. That is, either the transactions as shown above will produce a profit or exactly the reverse transactions (borrow \$, sell \$ spot, deposit DEM, sell DEM forward) will produce a profit. The supply and demand effect of this arbitrage activity is such as to move the rates back into line. If in fact this results in a forward rate which is out of line with the market’s “average” view, supply and demand pressure will tend to move the spot rate or the interest rates until this is no longer the case.

In more detail, the transactions might be as follows:

- (i) Borrow DEM 100 at an interest rate of v per annum. The principal and interest payment at maturity will be:

$$100 \times \left(1 + v \times \frac{\text{days}}{360}\right)$$

- (ii) Sell DEM 100 for USD at spot rate to give USD $(100 \div \text{spot})$.
- (iii) Invest USD $(100 \div \text{spot})$ at an interest rate of b per annum. The principal and interest returned at maturity will be:

$$(100 \div \text{spot}) \times \left(1 + b \times \frac{\text{days}}{360}\right)$$

- (iv) Sell forward this last amount at the forward exchange rate to give:

$$\text{DEM } (100 \div \text{spot}) \times \left(1 + b \times \frac{\text{days}}{360}\right) \times \text{forward outright}$$

Arbitrage activity will tend to make this the same amount as that in (i), so that:

$$\text{forward outright} = \text{spot} \times \frac{\left(1 + v \times \frac{\text{days}}{360}\right)}{\left(1 + b \times \frac{\text{days}}{360}\right)}$$

$$\text{Forward outright} = \text{spot} \times \frac{\left(1 + \text{variable currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}{\left(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}$$

**Calculation
summary**

Notice that the length of the year may be 360 or 365, depending on each currency.

Example 7.7

31-day DEM interest rate:	3%
31-day USD interest rate:	5%
Spot USD/DEM rate:	1.6876

$$\text{Then forward outright} = 1.6876 \times \frac{\left(1 + 0.03 \times \frac{31}{360}\right)}{\left(1 + 0.05 \times \frac{31}{360}\right)} = 1.6847$$

.03 ENTER $31 \times 360 \div 1 +$
 .05 ENTER $31 \times 360 \div 1 + \div$
 1.6876 \times

Forward swaps

Although forward outright are an important instrument, banks do not in practice deal between themselves in forward outright, but rather in forward “swaps”, where a forward swap is the difference between the spot and the forward outright. The reason for not dealing in outright will become clear later. The forward outright rate can therefore be seen as a combination of the current spot rate and the forward swap rate (which may be positive or negative) added together.

Key Point

$$\text{Forward outright} = \text{spot} + \text{forward swap}$$

Example 7.8

Spot USD/DEM: 1.6874 / 1.6879
Forward swap: 0.0145 / 0.0150
Forward outright: 1.7019 / 1.7029

Forward outright = spot + swap

In this case: 1.7019 = 1.6874 + 0.0145
1.7029 = 1.6879 + 0.0150

In the previous section, we saw that:

$$\text{forward outright} = \text{spot} \times \frac{\left(1 + \text{variable currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}{\left(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}$$

Since we know that:

$$\text{forward swap} = \text{forward outright} - \text{spot}$$

it follows that:

Calculation summary

$$\text{Forward swap} = \text{spot} \times \frac{\left(\text{variable currency interest rate} \times \frac{\text{days}}{\text{year}} - \text{base currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}{\left(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}$$

As before, the length of each year may be 360 or 365 days. If the year basis is the same for the two currencies and the number of days is sufficiently small, so that the denominator in the last equation is close to 1, the following approximation holds:

Calculation summary

Approximation

$$\text{Forward swap} \approx \text{spot} \times \text{interest rate differential} \times \frac{\text{days}}{\text{year}}$$

In reverse, one can calculate an approximate interest rate differential from the swap rate as follows:

Calculation summary

Approximation

$$\text{Interest rate differential} \approx \frac{\text{forward swap}}{\text{spot}} \times \frac{\text{year}}{\text{days}}$$

Example 7.9

31-day DEM interest rate: 3%
 31-day USD interest rate: 5%
 Spot USD/DEM rate: 1.6876

$$\text{Forward swap} = 1.6876 \times \frac{0.03 \times \frac{31}{360} - 0.05 \times \frac{31}{360}}{1 + 0.05 \times \frac{31}{360}}$$

$$= -0.0029 \text{ or } -29 \text{ points}$$

$$\text{Approximate swap} = 1.6876 \times (-0.02) \times \left(\frac{31}{360}\right) = -29 \text{ points}$$

Example 7.10 shows that the approximation is generally not accurate enough for longer periods.

Example 7.10

1-year DEM interest rate: 3%
 1-year USD interest rate: 10%
 Spot USD/DEM rate: 1.6876

$$\text{Forward swap} = 1.6876 \times \frac{0.03 \times \frac{365}{360} - 0.10 \times \frac{365}{360}}{1 + 0.10 \times \frac{365}{360}}$$

$$= -0.1087 \text{ or } -1087 \text{ points}$$

$$\text{Approximate swap} = 1.6876 \times (-0.07) \times \frac{365}{360} = -1198 \text{ points}$$

Swap prices are quoted as two-way prices in the same way as other prices. In theory, one could use a borrowing rate for Deutschemarks and a deposit rate for US dollars in Example 7.10, to calculate the swap prices where the bank buys US dollars from the customer against Deutschemarks for spot value, and simultaneously sells US dollars to the customer against Deutschemarks for value one month forward. One could then use the deposit rate for Deutschemarks and the borrowing rate for US dollars to determine the other side of the price. However, this would produce a rather large spread. It is more realistic to use middle prices throughout, to calculate a middle price for the swap, and then to spread the two-way swap price around this middle price. In practice, a dealer does not recalculate swap prices continually in any case, but takes them from the market just as the spot dealer takes spot prices.

Discounts and premiums

It can be seen from the formulas given above that when the base currency interest rate is higher than the variable currency rate, the forward outright exchange rate is always less than the spot rate. That is, the base currency is worth fewer forward units of the variable currency forward than it is spot. This can be seen as compensating for the higher interest rate: if I deposit money in the base currency rather than the variable currency, I will receive

more interest. However, if I try to lock in this advantage by selling forward the maturing deposit amount, the forward exchange rate is correspondingly worse. In this case, the base currency is said to be at a “discount” to the variable currency, and the forward swap price must be negative.

The reverse also follows. In general, given two currencies, the currency with the higher interest rate is at a “discount” (worth fewer units of the other currency forward than spot) and the currency with the lower interest rate is at a “premium” (worth more units of the other currency forward than spot). When the variable currency is at a premium to the base currency, the forward swap points are negative; when the variable currency is at a discount to the base currency, the forward swap points are positive.

When the swap points are positive, and the forward dealer applies a bid/offer spread to make a two-way swap price, the left price is smaller than the right price as usual. When the swap points are negative, he must similarly quote a “more negative” number on the left and a “more positive” number on the right in order to make a profit. However, the minus sign “−” is generally not shown. The result is that the larger number appears to be on the left. As a result, whenever the swap price *appears* larger on the left than the right, it is in fact negative, and must be subtracted from the swap rate rather than added.

Example 7.11

German interest rate:	3%
US interest rate:	5%
DEM is at a premium to USD	
USD is at a discount to DEM	
Forward swap points are negative.	

Swap prices are generally quoted so that the last digit of the price coincides with the same decimal place as the last digit of the spot price. For example, if the spot price is quoted to four decimal places (1.6874) and the swap price is “30 points,” this means “0.0030.”

Example 7.12

The Deutschemark is at a premium to the US dollar, and the swap rate is quoted as 30 / 28.

Spot USD/DEM:	1.6874 / 79
1-month swap:	30 / 28
1-month outright:	1.6844 / 1.6851

The spot US dollar will purchase DEM1.6874; the forward US dollar will purchase DEM1.6844. The dollar is therefore worth less in the future, and is thus at a forward discount.

Example 7.13

Spot USD/ITL:	1782.00 / 1784.00
1-month swap:	2.30 / 2.80
1-month outright:	1784.30 / 1786.80

The spot US dollar will purchase ITL 1782.00; the forward US dollar will purchase ITL 1784.30. The dollar is therefore worth more in the future and is thus at a forward premium.

If a forward swap price includes the word “par” it means that the spot rate and the forward outright rate are the same: “par” in this case represents zero. A/P is “around par”, meaning that the left-hand side of the swap must be subtracted from spot and the right-hand side added.

Example 7.14

Spot BEF:	31.98 / 00
1-year swap:	6 / 6 A/P
Forward outright:	31.92 / 32.06

This is often written $-6 / +6$, which means the same as $6 / 6$ A/P but indicates more clearly how the outrights are calculated.

Terminology

It is important to be careful about the terminology regarding premiums and discounts. The clearest terminology, for example, is to say that “the DEM is at a premium to the USD” or that “the USD is at a discount to the DEM;” then there is no ambiguity. If, however, a dealer says that “the USD/DEM is at a discount,” he generally means that the *variable* currency, DEM, is at a discount and that the swap points are to be added to the spot. Similarly, if he says that “the GBP/FRF is at a premium,” he means that the *variable* currency, FRF, is at a premium and that the points are to be subtracted from the spot. If there is no qualification, he is generally referring to the variable currency, not the base currency.

A forward swap position

In order to see why a bank trades in forward swaps rather than forward outright, consider how the following swap and outright rates change as the spot rate and interest rates move:

spot rate	USD interest rate	DEM interest rate	31-day forward outright	31-day forward swap
1.6876	5.0%	3.0%	1.6847	– 0.0029
1.6976	5.0%	3.0%	1.6947	– 0.0029
1.6976	5.5%	3.0%	1.6940	– 0.0036

A movement of 100 points in the exchange rate from 1.6876 to 1.6976 has not affected the forward swap price (to 4 decimal places). However, a change in the interest rate differential from 2.0 percent to 2.5 percent has changed it significantly. Essentially, a forward swap is an interest rate instrument rather than a currency instrument; when banks take forward positions, they are

taking an interest rate view rather than a currency view. If bank dealers traded outright, they would be combining two related but different markets in one deal, which is less satisfactory.

When a bank quotes a swap rate, it quotes in a similar manner to a spot rate. The bank buys the base currency *forward* on the left, and sells the base currency *forward* on the right.

The forward swap deal itself is an exchange of one currency for another currency on one date, to be reversed on a given future date. Thus, for example, when the bank sells USD outright to a counterparty, it may be seen as doing the following:

Bank's spot dealer sells USD spot	spot deal
Bank's forward dealer buys USD spot	} forward swap deal
Bank's forward dealer sells USD forward	
Bank sells USD forward outright	net effect

Therefore on a USD/DEM 1-month forward swap quote of 30 / 28, the bank quoting the price does the following:

30	/	28
sells USD spot		buys USD spot
buys USD forward		sells USD forward

A forward foreign exchange swap is therefore a temporary purchase or sale of one currency against another. An equivalent effect could be achieved by borrowing one currency for a given period, while lending the other currency for the same period. This is why the swap rate formula reflects the interest rate differential (generally based on Eurocurrency interest rates rather than domestic interest rates) between the two currencies, converted into foreign exchange terms.

If a forward dealer has undertaken a similar deal to the one above – bought and sold USD (in that order) as a speculative position – what interest rate view has he taken? He has effectively borrowed dollars and lent Deutschemarks for the period. He probably expects USD interest rates to rise (so that he can relend them at a higher rate) and/or DEM rates to fall (so that he can reborrow them at a lower rate). In fact, the important point is that the interest differential should move in the USD's favour. For example, even if USD interest rates fall rather than rise, the dealer will still make a profit as long as DEM rates fall even further.

Although only one single price is dealt (the swap price), the transaction has two separate settlements:

- a settlement on the spot value date
- a settlement on the forward value date

There is no net outright position taken, and the spot dealer's spread will not be involved, but some benchmark spot rate will nevertheless be needed in order to arrive at the settlement rates. As the swap is a representation of the interest rate differential between the two currencies quoted, as long as the “near” and “far” sides of the swap quotation preserve this differential, it

does not generally make a significant difference which exact spot rate is used as a base for adding or subtracting the swap points. The rate must, however, generally be a current rate. This is discussed further below; see “Historic rate rollovers” and “Discounting future foreign exchange risk.”

Example 7.15

Spot USD/DEM:	1.6874 / 1.6879
31-day DEM interest rate:	3.0%
31-day USD interest rate:	5.0%
31-day forward swap:	30 / 28

Our bank's dealer expects USD interest rates to rise. He therefore asks another bank for its price, which is quoted as 30 / 28. Our dealer buys and sells USD 10 million at a swap price of 30 (that is, -0.0030). The spot rate is set at 1.6876 and the forward rate at 1.6846. The cashflows are therefore:

<i>Spot</i>	<i>31 days forward</i>
buy USD 10,000,000	sell USD 10,000,000
sell DEM 16,876,000	buy DEM 16,846,000

Immediately after dealing, USD rates in fact fall rather than rise, but DEM rates also fall, as follows:

Spot USD/DEM:	1.6874 / 1.6879
31-day DEM interest rate:	2.5%
31-day USD interest rate:	4.75%
31-day forward swap:	34 / 32

Our dealer now asks another counterparty for a price, is quoted 34 / 32, and deals to close out his position. Thus he now sells and buys USD at a swap price of 32 (that is, -0.0032). The spot rate is set at 1.6876 again and the forward rate at 1.6844. The new cashflows are:

<i>Spot</i>	<i>31 days forward</i>
sell USD 10,000,000	buy USD 10,000,000
buy DEM 16,876,000	sell DEM 16,844,000

The net result is a profit of DEM 2,000, 31 days forward. The dealer has made a profit because the interest differential between DEM and USD has widened from 2.0% to 2.25%, even though it did not widen in the way he expected.

In general:

A forward dealer expecting the interest rate differential to move in favour of the base currency (for example, base currency interest rates rise or variable currency interest rates fall) will “buy and sell” the base currency. This is equivalent to borrowing the base currency and depositing in the variable currency.

And vice versa

Key Point

Historic rate rollovers

We have mentioned above that the settlement rates (spot and forward) for a forward swap deal must generally be based on a current market spot rate. This is because many central banks require that banks under their supervision use only current rates. Example 7.16 illustrates why a corporate customer might wish to use a historic rate rather than a current rate, and the effect.

Example 7.16

In June, a German company sells USD 10 million forward outright for value 15 August against DEM, at a forward outright rate of 1.5250. This deal is done to convert money the company expects to receive from export sales. On 13 August, the company realizes that it will not receive the money until a month later. It therefore rolls over the foreign exchange cover by using a forward swap – buying USD spot and selling one month forward.

On 13 August, the exchange rates are as follows:

Spot USD/DEM:	1.6874 / 79
31-day forward swap:	30 / 28

The company therefore buys and sells USD at 1.6876 (spot) and $1.6876 - 0.0030 = 1.6846$ (forward).

The company's cashflows will then be:

15 August	15 September
sell USD 1,000,000	
buy DEM 1,525,000	
buy USD 1,000,000	sell USD 1,000,000
<u>sell DEM 1,687,600</u>	<u>buy DEM 1,684,600</u>
Net:	sell USD 1,000,000
sell DEM 162,600	buy DEM 1,684,600

The overall net result is that the company sells USD 1 million against DEM 1,522,000 (= DEM 1,684,600 – DEM 162,600) – an all-in rate of 1.5220 which is effectively the original rate dealt of 1.5250 adjusted by the swap price of 30 points. The company may however have a cashflow problem on 15 August, because there is a cash outflow of DEM 162,600 then but no inflow. The company might therefore prefer to request the bank to base the swap on the “historic” rate of 1.5250 – dealing instead at 1.5250 spot and 1.5220 forward.

The cashflows would then be:

15 August	15 September
sell USD 1,000,000	
buy DEM 1,525,000	
buy USD 1,000,000	sell USD 1,000,000
<u>sell DEM 1,525,000</u>	<u>buy DEM 1,522,000</u>
Net:	sell USD 1,000,000
	buy DEM 1,522,000

The overall net result is the same as before, but there is no cashflow problem. Underlying this arrangement, however, is an effective loan from the bank to the com-

pany of DEM 162,600 for 31 days. If the bank is, exceptionally, prepared to base the swap on a historic rate, it needs to charge the company interest on this hidden loan. This interest would normally be incorporated into a less favourable swap price.

The reason many central banks discourage historic rate rollovers is that they may help a bank's customer to conceal foreign exchange losses. If a customer has taken a speculative position which has made a loss, a historic rate rollover enables it to roll the loss over to a later date rather than realize it.

Covered interest arbitrage

The link between interest rates and forward swaps allows banks and others to take advantage of different opportunities in different markets. This can be seen in either of two ways. First, suppose that a bank needs to fund itself in one currency but can borrow *relatively* cheaply in another. It can choose deliberately to borrow in the second currency and use a forward swap to convert the borrowing to the first currency. The reason for doing this would be that the resulting all-in cost of borrowing is slightly less than the cost of borrowing the first currency directly.

Second, even if the bank does not need to borrow, it can still borrow in the second currency, use a forward swap to convert the borrowing to the first currency and then make a deposit directly in the first currency. The reason for doing this would be that a profit can be locked in because the swap price is slightly out of line with the interest rates.

Taking advantage of such a strategy is known as “covered interest arbitrage”.

Example 7.17

USD/DEM	spot:	1.4810 / 1.4815
	3-month swap:	116 / 111
USD	3-month interest rates:	7.43% / 7.56%
DEM	3-month interest rates:	4.50% / 4.62%

Suppose that the 3-month period is 92 days and the bank needs to borrow DEM 10 million. It deals on rates quoted to it as above by another bank.

- Bank borrows USD 6,749,915.63 for 92 days from spot at 7.56%.
- At the end of 92 days, bank repays principal plus interest calculated as:

$$\text{principal USD 6,749,915.63 plus interest USD 6,749,915.63} \times 0.0756 \times \frac{92}{360}$$

$$= \text{USD 6,880,324.00}$$
- Bank “sells and buys” USD against DEM at a swap price of 111, based on a spot of 1.4815:

Bank sells USD 6,749,915.63 / buys DEM 10,000,000.00 spot at 1.4815

Bank buys USD 6,880,324.00 / sells DEM 10,116,828.41 3 months forward at 1.4704

The net USD flows balance to zero.

The effective cost of borrowing is therefore interest of DEM 116,828.41 on a principal sum of DEM 10,000,000 for 92 days:

$$\frac{116,828.41}{10,000,000} \times \frac{360}{92} = 4.57\%$$

The net effect is thus a DEM 10 million borrowing at 4.57% – 5 basis points cheaper than the 4.62% at which the bank could borrow directly.

If the bank is in fact not looking for funds, but is able to deposit DEM at higher than 4.57%, it can instead “round trip”, locking in a profit.

Three points to notice in Example 7.17 are:

1. The example assumes that we are not the bank quoting the price; we are taking another bank's rates to borrow at 7.56 percent and swap at 111 points. If we were able to trade on our own prices, the result would be even better.
2. When a swap is dealt, the amount of the deal (e.g. USD 6,749,915.63) is usually the same at both ends of the deal, spot and forward. In the example above, the amounts are mismatched, with USD 6,880,324.00 at the far end in order to match the cashflows exactly with the underlying flows arising from the borrowing. It is generally acceptable in the market to use mismatched amounts in this way as long as the mismatch is not great.
3. When dealing on a forward swap rather than a forward outright, it is the swap price that is dealt rather than the spot price; the spot price is needed only for settlement. The spot dealer is not involved and the spot spread is not involved. In general therefore, the spot and forward settlement prices could be 1.4815 and 1.4704 (as in Example 7.17) or 1.4810 and 1.4699 or something between. The spot rate must be a current market rate and the difference between the spot and forward settlement prices must be the correct swap price of 111 points. Conventionally, an approximate mid-price is taken for the spot.

However, in Example 7.17, because the amounts are mismatched, it is usual to use for the *whole* deal, whichever side of the spot price would normally be used for the mismatch amount. The size of the mismatch in this example is a forward sale by the quoting bank of USD 130,408.37 (= 6,880,324.00 – 6,749,915.63). The quoting bank will wish to deal on the right for this, based on a spot of 1.4815. This is therefore usually the spot rate used for the deal. Although this approach is common, it does not in fact necessarily benefit the quoting bank. In Example 7.17, for instance, the quoting bank is actually slightly worse off using a spot of 1.4815 and would instead benefit from using 1.4810. In general, this depends on which of the two currencies' interest rates is higher; the effect is in any case generally not great.

The formula we saw earlier for calculating a forward outright from interest rates was:

$$\text{forward swap} = \text{spot} \times \frac{\left(1 + \text{variable currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}{\left(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{year}}\right)}$$

This can be turned round to give the result of the covered interest arbitrage from the swap price:

Covered interest arbitrage

Creating the variable currency interest rate:

variable currency rate =

$$\left[\left(1 + \text{base currency rate} \times \frac{\text{days}}{\text{base year}} \right) \times \frac{\text{outright}}{\text{spot}} - 1 \right] \times \frac{\text{variable year}}{\text{days}}$$

or

Creating the base currency interest rate:

base currency rate =

$$\left[\left(1 + \text{variable currency rate} \times \frac{\text{days}}{\text{variable year}} \right) \times \frac{\text{spot}}{\text{outright}} - 1 \right] \times \frac{\text{base year}}{\text{days}}$$

**Calculation
summary**

Example 7.18

Using the same data as in Example 7.17, we have:

variable currency interest rate created

$$= \left[\left(1 + 7.56\% \times \frac{92}{360} \right) \times \frac{1.4704}{1.4815} - 1 \right] \times \frac{360}{92}$$

$$= 4.57\%$$

.0756 ENTER 92 x 360 ÷ 1 + 1.4704 x 1.4815 ÷ 1 - 360 x 92 ÷

CROSS-RATE FORWARDS

Outrights

A forward cross-rate is a forward exchange rate between two currencies other than the US dollar. These are calculated in a similar way to spot cross-rates. To calculate a forward outright cross-rate between two indirect rates, divide opposite sides of the forward rates against the US dollar.

Example 7.19

Spot USD/MYR:	2.4782 /	92
6-month swap:	90 /	95
6-month outright:	2.4872 /	2.4887
Spot USD/DEM:	1.6874 /	79
6-month swap:	155 /	150
6-month outright:	1.6719 /	1.6729

Therefore 6-month DEM/MYR is:

$$\frac{2.4872}{1.6729} = 1.4868: \text{how the quoting bank buys DEM, sells MYR}$$

$$\frac{2.4887}{1.6719} = 1.4885: \text{how the quoting bank sells DEM, buys MYR}$$

The forward outright cross-rate is therefore 1.4868 / 1.4885.

To calculate a forward outright cross-rate between two direct rates, again divide opposite sides of the forward rates against the US dollar. Similarly, to calculate a forward outright cross-rate between one direct rate and one indirect rate, multiply the same sides.

Example 7.20

Spot GBP/USD:	1.6166 /	71
1-month swap:	14 /	9
1-month outright:	1.6152 /	1.6162
Spot AUD/USD:	0.7834 /	39
1-month swap:	60 /	55
1-month outright:	0.7774 /	0.7784

Therefore 1-month GBP/AUD is:

$$\frac{1.6152}{0.7784} = 2.0750: \text{how the quoting bank buys GBP, sells AUD}$$

$$\frac{1.6162}{0.7774} = 2.0790: \text{how the quoting bank sells GBP, buys AUD}$$

The forward outright cross-rate is therefore 2.0750 / 2.0790.

Swaps

To calculate cross-rate forward swaps, the process above must be taken a step further:

1. calculate the spot cross-rate
2. calculate the two individual forward outrights
3. from (2) calculate the forward outright cross-rate
4. from (1) and (3) calculate the cross-rate swap

Example 7.21

Using the same details as in Example 7.19, the DEM/MYR cross-rate swap can be calculated as follows:

From Example 7.19, the forward outright rate is: 1.4868 / 1.4885

From Example 7.2, the spot rate is: $\frac{1.4682}{1.4692}$

Therefore the forward swap is (outright – spot): 0.0186 / 0.0193

That is, 186 / 193.

SHORT DATES

Value dates earlier than one month are referred to as “short dates”. There are certain “regular” dates usually quoted, and the terminology used is the same as in the deposit market, as follows:

Overnight A deposit or foreign exchange swap from today until “tomorrow.”

Tom-next A deposit or foreign exchange swap from “tomorrow” to the “next” day (spot).

Spot-next A deposit or foreign exchange swap from spot until the “next” day.

Spot-a-week A deposit or foreign exchange swap from spot until a week later.

Tomorrow means “the next working day after today” and **next** means “the next working day following.”

When referring to outright deals rather than swaps, one refers to **value today**, **value tomorrow**, **value spot-next**, **value a week over spot**.

Terminology

In considering swaps and outright forwards for short dates later than the spot date, exactly the same rules apply as in calculating longer dates. However, confusion can arise in considering the prices for dates earlier than spot – that is, value today and tomorrow. The rules are still the same in that the bank buys the base currency on the far date on the left and sells the base currency on the far date on the right. In other words, the bank always “sells and buys” (in that order) the base currency on the left and “buys and sells” the base currency on the right – regardless of whether it is before or after spot. The confusion can arise because the spot value date – effectively the baseline date for calculation of the outright rate – is the near date when calculating most forward prices. For value today and tomorrow, the spot date becomes the far date and the outright date is the near date.

Example 7.22

Spot USD/DEM: 1.5505 / 10

Overnight swap: 1 / $\frac{3}{4}$

Tom-next swap: $\frac{1}{2}$ / $\frac{1}{4}$

1-week swap: 7 / 5

- (i) Suppose a customer wishes to buy DEM for outright value one week after spot. The bank spot dealer sells DEM for value spot on the left at 1.5505. The bank forward dealer sells DEM for value on the “far” date (= one week after spot) also on the left at a swap difference of 7 points. Therefore the bank sells DEM out-

right one week after spot at $1.5505 - 0.0007 = 1.5498$. The other side of the one week outright price is $1.5510 - 0.0005 = 1.5505$.

- (ii) Suppose the customer wishes to buy DEM for outright value tomorrow. This is equivalent to *buying* DEM for value spot and, at the time, undertaking a swap to *buy* DEM for value tomorrow and *sell* DEM back for value spot.

Again, the bank spot dealer buys USD for value spot on the left at 1.5505. However, the bank forward dealer sells USD for value on the “far” date (= *spot* this time) on the right at a swap difference of $\frac{1}{4}$ point. Furthermore (because DEM interest rates are lower than USD rates) the USD is at a discount to the DEM: the “bigger number” $\frac{1}{2}$ is on the left. The USD is therefore worth less on the “far” date and more on the “near” date. The swap difference is therefore *added* to the spot rate to give an outright value tomorrow price of $1.5505 + \frac{1}{4} = 1.550525$. The other side of the value tomorrow outright price is $1.5510 + \frac{1}{2} = 1.55105$.

A simple rule to remember for the calculation of dates *earlier* than spot is “*reverse* the swap points and proceed exactly as for a forward later than spot.” In Example 7.22, this would mean reversing $\frac{1}{2} / \frac{1}{4}$ to $\frac{1}{4} / \frac{1}{2}$. The outright value tomorrow price is then $(1.5505 + \frac{1}{4}) / (1.5510 + \frac{1}{2})$, obtained by adding the swap points to the spot rate because the “bigger” swap number is now on the right. **However, it is important always to remember to make this reversal in your head only! Never actually quote the price in reverse!**

“Overnight” prices are the only regular swap prices not involving the spot value date. To calculate an outright value today price, it is therefore necessary to combine the “overnight” price with the “tom-next” price:

- (iii) Suppose the customer wishes to buy DEM for outright value today. This is equivalent to three separate transactions: *buying* DEM for value spot, undertaking a swap to *buy* DEM for value tomorrow and *sell* DEM back for value spot (“tom-next”) and undertaking another swap to *buy* DEM for value today and *sell* DEM back for value tomorrow (“overnight”). The price is therefore $1.5505 + \frac{1}{4} + \frac{3}{4} = 1.5506$.

The “rules” can be thought of in terms of premiums and discounts, which apply in the same way as in forwards after spot. The swaps in the previous example show a USD discount because DEM interest rates are lower than USD interest rates. Consequently, if the customer buys DEM value today and not value spot, he/she will receive the currency with the lower interest rate two days early. The extra point which he/she receives from the bank reflects this.

Deals cannot always be done for value today. For example, when London and European markets are open, the Japanese banks have already closed their books for today, so deals in yen can only be done for value tomorrow. Similarly, in London, most European currencies can only be dealt early in the morning for value today, because of the time difference and the mechanical difficulties of ensuring good value. Even the market for value “tomorrow” generally closes during the morning.

Some further examples follow. “Overnight,” “tom-next,” “spot-next” and “spot-a-week” are often abbreviated as O/N, T/N, S/N and S/W respectively.

Example 7.23

GBP/USD spot rate: 1.5103 / 13

O/N: $\frac{1}{4}$ / $\frac{1}{2}$

T/N: $\frac{1}{4}$ / $\frac{1}{2}$

S/N: $\frac{1}{4}$ / $\frac{1}{2}$

The bank's customers can make purchases and sales as follows:

Value S/N: Outright purchase of USD and sale of GBP:
 1.5103 + 0.000025 = 1.510325

 Outright sale of USD and purchase of GBP:
 1.5113 + 0.00005 = 1.51135

Value tomorrow: Outright purchase of USD and sale of GBP:
 1.5103 – 0.00005 = 1.51025

 Outright sale of USD and purchase of GBP:
 1.5113 – 0.000025 = 1.511275

Value today: Outright purchase of USD and sale of GBP:
 1.5103 – 0.0001 = 1.5102

 Outright sale of USD and purchase of GBP:
 1.5113 – 0.00005 = 1.51125

CALCULATION SUMMARY

It may be helpful to collect together here various “rules” which apply to calculating forwards:

1. The currency with higher interest rates (= the currency at a “discount”) is worth less in the future.

The currency with lower interest rates (= the currency at a “premium”) is worth more in the future.

2. The bank quoting the price buys the base currency / sells the variable currency on the far date on the left.

The bank quoting the price sells the base currency / buys the variable currency on the far date on the right.

For outright forwards later than spot

3. The right swap price is added to (or subtracted from) the right spot price.

The left swap price is added to (or subtracted from) the left spot price.

4. If the swap price is larger on the right than the left, add it to the spot price.



Key Points

If the swap price is larger on the left than the right, subtract it from the spot price.

For outright deals earlier than spot

5. Calculate as if the swap price were reversed and then follow (3) and (4).

In general

6. Of the two prices available, the customer gets the worse one. Thus if the swap price is 3 / 2 and the customer knows that the points are “in his favour” (the outright will be better than the spot), the price will be 2. If he knows that the points are “against him” (the outright will be worse than the spot), the price will be 3.
7. The effect of combining the swap points with the spot price will always be to widen the spread, never to narrow it.

VALUE DATES

Swap rates are normally quoted for “regular” dates – for example 1, 2, 3, 6 and 12 months forward. They are quoted over the spot date. This means that the one-month swap rates are calculated for one calendar month after the present spot date. If the current spot date is 21 April, the one month forward date will be 21 May. If the forward delivery date falls on a weekend or holiday, the value date becomes the next working day. No adjustment in the forward value date is made for any weekends or public holiday *between* the spot date and the forward delivery date.

An exception to these rules is when the spot value date is at or close to the end of the month. If the spot value date is the last working day of a month, the forward value date is the last working day of the corresponding forward month; if necessary, the forward value date is brought *back* to the nearest previous business day in order to stay in the same calendar month, rather than moved forward to the beginning of the next month. This is referred to as dealing “end/end.”

Example 7.24

Ordinary Run

Dealing date:	Friday 14 April
Spot date:	Tuesday 18 April (2 working days forward)
1 month:	Thursday 18 May
2 months:	Monday 19 June (18 June is a Sunday)
3 months:	Tuesday 18 July
4 months:	Friday 18 August

End/End

Dealing date:	Wednesday 26 June
Spot date:	Friday 28 June (last working day of June)
1 month:	Wednesday 31 July (last working day of July)
2 months:	Friday 30 August (last working day of August)
etc.	

Similarly, even if the spot value date is earlier than the last working day of the month, but the forward value date would fall on a non-working day, this is still brought back rather than moved later, if necessary to keep it in the appropriate month.

Example 7.25

Dealing date:	Monday 28 August
Spot date:	Wednesday 30 August (not the last working day)
1 month:	Friday 29 September (30 September is a Saturday)
2 months:	Monday 30 October
etc.	

If a bank deals in any month that is not a regularly quoted date, for example, for four or five months' maturity, this is called an "in-between" month because it is between the regular dates. A forward deal may in fact be arranged for value on *any* day which is a working day in both currencies. Dates which do not fit in with calendar month dates are called "broken dates" or "odd dates." The forward swap points are generally calculated by straightline interpolation between the nearest whole month dates either side.

FORWARD-FORWARDS

A forward-forward swap is a swap deal between two forward dates rather than from spot to a forward date – for example, to sell US dollars one month forward and buy them back in three months' time. In this case, the swap is for the two-month period between the one-month date and the three-month date. A company might undertake such a swap because it has previously bought dollars forward but wishes now to defer the transaction by a further two months, as it will not need the dollars as soon as it thought.

From the bank's point of view, a forward-forward swap can be seen as two separate swaps, each based on spot.

Example 7.26

USD/DEM spot rate:	1.5325 / 35
1-month swap:	65 / 61
3-month swap:	160 / 155

If our bank's counterparty wishes to sell USD one month forward, and buy them three months forward, this is the same as undertaking one swap to buy USD spot

and sell USD one month forward, and another swap to sell USD spot and buy USD three months forward.

As swaps are always quoted as how the quoting bank buys the base currency forward on the left, and sells the base currency forward on the right, the counterparty can “buy and sell” USD “spot against one month” at a swap price of -65, with settlement rates of spot and (spot – 0.0065). He can “sell and buy” USD “spot against three months” at a swap price of -155 with settlement rates of spot and (spot – 0.0155). He can therefore do both simultaneously – “sell and buy” USD “one month against three months” – at settlement rates of (spot – 0.0065) and (spot – 0.0155), which implies a difference between the two forward prices of (-155) – (-65) = -90 points.

Conversely, the counterparty can “buy and sell” USD “one month against three months” at a swap price of (-160) – (-61) = -99 points. The two-way price is therefore -99 / -90, quoted as usual without the “-” signs, as 99 / 90.

As with a swap from spot to a forward date, the two settlement prices in a forward-forward must be based on a current market rate. In Example 7.26, using the middle spot rate, the settlement rates could be 1.5265 (= 1.5330 – 0.0065) for 1 month forward and 1.5175 (= 1.5330 – 0.0155) for 3 months forward.

These settlement rates would enable our forward dealer to cover his position exactly with another bank. We could, for example, ask another bank for a 1-month swap price to cover the first leg of the forward-forward. Assuming prices have not moved, we could deal at -65 points with settlement rates of 1.5330 (spot) and 1.5265 (1 month). We could then cover the second leg with a 3-month swap at another bank’s price of -155, with settlement rates of 1.5330 (spot) and 1.5175 (3 months). The spot settlements would offset each other and the forward settlements would exactly offset the settlements with our own counterparty.

In practice, however, forward dealers often base the settlement rate for the first leg on a middle rate for spot and a middle rate for the near forward date. In the example above, this would give a settlement rate of 1.5330 (middle) – 0.0063 (middle) = 1.5267. The settlement rate for the second leg would then be 1.5267 – 0.0090 = 1.5177. The difference between the two settlement rates is still the -90 points agreed, but the settlement rates are slightly different.

The calculation rule to create the forward-forward price after spot is as follows:

Calculation
summary

Forward-forward price after spot	
Left side = (left side of far-date swap) – (right side of near-date swap)	
Right side = (right side of far-date swap) – (left side of near-date swap)	

Note that the bid-offer spread of the resulting price is the sum of the two separate bid-offer spreads.

Care needs to be taken with swaps from before spot to after spot:

Example 7.27

USD/DEM spot rate	1.5325 / 35
T/N swap:	3 / 2
3-month swap:	160 / 155

If a counterparty requests a price to sell and buy USD tomorrow against 3 months after spot, this can be seen as a price to sell tomorrow and buy spot at (-2) points with settlement rates of (spot + 0.0002) and spot, and a price to sell spot and buy 3 months later at (-155) points with settlement rates of spot and (spot - 0.0155). The total price is therefore the difference between (spot - 0.0155) and (spot + 0.0002), which is $(-155) - (+2) = -157$. The other side of the price is $(-160) - (+3) = -163$. The two-sided price is therefore 163 / 157.

TIME OPTIONS

When a bank makes a forward outright deal with a company, it will quote a rate for a fixed date, which means the company must deliver one currency and receive another on that date.

If the company has a commitment in the future but does not know the exact delivery date, it has an alternative means of covering this exposure in the traditional foreign exchange market, using “time options.” These allow the company to deal now, but to choose the maturity date later, within a specified period. Delivery must take place at some point during that period, however, for the amount and rate agreed.

It is important not to confuse “time options” in this sense with “currency options” which are covered later. “Currency options” entail the up-front payments of an “insurance premium”, in return for which the customer has the right to choose whether or not to deal at all.

In pricing a time option, the bank will always assume that the company will take delivery of the currency at the worst possible time for the bank. Therefore the company will always be charged the worst possible forward rate within the period of the time option.

Example 7.28

USD/DEM	Spot rate:	1.7950 / 60
	6-month swap:	485 / 475
	7-month swap	560 / 550

If a customer wants to buy DEM with an option for delivery between six and seven months, the bank will assume in pricing the option that delivery will be after seven months (560 points against the customer). However, if the customer wants to sell DEM with an option for delivery between six and seven months, the bank will assume that delivery will be after six months (only 475 points in the customer's favour). The time option price will therefore be $(1.7950 - 0.0560) / (1.7960 - 0.0475) = 1.7390 / 1.7485$.

The advantage of a time option to a company is its flexibility. The company can lock in a fixed exchange rate at which it knows it can deal. There is no exposure to interest rate changes which would affect it if commitments were covered with a forward outright which subsequently needed to be adjusted by means of forward swaps. The disadvantage is the cost, given the wide bid/offer spread involved, particularly if the time period of the option is wide.

Key Point

A time option price is the best for the bank / worst for the customer over the time option period

LONG-DATED FORWARDS

The formula we have already seen for a forward outright less than one year is:

$$\text{Forward outright} = \text{spot} \times \frac{\left(1 + \text{variable interest rate} \times \frac{\text{days}}{\text{year}}\right)}{\left(1 + \text{base interest rate} \times \frac{\text{days}}{\text{year}}\right)}$$

This is derived from the fact that the interest on a Eurocurrency deposit or loan is paid on a simple basis. For deposits and loans over one year, the interest must be compounded. The formula for a forward over one year will be, correspondingly:

where N is the number of years, and the interest rates are quoted on the basis of a true calendar year rather than a 360-day year. This theoretical formula

$$\text{Forward outright} = \text{spot} \times \frac{(1 + \text{variable interest rate})^N}{(1 + \text{base interest rate})^N}$$

is not precise in practice for two reasons. First, this compounding does not take account of reinvestment risk. This problem could be overcome by using zero-coupon yields for the interest rates. More importantly, the market in long-dated forwards is not very liquid and spreads are very wide. The prices available in practice therefore depend partly on banks' individual positions and hence their interest in quoting a price.

Calculation summary

For long-dated forwards

$$\text{Forward outright} = \text{spot} \times \frac{(1 + \text{variable interest rate})^N}{(1 + \text{base interest rate})^N}$$

SYNTHETIC AGREEMENTS FOR FORWARD EXCHANGE (SAFEs)

A SAFE is essentially a form of off-balance sheet forward-forward foreign exchange swap. There are two versions of a SAFE – an FXA and an ERA.

An FXA (foreign exchange agreement) exactly replicates the economic effect of a forward-forward in the cash market which is reversed at the first forward date to take a profit or loss. A price for an FXA is quoted in the same way as a forward-forward, and is dealt in the same way. A forward-forward swap results in two cash settlements, each for the whole notional amount of the deal. An FXA, however, is settled in a manner analogous to the way an FRA is settled. On the nearer of the two forward dates, a settle-

ment amount is paid from one party to the other to reflect the movement in rates between dealing and settlement. The settlement amount is calculated using agreed settlement rates, such as the BBA (British Bankers' Association) settlement rates published on Reuters. The settlement formula ensures that the result is the same as the profit or loss would be with a cash forward-forward. To do so, the formula takes account of the movement in the spot rate from dealing to settlement, as well as the movement in the forward swap points. This is because although a forward dealer takes a position on the basis of his expectations of swap movements his profit / loss is also affected to some extent by spot rate movements. The reason for this is discussed in Discounting Future Foreign Exchange Risk later. As with an FRA, the settlement formula involves an element of discounting, because the settlement is made at the beginning of the forward-forward period.

An ERA (exchange rate agreement) price is exactly the same as an FXA price, and allows for a discounted settlement to be made at the beginning of the forward-forward period in the same way as an FXA. The ERA settlement, however, deliberately takes no account of the movement in the spot rate, and the settlement formula is correspondingly simpler. The two instruments can therefore be compared as follows:

- An FXA replicates exactly the effect of a cash forward-forward. In a trading strategy that requires a forward-forward therefore, an FXA is an alternative.
- An ERA may be used to trade movements in forward swap prices when the trader specifically wishes the result to be unaffected by movements in the spot rate. This does not exactly replicate a forward-forward.

Key Point

Advantages of SAFEs

The advantages of a SAFE lie in the problems which arise on credit lines and balance sheet utilization. In the absence of an appropriate netting agreement, when a bank deals forward-forward its credit lines to and from the counterparty are generally utilized to the extent of twice the notional credit amount applied to the deal – once for each leg of the forward-forward. Between dealing date and the first settlement date, however, the risk may be far less than this, because the two legs of the deal largely offset each other. The credit exposure allocated to a SAFE, as a contract for differences, will be far less. The capital utilization requirements are similarly reduced. Rather than having two deals on the balance sheet, there are none.

A further advantage of an ERA arises if a forward trader wishes to trade the forward swap points without needing to hedge the effect of potential spot rate movements (although this does not avoid the effect that a significant spot rate movement may have on a swap price itself).

FXAs (foreign exchange agreements)

In order to understand the FXA settlement formula, it is helpful to look at the actual cashflows which would arise in a cash forward-forward deal which is reversed two working days before the first forward date. It is then possible to build up the transactions and cashflows implied in the FXA settlement formula.

Example 7.29

Our dealer sells and buys USD10 million, 1 month against 3 months, when rates are as follows:

USD/DEM spot:	1.7400
1-month swap (31 days):	50 / 51
3-month swap (92 days):	153 / 156

We ask for a price from another bank and deal at 106 points (= 156 – 50). The settlement rates will be 1.7450 (= 1.7400 + 0.0050) and 1.7556 (= 1.7400 + 0.0156). Our cashflows will be as follows:

<i>1 month</i>	<i>3 months</i>
– USD 10,000,000	+ USD 10,000,000
+ DEM 17,450,000	– DEM 17,556,000

After 1 month, our dealer reverses the position by buying and selling USD10 million spot against 2 months. Rates are now as follows and he deals on another bank's prices at 1.7500 and 1.7618:

USD/DEM spot:	1.7500
2-month swap (61 days):	118 / 120
2-month DEM interest rate:	5.3%

Our cashflows are now as follows:

<i>1 month</i>	<i>3 months</i>
– USD 10,000,000	+ USD 10,000,000
+ DEM 17,450,000	– DEM 17,556,000
+ USD 10,000,000	– USD 10,000,000
– DEM 17,500,000	+ DEM 17,618,000
Net: – DEM 50,000	+ DEM 62,000

Having reached the 1-month date and ascertained these cashflows, we are able effectively to move the cashflow of + DEM 62,000 from 3 months to now, by borrowing the present value of that amount. That is, we can borrow the amount

$\frac{\text{DEM } 62,000}{(1 + 0.053 \times \frac{61}{360})} = \text{DEM } 61,448$ for 2 months. If we do this, our cashflows become:

<i>1 month</i>	<i>3 months</i>
– USD 10,000,000	+ USD 10,000,000
+ DEM 17,450,000	– DEM 17,556,000
+ USD 10,000,000	– USD 10,000,000
– DEM 17,500,000	+ DEM 17,618,000
+ DEM 61,448	– DEM 62,000
Net: + DEM 11,448	–

After 1 month, we can therefore crystallize the result of the deals as +DEM 11,448.

In Example 7.29, the amount of + DEM 62,000 can be seen as arising from:

$$10 \text{ million} \times (1.7618 - 1.7556) \\ = -10 \text{ million} \times [(1.7450 - 1.7500) + (0.0106 - 0.0118)]$$

Therefore the amount + DEM 61,448 can be seen as:

$$- \frac{10 \text{ million} \times [(1.7450 - 1.7500) + (0.0106 - 0.0118)]}{\left(1 + 5.3\% \times \frac{61}{360}\right)}$$

Similarly, the amount of – DEM 50,000 can be seen as arising from:

$$+ 10 \text{ million} \times (1.7450 - 1.7500)$$

The net result of + DEM 11,448 therefore comes from:

$$-10 \text{ million} \times \left[\frac{(1.7450 - 1.7500) + (0.0106 - 0.0118)}{\left(1 + 5.3\% \times \frac{61}{360}\right)} \right] \\ + 10 \text{ million} \times [1.7450 - 1.7500]$$

In an FXA, we achieve exactly the same result as in Example 7.29, but without any of the transactions shown. Instead, we agree with a counterparty to make a settlement based on the same settlement formula, which can be expressed as:

The FXA settlement amount

$$A_2 \times \left[\frac{(\text{OER} - \text{SSR}) + (\text{CFS} - \text{SFS})}{\left(1 + L \times \frac{\text{days}}{\text{year}}\right)} \right] - A_1 \times [\text{OER} - \text{SSR}]$$

paid by the “seller” to the “buyer” of the FXA (or vice versa if it is a negative amount), where the “buyer” is the party which buys the base currency on the first date and sells it on the second date.

**Calculation
summary**

and where:

A_1 = the base currency amount transacted at the beginning of the swap.

A_2 = the base currency amount transacted at the end of the swap.

(A_1 and A_2 may be the same amount, as in the above example, but do not need to be.)

OER = the outright exchange rate, when the FXA is transacted, to the beginning of the swap.

SSR = the settlement spot rate two working days before the swap period.

CFS = the contract forward spread – that is, the swap price agreed when the FXA is transacted.

SFS = the settlement forward spread – that is, the swap price used for settlement two working days before the swap period.

L = variable currency LIBOR for the swap period, two working days before the swap period.

days = the number of days in the swap period.

year = the year basis for the variable currency.

Example 7.30

Based on the same data as in Example 7.29, we have:

$$\begin{aligned}
 A_1 &= 10,000,000 \\
 A_2 &= 10,000,000 \\
 OER &= 1.7450 (= 1.7400 + 0.0050) \\
 SSR &= 1.7500 \\
 CFS &= 0.0106 (= 0.0156 - 0.0050) \\
 SFS &= 0.0119 \\
 L &= 0.053 \\
 \text{days} &= 61 \\
 \text{year} &= 360
 \end{aligned}$$

The settlement amount is therefore:

$$\begin{aligned}
 &10,000,000 \times \left[\frac{(1.7450 - 1.7500) + (0.0106 - 0.0119)}{\left(1 + 0.053 \times \frac{61}{360}\right)} \right] \\
 &\quad - 10,000,000 \times [1.7450 - 1.7500] \\
 &= -12,439.26
 \end{aligned}$$

That is, DEM12,439.26 is paid to the seller of the FXA.

1.745 ENTER 1.75 - .0106 + .0119 -
 .053 ENTER 61 × 30 ÷ 1 + ÷ 10,000,000 ×
 1.745 ENTER 1.75 - 10,000,000 × -

Note that in the case of the FXA, the settlement is made on the basis of a published settlement forward spread which should be a mid-rate (0.0119 rather than 0.0118, for example) – hence the slightly improved result for our dealer.

To see that this is a “fair” settlement, consider that the counterparty (the buyer of the FXA) can cover its position at no risk by dealing as follows:

- At the outset, sell and buy USD10 million one month against three months.
- After one month, buy and sell USD10 million spot against two months.
- Also after one month, borrow DEM61,448 for two months at LIBOR (5.3%).

ERAs (exchange rate agreements)

An FXA exactly replicates a forward-forward swap in that the final settlement is economically the equivalent of transacting a forward-forward swap – with either equal or unequal amounts at each end of the swap. An FXA is affected by changes in both spot and forward swap rates.

An ERA is similar but is only affected by changes in the swap rate. Thus the settlement amount is simply the notional amount of the contract multiplied by the change in the swap rate. As the settlement is made at the beginning of the swap period rather than at the end, it is discounted back to

a present value in the same way as an FRA settlement or an FXA settlement. With the same notation as with an FXA, the settlement amount is therefore:

The ERA settlement amount

$$A \times \left(\frac{\text{CFS} - \text{SFS}}{1 + L \times \frac{\text{days}}{\text{year}}} \right)$$

**Calculation
summary**

In this case, only one notional amount A is involved. Again, the settlement amount is paid to the buyer of the ERA if it is positive.

Example 7.31

Our dealer expects US interest rates to fall and, by selling an ERA for USD10 million, takes a speculative position. Rates are the same as in Example 7.30:

USD/DEM spot: 1.7400
1-month swap: 50 / 51
3-month swap: 153 / 156

After one month, the rates are as follows:

USD/DEM spot: 1.7500
2-month settlement forward spread: 0.0119
2-month DEM LIBOR: 5.3%

As before, we have:

A = 10,000,000
CFS = 0.0106 (= 0.0156 – 0.0050)
SFS = 0.0119
L = 0.053
days = 61
year = 360

The settlement amount is:

$$10,000,000 \times \frac{(0.0106 - 0.0119)}{\left(1 + 0.053 \times \frac{61}{360}\right)} = -12,884.29$$

That is, DEM 12,884.29 is paid to our dealer as the seller of the ERA. Our dealer has made a profit because the forward price has moved as he expected.

$$0.0106 \text{ ENTER } .0119 - .053 \text{ ENTER } 61 \times 360 \div 1 + \div 10,000,000 \times$$

ARBITRAGING AND CREATING FRAs

In addition to using SAFEs for taking positions as an alternative to forward-forwards, FXAs can be used to arbitrage between FRAs in different currencies, or to create synthetic FRAs. Because theoretical forward swap prices and FRAs are both linked mathematically to the current Eurointerest rates, it should be possible to round-trip between FRAs and forward-forwards at zero cost – or, if

prices are out of line, to make an arbitrage profit. Because FXAs are economically equivalent to forward-forwards, this round-trip can be made using FRAs and FXAs instead. Essentially, the following holds:

Key Point

$$\text{Buy FRA in currency A} = \text{buy FRA in currency B} \\ + \text{buy FXA in exchange rate A/B}$$

In this way, it is possible to create an FRA in one currency – say a currency in which FRAs are not finely priced – from an FRA in another currency. Similarly:

Key Point

$$\text{Buy FRA in currency A} + \text{sell FRA in currency B} \\ + \text{sell FXA in exchange rate A/B} = 0$$

In this way, it is possible to arbitrage between FRAs in two currencies using the FXA market, if prices are out of line.

Example 7.32 works through the details of how a SEK FRA can be created from a USD FRA. In order to clarify all the underlying cashflows – which are largely absorbed into one settlement figure when an FXA is used – the example uses cash forward-forwards instead of an FXA. We have then repeated the example using an FXA. In both cases, we have used middle rates for simplicity and clarity, rather than separate bid and offer rates.

Example 7.32

Rate structure at the start:

	<i>USD/SEK</i>	<i>USD%</i>	<i>SEK%</i>
Spot:	6.0000		
3 months (91 days):	6.0598	6.00	10.00
6 months (182 days):	6.1178	6.00	10.00

The 3 v 6 USD FRA rate is:

$$\left[\frac{\left(1 + 0.06 \times \frac{182}{360}\right)}{\left(1 + 0.06 \times \frac{91}{360}\right)} - 1 \right] \times \frac{360}{(182 - 91)} = 5.91\%$$

The theoretical 3 v 6 SEK FRA rate is:

$$\left[\frac{\left(1 + 0.10 \times \frac{182}{360}\right)}{\left(1 + 0.10 \times \frac{91}{360}\right)} - 1 \right] \times \frac{360}{(182 - 91)} = 9.753\%$$

Outline structure

Assuming that it is not possible to obtain satisfactory FRA prices in SEK from a bank, we will create a synthetic SEK FRA by combining a USD FRA with various forward foreign exchange deals. This is structured as follows:

- (i) Assume a borrowing in USD for 3 months, starting in 3 months' time. Arrange now a USD FRA 3 v 6 on this USD borrowing
- (ii) Convert the resulting fixed-cost USD borrowing to a fixed-cost SEK borrowing by a forward-forward swap:
selling USD / buying SEK now for value in 3 months' time
buying USD / selling SEK now for value in 6 months' time
- (iii) After 3 months, instead of the dollar borrowing which has been assumed, this borrowing is synthesized from a 3-month SEK borrowing and more forward deals. To achieve this, it is necessary after 3 months to:
- (iv) Convert the SEK borrowing to a USD borrowing by:
buying USD / selling SEK for value spot
selling USD / buying SEK for value 3 months forward

Detailed structure

- (i) Assume a borrowing of USD $\frac{1,000,000}{6.0598} = \text{USD } 165,021.95$ in 3 months' time.
This will achieve SEK 1,000,000 if it is converted forward now.

Arrange a 3 v 6 FRA at 5.91% on USD 165,021.95. Although the FRA settlement will in fact be after 3 months on a discounted basis, the economic effect will be as if we had borrowed at 5.91% for the period, with the following total repayment at the end of 6 months (we can in fact achieve exactly this result by investing or borrowing the FRA settlement amount from 3 months to 6 months at LIBOR):

$$\text{USD } 165,021.95 \times \left(1 + 0.0591 \times \frac{91}{360}\right) = \text{USD } 167,487.24$$

- (ii) 3 months forward: sell USD 165,021.95 / buy SEK 1,000,000.00 (at 6.0598)
6 months forward: buy USD 167,487.24 / sell SEK 1,024,653.44 (at 6.1178)

After 3 months from the start, suppose the following new rate structure:

	USD/SEK	USD%	SEK%
Spot:	7.0000		
3 months (91 days):	7.1223	5.00	12.00

- (i) The discounted FRA settlement amount is

$$\frac{\text{USD } 165,021.95 \times (0.0591 - 0.05) \times \frac{91}{360}}{\left(1 + 0.05 \times \frac{91}{360}\right)} = \text{USD } 374.86$$

We borrow this settlement amount (which is to be paid by us because interest rates have fallen) for 3 months at 5.00% to give an all-in settlement cost to be paid at the end of:

$$\text{USD } 374.86 \times \left(1 + 0.05 \times \frac{91}{360}\right) = \text{USD } 379.60$$

- (iii) Borrow SEK $(165,021.95 \times 7.0000) = \text{SEK } 1,155,153.65$ for 3 months

Total repayment after 3 months would be:

$$\text{SEK } 1,155,153.65 \times \left(1 + 0.12 \times \frac{91}{360}\right) = \text{SEK } 1,190,193.31$$

(iv) Spot: sell SEK 1,155,153.65 / buy USD 165,021.95 (at 7.0000)

3 months forward: buy SEK 1,190,193.31 / sell USD 167,108.00 (at 7.1223)

Resulting cashflows

	SEK		USD
After 3 months:	+ 1,000,000.00	(ii) First hedge	– 165,021.95
	+ 1,155,153.65	(iii) SEK loan	
	<u>– 1,155,153.65</u>	(iv) Second hedge	+ 165,021.95
	+ 1,000,000.00		–
After 6 months:		(i) FRA settlement	– 379.60
	– 1,024,653.44	(ii) First hedge	+ 167,487.24
	– 1,190,193.31	(iii) Loan repayment	
	<u>+ 1,190,193.31</u>	(iv) Second hedge	– 167,108.00
	– 1,024,653.44		–

The resulting effective cost of borrowing the SEK 1,000,000 is:

The final step

$$\frac{24,653.44}{1,000,000} \times \frac{360}{91} = 9.753\%, \text{ which is the theoretical SEK FRA cost.}$$

So far, we have created a forward cash borrowing in SEK at 9.75%, rather than an FRA. The final step is to remove the cash element, by subtracting from step (iii) in the structure the originally intended notional principal cash amount of SEK 1,000,000. This reduces the amount in step (iii) to SEK 155,153.65. The repayment on this amount is:

$$\text{SEK } 155,153.65 \times \left(1 + 0.12 \times \frac{91}{360}\right) = \text{SEK } 159,859.98$$

This leaves the following cashflows:

	SEK		USD
After 3 months:	+ 1,000,000.00	(ii) First hedge	– 165,021.95
	+ 155,153.65	(iii) SEK loan	
	<u>– 1,155,153.65</u>	(iv) Second hedge	+ 165,021.95
	–		–
After 6 months:		(i) FRA settlement	– 379.60
	– 1,024,653.44	(ii) First hedge	+ 167,487.24
	– 159,859.98	(iii) Loan repayment	
	<u>+ 1,190,193.31</u>	(iv) Second hedge	– 167,108.00
	+ 5,679.89		–

If we discount this resulting net inflow back three months at 12.00%, we have:

$$\frac{\text{SEK } 5,679.89}{\left(1 + 0.12 \times \frac{91}{360}\right)} = \text{SEK } 5,513$$

This is the same settlement amount as if we had been able to buy a SEK FRA at 9.753%:

$$\frac{\text{SEK } 1,000,000 \times (0.12 - 0.09753) \times \frac{91}{360}}{\left(1 + 0.12 \times \frac{91}{360}\right)} = \text{SEK } 5,513$$

In practice, because of the various bid/offer spreads involved, the effective cost would be higher than this – although the spreads on the FX forward deals are reduced slightly because they are swap deals rather than forward outright deals, so that the bid/offer spread is not paid on the spot price as well as on the forward swaps. The question would therefore be whether this synthetic FRA rate is more attractive than a straightforward FRA in SEK.

Because an FXA replicates the effects of a cash forward-forward foreign exchange deal, we can repeat the above example using an FXA.

Example 7.33

With the same data as in Example 7.32, we wish to create a synthetic SEK FRA as follows:

buy USD FRA at 5.91% on USD 165,021.95
sell USD/SEK FXA at + 580 points (= 6.1178 – 6.0598)

With the same notation as in the previous section, we have:

A_1 = USD 165,021.95
 A_2 = USD 167,487.24
OER = 6.0598
SSR = 7.0000
CFS = 0.0580
SFS = 0.1223 (= 7.1223 – 7.0000)
L = 0.12
days = 91
year = 360

The FXA settlement is then:

$$167,487.24 \times \left[\frac{(6.0598 - 7.0000) + (0.0580 - 0.1223)}{(1 + 0.12 \times \frac{91}{360})} \right] - 165,021.95 \times [6.0598 - 7.0000] = -8,134.23$$

That is, SEK 8,134.23 paid to us as the seller of the FXA.

The FRA settlement amount is:

$$\frac{165,021.95 \times (0.05 - 0.0591) \times \frac{91}{360}}{(1 + 0.05 \times \frac{91}{360})} = -\text{USD } 374.86$$

That is, 374.86 paid by us as buyer of the FRA. This can be converted spot to SEK 2,624.02 (= 374.86 × 7.0000).

The net settlement received from the two deals is thus SEK 8,134.23 – SEK 2,624.02 = SEK 5,510.

Apart from rounding differences, this is the same result as in Example 7.32.

In the section Relationship with the Money Markets we gave the covered interest arbitrage formula for creating one interest rate from another:

variable currency interest rate created =

$$\left[\left(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{base year}} \right) \times \frac{\text{outright}}{\text{spot}} - 1 \right] \times \frac{\text{variable year}}{\text{days}}$$

This formula can be adapted for forward periods as follows:

Calculation
summary

Variable currency FRA rate =

$$\left[\left(1 + \text{base currency FRA rate} \times \frac{\text{days}}{\text{year}} \right) \times \frac{\text{outright to far date}}{\text{outright to near date}} - 1 \right] \times \frac{\text{variable year}}{\text{days}}$$

In the example above, this would give the SEK FRA rate created as:

$$\left[\left(1 + 0.0591 \times \frac{91}{360} \right) \times \frac{6.1178}{6.0598} - 1 \right] \times \frac{360}{91} = 9.75\%$$

Expressing this for the base currency gives:

Calculation
summary

Base currency FRA rate =

$$\left[\left(1 + \text{base currency FRA rate} \times \frac{\text{days}}{\text{variable year}} \right) \times \frac{\text{outright to near date}}{\text{outright to far date}} - 1 \right] \times \frac{\text{base year}}{\text{days}}$$

In the same way, we can reverse the arbitrage process and create a forward-forward foreign exchange swap (or FXA) from two FRAs (or forward-forwards or futures). Again, the formula given in the section Forward Swaps for a normal foreign exchange swap can be adapted directly for forward periods:

Calculation
summary

Forward-forward swap =

$$\text{outright to near date} \times \frac{\left(\text{variable currency FRA} \times \frac{\text{days}}{\text{year}} - \text{base currency FRA} \times \frac{\text{days}}{\text{year}} \right)}{\left(1 + \text{base currency FRA} \times \frac{\text{days}}{\text{year}} \right)}$$

DISCOUNTING FUTURE FOREIGN EXCHANGE RISK

We have suggested on several occasions that there is a risk for a forward foreign exchange position which arises out of potential spot exchange rate movements rather than just the movements in the forward swap price.

Example 7.34

Our forward dealer sells and buys USD 1 million against NLG, 3 months against 6 months, at the following rates, because he expects the NLG's forward discount to increase (a fall in USD interest rates relative to NLG rates).

	USD/NLG	USD%	NLG%
Spot:	2.0000		
3 months (90 days):	2.0098	8.0	10.0
6 months (180 days):	2.0172	8.8	10.6

The cashflows arising from the deal are as follows:

90 days	180 days
– USD 1,000,000	+ USD 1,000,000
+ NLG 2,009,800	– NLG 2,017,200

Now consider an immediate move in rates to the following:

	USD/NLG	USD%	NLG%
Spot:	2.1000		
3 months:	2.1103	8.0	10.0
6 months:	2.1181	8.8	10.6

Even though interest rates have not moved, the NLG's forward discount has increased slightly, because the change in the spot rate from 2.0000 to 2.1000 has "magnified" the forward points. The dealer therefore expects to have made a profit.

In fact, however, the bank as a whole has made a loss. Suppose that the deal is now closed out at current rates. The cashflows become:

Spot	90 days	180 days
–	– USD 1,000,000	+ USD 1,000,000
–	+ NLG 2,009,800	– NLG 2,017,200
–	+ USD 1,000,000	– USD 1,000,000
–	<u>– NLG 2,110,300</u>	<u>+ NLG 2,118,100</u>
	– NLG 100,500	+ NLG 100,900

The net effect appears to be a profit of NLG 400 (= NLG 100,900 – NLG 100,500). However, the outflow of NLG 100,500 after 90 days must be financed for 3 months before the inflow of NLG 100,900. The forward-forward rate for this financing is:

$$\left(\frac{1 + 0.106 \times \frac{180}{360}}{1 + 0.10 \times \frac{90}{360}} - 1 \right) \times \frac{360}{(180 - 90)} = 10.9268\%$$

The cost of funding is therefore:

$$\text{NLG } 100,500 \times 10.9268\% \times \frac{90}{360} = \text{NLG } 2,745$$

This financing cost is a real loss, which has arisen because of the spot rate movement. It is therefore possible for the dealer to make a profit by correctly anticipating interest rate movements, but for this profit to be more than offset by the spot rate movements.

The exposure – known as the forward "tail" – arises because of the time difference between the cashflows. It is possible to compensate for this effect by hedging the NPV of the original cashflows.

The NPV of the USD cashflows is:

$$\frac{\text{USD } 1,000,000}{(1 + 0.08 \times \frac{90}{360})} + \frac{\text{USD } 1,000,000}{(1 + 0.088 \times \frac{180}{360})} = - \text{USD } 22,538$$

On an NPV basis, the dealer is therefore short of USD 22,538.

A possible hedge is therefore to buy USD 22,538 for value spot at the same time as the original deal. The cashflows if the deal and the hedge were closed out would then be:

<i>Spot</i>	<i>90 Days</i>	<i>180 Days</i>
+ USD 22,538	– USD 1,000,000	+ USD 1,000,000
– NLG 45,076	+ NLG 2,009,800	– NLG 2,017,200
– USD 22,538	+ USD 1,000,000	– USD 1,000,000
<u>+ NLG 47,330</u>	<u>– NLG 2,110,300</u>	<u>+ NLG 2,118,100</u>
+ NLG 2,254	– NLG 100,500	+ NLG 100,900

It is possible to deposit the NLG 2,254 profit from spot until three months at 10%, for total proceeds of:

$$\text{NLG } 2,254 \times \left(1 + 0.10 \times \frac{90}{360}\right) = \text{NLG } 2,310$$

This gives a net cashflow after 3 months of:

$$\text{NLG } 2,310 - \text{NLG } 100,500 = - \text{NLG } 98,190$$

At the forward-forward NLG rate of 10.9268%, it is possible to fund this net NLG 98,190 from 3 months until 6 months for a total repayment of:

$$\text{NLG } 98,190 \times \left(1 + 0.109268 \times \frac{90}{360}\right) = \text{NLG } 100,872$$

This amount is offset by the NLG 100,900 profit after 6 months (the difference is due only to rounding).

Discounting in this way is effective because forward swap deals are equivalent to deposits and loans in the two currencies. Clearly deposits and loans in the domestic currency (NLG in this case) are unaffected by exchange rate movements. Deposits and loans in a foreign currency (USD in this case) are, however, equivalent to their net present value, which is directly affected by the exchange rate. On the other hand, the hedge will only be exact to the extent that the interest rates used to discount the cashflows reflect the interest rates implicit in the forward exchange rates. The amount of the hedge required will also change as interest rates change, so that a hedged position will not remain hedged. In practice the exposure might, for example, be reassessed daily.

It is worth noting that a dealer who wishes to avoid this tail effect can largely do so by trading ERAs rather than forward-forwards, as ERAs are settled on the movement in forward points only. The dealer would, however, still be vulnerable to the “magnification” effect mentioned.

In forward transaction exposures, other elements, such as interest to be paid and received (rather than only interest already accrued), and the principal of a loan or deposit itself, should also be hedged.

EXERCISES

61. The Eurosterling interest rate for one year (exactly 365 days) is 13%. The EuroDeutschemark interest rate for the same period is 9%. The spot rate today is GBP/DEM 2.5580 / 90.

What would you expect the GBP/DEM swap price to be for one year forward? (Ignore the buy-sell spread and calculate the middle price only.)



62.	Spot	3-month forward swap
USD/DEM	1.5140 / 45	29 / 32
USD/FRF	5.1020 / 40	246 / 259
GBP/USD	1.9490 / 00	268 / 265



Based on the prices above, what are the two-way prices for:

- DEM/FRF spot? Which side does the customer buy FRF?
 - GBP/FRF spot? Which side does the customer sell GBP?
 - USD/FRF 3 months forward outright?
 - GBP/USD 3 months forward outright?
 - GBP/FRF 3 months forward outright? Which currency has higher interest rates?
 - DEM/FRF 3 months forward outright? Which currency has higher interest rates?
 - DEM/FRF 3 months forward swap?
- 63.
- | | Spot | O/N | T/N | S/W | 1 month |
|---------|-------------|-------------|-----------|---------|---------|
| USD/FRF | 5.1020 / 40 | 2.0 / 2.5 | 2.3 / 2.9 | 18 / 20 | |
| GBP/USD | 1.9490 / 00 | 10.6 / 10.1 | 3.5 / 3.3 | 23 / 22 | 96 / 94 |

Based on the prices above, what are the two-way prices for:

- USD/FRF forward outright value one week after spot?
 - USD/FRF forward outright value tomorrow?
 - USD/FRF forward outright value today? Which side does the customer buy FRF?
 - GBP/USD forward outright value today? Which side does the customer buy GBP?
 - GBP/USD forward-forward swap from one week after spot to one month after spot? Which side does the customer buy and sell GBP (in that order)?
 - GBP/USD forward-forward swap from tomorrow to one month after spot? Which side does the customer buy and sell GBP (in that order)?
64. You are a bank FX dealer. Looking at the Reuters screen, you see the following rates:

USD/FRF spot:	6.2695 / 15
T/N:	2.5 / 1.5
3 months:	310 / 290
6 months:	550 / 510
USD/NOK spot:	6.7620 / 40
T/N:	0.2 / 0.5
3 months:	15 / 15 A/P
6 months:	50 / 100

- a. Some time ago, your customer sold NOK receivables forward into FRF, and that deal matures on the date which is now the 3-month forward date. However, he now discovers that these receivables will be delayed by three months because of late delivery of the goods. He therefore needs to adjust the forward deal. What forward-forward swap price do you quote him? He asks for a two-way price and prefers to have it quoted in terms of number of FRF per 1 NOK. Which side of the price do you deal on?
 - b. Your customer has another deal to sell NOK and buy FRF, also previously undertaken, also maturing on the three-month forward date. He discovers first thing in the morning that he needs the FRF by tomorrow and that he will have enough NOK in his account tomorrow to cover this. He therefore uses another forward-forward deal to adjust this second deal in order to take delivery of it tomorrow. Again, what two-way price do you quote, and on which side do you deal?
- 65.** Today is Friday 19 April. You are a bank FX dealer. You look at your Reuters screen and see the following rates quoted:

USD/FRF	Spot:	5.2580 / 00
	S/W:	25 / 23
	1 month:	100 / 90
	2 months:	195 / 175
GBP/USD	Spot:	1.9157 / 67
	O/N:	- 0.4 / +0.1
	T/N:	1.5 / 1
	S/W:	11 / 9
	1 month:	50 / 45
	2 months:	105 / 95

- a. Some time ago, your customer sold GBP forward against FRF for delivery on 3 June. He now discovers that he will need the FRF on 30 April instead. He therefore asks you for a swap price to adjust the deal's maturity date. What price do you quote (in terms of FRF per 1 GBP)?
- b. Some time ago, you bought GBP from your customer against USD, and the deal matures today. He discovers (early enough) that he does not have the GBP in his account, and will not have them until 30 April. He asks you for a two-way price to swap the deal from today until 30 April. What price do you quote? On which side of the price do you deal?

- c. He has discovered that he is not in fact going to receive the GBP in (b) at all, and decides to reverse the contract he made some time ago. He therefore asks you for a two-way outright value today price. What price do you quote? On which side of the price do you deal?

66. You are a bank FX dealer. You look at your Reuters screen and see the following rates quoted:

USD/DEM	spot:	1.6012 / 22
USD/ITL	spot:	1633.25 / 25
USD/ITL	6-month swap:	2237 / 2287
USD	6-month interest rates:	5.25 / 5.375%
ITL	6-month interest rates:	8.00 / 8.25%

The 6-month value date is 182 days after spot value date.

- a. Your customer needs to convert his ITL receivables into USD in six months' time. What two-way forward outright price would you quote for this? Is the ITL at a discount or a premium to the USD? On which side of the price would you deal?
- b. He is not necessarily in a hurry to do this transaction, because he thinks that the spot exchange rate will get better for him. He expects the USD/DEM to strengthen to 1.6150 over the next two days, but believes that the ITL is likely to move from its present level of 1020 against the DEM to 1005.

He also believes that USD interest rates will fall 0.75% but that ITL rates will probably rise 1.0% at the same time.

Should he sell the ITL forward now, or wait two days?

67. An investor has DEM15 million to invest for three months. He has a choice between two possible investments – either a DEM deposit, or USD Eurocommercial paper which could be hedged back into DEM (covered interest arbitrage). The Eurocommercial paper would yield LIBOR + 4 basis points. If he invests via the Eurocommercial paper, what is his absolute all-in rate of return, and what is this relative to Deutschemark LIBOR?

Spot USD/DEM	1.6730 / 40
3-month swap:	173 / 168
3-month USD%:	$8\frac{1}{4}$ / $8\frac{3}{8}$ %
3-month DEM%:	$4\frac{1}{8}$ / $4\frac{1}{4}$ %

Assume that a mismatch of principal amounts is possible in the foreign exchange swap, and that the 3-month period has 91 days.

68. Market rates now are as follows for EuroUSD and EuroSEK:

	USD/SEK	USD%	SEK%
Spot	6.0000 / 10		
3 months (91 days)	590 / 605	5.87 / 6.00	9.87 / 10.00
6 months (182 days)	1274 / 1304	5.75 / 5.87	10.12 / 10.25
9 months (273 days)	1832 / 1872	5.75 / 5.87	10.00 / 10.12
FRA 3 v 9		5.70 / 5.75	

Market rates three months later are as follows:



	USD/SEK	USD%	SEK%
Spot	6.2060 / 65		
3 months (91 days)	649 / 664	6.00 / 6.12	10.25 / 10.37
6 months (182 days)	1394 / 1424	5.87 / 6.00	10.50 / 10.62
9 months (274 days)	2014 / 2054	5.87 / 6.00	10.37 / 10.50

What would be the effective synthetic forward-forward 3 v 9 cost for SEK for a borrower, created from an FRA 3 v 9 for USD, and all necessary forward foreign exchange deals, taking into account all the relevant bid / offer spreads? Show all the deals necessary based on an amount of SEK 1 million and assume that you are a price-taker.

69. You undertake the following two USD/DEM forward swap deals for value spot against 6 months (182 days):

- You buy and sell USD 10 million at 1.6510 and 1.6350.
- You sell and buy USD 10 million at 1.6495 and 1.6325.

6-month interest rates are as follows:

USD:	6.5%
DEM:	4.5%

What is the NPV of your position?

70. You are USD-based and have the following transactions on your books:

- A 6-month (182 days) forward purchase of DEM 10 million.
- A 12-month (365 days) forward sale of DEM 10 million.
- A borrowing from a counterparty of DEM 10 million at 7% for 12 months (365 days; all the interest paid at maturity).
- A deposit placed with a counterparty of DEM 10 million at 6.5% for 3 months (91 days).

Rates are currently as follows:

	USD/DEM	USD%	DEM%
Spot:	1.8000		
3 months:	1.8040	6.5	7.4
6 months:	1.8088	6.5	7.5
12 months:	1.8170	7.0	8.0

- Suppose that the spot exchange rate moves to 2.0000 but interest rates are unchanged. What is the effect on the profit and loss account, not considering discounting?
- What is the effect considering discounting, and what spot USD/DEM deal would provide a hedge against this risk?



Swaps and Options



“Any swap is effectively an exchange of one set of cashflows for another, considered to be of equal value. The concept of a basic interest rate swap is similar to an FRA, but is applied to a series of cashflows over a longer period of time rather than to a single period.”

Interest Rate and Currency Swaps

Basic concepts and applications

Pricing

Valuing swaps

Hedging an interest rate swap

Amortising and forward-start swaps

Currency swaps

Exercises

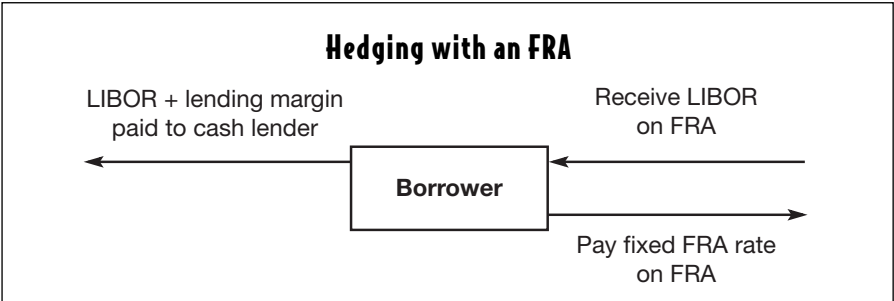
BASIC CONCEPTS AND APPLICATIONS

Any swap is effectively the exchange of one set of cashflows for another considered to be of equal value. The concept of a basic interest rate swap in particular is very similar to an FRA, but is applied to a series of cashflows over a longer period of time, rather than to a single borrowing period.

Hedging borrowing costs

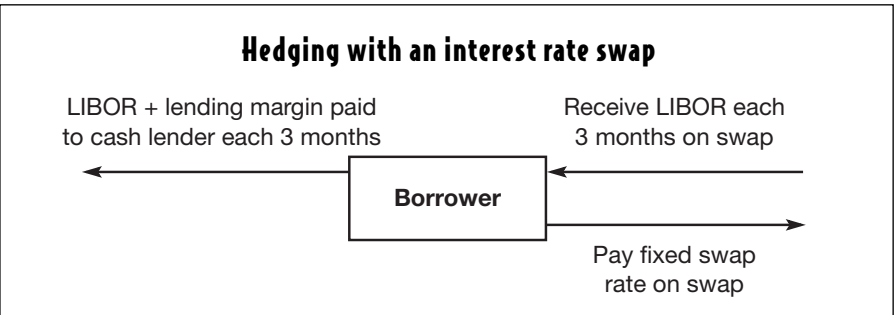
Consider, for example, the case we examined in the chapter on FRAs, of a borrower who uses an FRA to hedge the cost of a single three-month borrowing due to begin in two months' time, by buying a 2 v 5 FRA:

Fig 8.1



In this case, the borrower's *net* cost is (fixed FRA rate + lending margin).
Now consider the case of a borrower who takes a 5-year borrowing now and on which she will pay LIBOR refixed at 3-monthly intervals throughout the life of the borrowing. The cost of the first 3-month period is already fixed at the current LIBOR. The borrower could fix the cost of the second 3-month period of the borrowing with a 3 v 6 FRA. She could also fix the cost of the third 3-month period with a 6 v 9 FRA, and so on. However, if she wishes to hedge the cost of all the 3-month LIBOR settings throughout the 5 years, she would use an interest rate swap, which achieves exactly this (see Figure 8.2).

Fig 8.2



In this case, the borrower's net cost is (fixed swap rate + lending margin). The fixed rate quoted to the borrower by the swap dealer applies throughout the

5-year period of the swap. The fixed rate may be paid for example 3-monthly, 6-monthly or annually, depending on the exact terms of the swap. The floating rate may be 1-month LIBOR, 3-month LIBOR, 6-month LIBOR etc. A basic interest rate swap is quoted as a fixed interest rate on one side and LIBOR exactly on the other (rather than say, LIBOR + margin or LIBOR – margin). The fixed and floating payments may not have the same frequency. For example the fixed rate may be paid annually, but the floating rate may be based on 6-month LIBOR and paid semi-annually.

As with an FRA, an interest rate swap involves no exchange of principal. Only the interest flows are exchanged and, in practice, again as with an FRA, these are netted rather than transferred gross in both directions. An important mechanical difference is that the settlement amount in a swap is generally paid at the end of the relevant period (rather than at the beginning on a discounted basis as in an FRA).

The motivation for the borrower in the example above may be that she formerly expected interest rates to fall (and therefore took a floating rate borrowing), but now expects interest rates to rise – that is, she has changed her view. An alternative motivation could be that, regardless of her view, she has existing floating-rate funding but for commercial purposes needs fixed-rate funding (for example, a company funding a long-term project).

An interest rate swap is an exchange of one set of interest flows for another, with no exchange of principal

Key Point

Relative advantage in borrowing

One particular driving force behind the swap market is the existence of cost discrepancies between different funding methods, as shown by the following example:

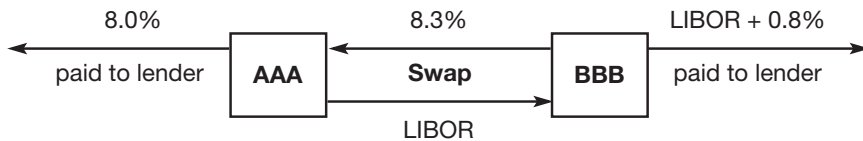
Example 8.1

Each of the following two companies wishes to borrow for 5 years and each has access to both fixed-rate borrowing and floating-rate borrowing:

Company AAA has access to floating-rate borrowing at LIBOR + 0.1% and also has access to fixed-rate borrowing at 8.0%. Company AAA would prefer floating-rate borrowing.

Company BBB has access to floating-rate borrowing at LIBOR + 0.8% and also has access to fixed-rate borrowing at 9.5%. Company BBB would prefer fixed-rate borrowing.

If AAA borrows at LIBOR + 0.1% and BBB borrows at 9.5%, each company achieves what it requires. There is however a structure which achieves the same result at a lower cost. This is for AAA to take a fixed-rate borrowing at 8.0%, BBB to take a floating-rate borrowing at LIBOR + 0.8% and for AAA and BBB to transact a swap at 8.3% (which we will assume to be the current market rate for a swap) against LIBOR as follows:



The net cost for AAA is: $(8.0\% - 8.3\% + \text{LIBOR}) = \text{LIBOR} - 0.3\%$

The net cost for BBB is: $(\text{LIBOR} + 0.8\% + 8.3\% - \text{LIBOR}) = 9.1\%$

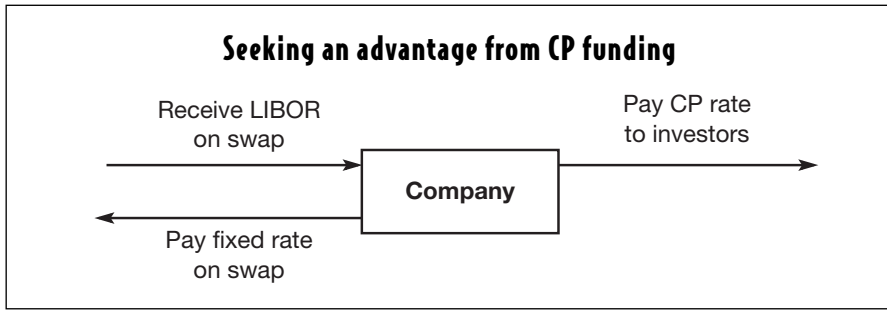
In this way, each company achieves what it requires, but at a cost which is 0.4% lower than it would achieve by the “straightforward” route.

In the example above, BBB’s borrowing costs are higher than AAA’s whether we are comparing fixed-rate or floating-rate borrowing. However, the difference between their costs is greater in the fixed-rate market (9.5% compared with 8.0%) than in the floating-rate market ($\text{LIBOR} + 0.8\%$ compared with $\text{LIBOR} + 0.1\%$). It is this discrepancy – which does arise in practice – which makes the structure of the example possible. Company BBB has an absolute cost disadvantage compared with company AAA in either market. However, BBB’s *relative* advantage is in the floating-rate market and AAA’s *relative* advantage is in the fixed-rate market. It is therefore more cost-efficient for each company to borrow where it has a relative advantage and then swap.

In practice, each company is likely to deal with a bank rather than another company. The bank, whose role is to make a market in swaps, is unlikely to deal with two offsetting counterparties in this way at the same moment for the same period and the same amount. Nevertheless, the simplified example above demonstrates what is an important force behind the existence of the swap market.

Typically, the structure described above might involve company AAA in issuing a fixed-rate bond and simultaneously arranging the swap. The bond issue would give rise to various costs – issuing fees, underwriting fees etc. – which would not arise in a straightforward floating-rate borrowing, and these need to be taken into account by AAA in calculating its all-in net floating-rate cost after the swap.

This “arbitrage” between different borrowing markets can be extended. Suppose that a company arranges to borrow at a floating rate based on something other than LIBOR. For example, the company might be in a position to issue commercial paper. If it believes that its cost of borrowing through CP will average less than its cost would be based on a margin over LIBOR over 5 years, it could use a rolling CP borrowing programme instead. Clearly this leaves the company vulnerable to the risk that its CP costs may increase relative to LIBOR because the market’s perception of the company’s credit rating worsens, or because investors’ demand for CP falls. The company may nevertheless be prepared to take this risk in return for a possible advantage.

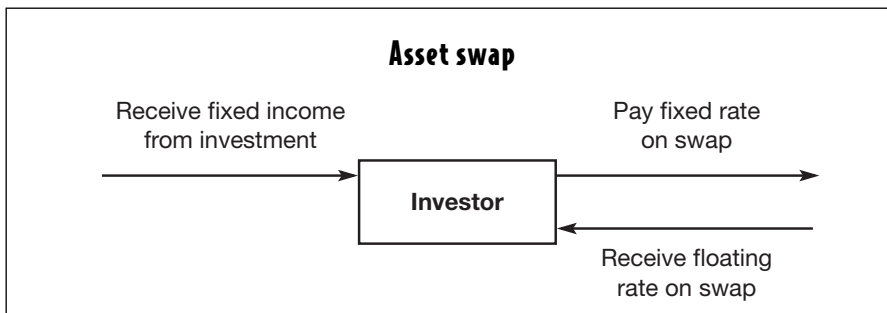
**Fig 8.3**

The net result of such an arrangement will be a borrowing cost of (fixed swap rate – (LIBOR – CP rate)). As long as the CP rate is below LIBOR, the borrowing cost will be less than the fixed swap rate. If the CP rate rises relative to LIBOR however, so will the all-in cost.

Asset swap

The examples we have used so far have been based on an underlying borrowing. That is, the “end-user” undertaking the swap has an underlying borrowing and wishes to change the character of the borrowing from fixed-rate to floating-rate or vice versa.

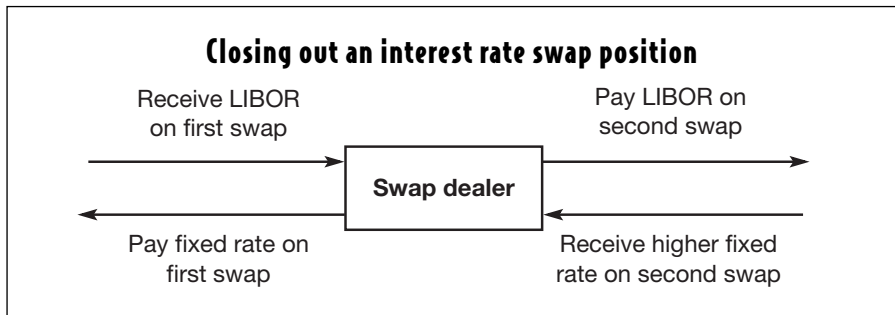
A swap can, however, be used by an investor just as well as by a borrower. An investor might for example buy an FRN and also transact a swap to receive a fixed interest rate and pay LIBOR. The result would be a synthetic fixed rate investment. A swap with an underlying asset like this is called an asset swap, while a swap with an underlying liability is called a liability swap. The swap itself is the same in both cases; the “asset” or “liability” tag refers to the package of which it forms a part.

**Fig 8.4**

Speculation

As with any instrument, a swap may be used for speculation as well as hedging.

A dealer deliberately taking a position with a swap is speculating that long-term yields will move up or down. If he expects yields to rise, for example, he will undertake a swap where he is paying out the fixed interest rate and receiving LIBOR. If he is correct, he can later offset this with a swap, for the same period, where he is paying LIBOR and receiving the fixed interest rate – which will then be at a higher level, giving him a profit:

Fig 8.5

Basis swap

As described so far, an interest rate swap involves the payment of a fixed interest rate in one direction and a floating interest rate in the other direction. This is known as a “coupon swap”. A swap can alternatively involve two differently defined floating rates – for example, the payment of LIBOR and the receipt of a rate based on commercial paper rates. Such a floating-floating swap is called a “basis swap”. “Index swaps” can also be constructed, where the flows in one or other direction are based on an index (such as a stock index, for example).

PRICING

Day/year conventions

The day/year conventions for calculating interest payments on swaps are largely the same as we have already seen for money market and bond calculations – that is, ACT/360, ACT/365, ACT/ACT, 30(E)/360 or 30(A)/360. There are a few points to mention, however.

Modified following

In the bond market, we have already seen that if a regular coupon date falls on a weekend or holiday, the payment is generally delayed until the next working day but the amount of the payment is not changed.

In the money market, in the same circumstances, the payment is delayed until the next working day unless this would move into the following calendar month, in which case the payment is made on the previous working day. In either case, the amount of the payment is calculated to the actual payment date, not the regular date.

The usual convention in the swap market is the same as in the money market and is called the “modified following” method. This can mean that the cashflows in a swap and a bond, which are intended to match precisely, are in fact slightly different in timing and amount.

ACT/ACT

In a swap, calculation of an interest payment on an ACT/ACT basis is split between that part of the interest period falling in a leap year, which is divided by 366, and the remainder, which is divided by 365.

Example 8.2

The fixed leg of a swap is based on 10% ACT/ACT (annual). What is the amount of the fixed payment for the period 15 October 1999 to 15 October 2000?

15 October 1999 to 31 December 1999 (inclusive): 78 days
 1 January 2000 to 14 October 2000 (inclusive): 288 days

$$\text{Interest amount is therefore } \left(10\% \times \frac{78}{365}\right) + \left(10\% \times \frac{288}{366}\right)$$

ACT/365

In swap documentation (as in standard ISDA documentation, for example), the expression “ACT/365” is sometimes used as an alternative for ACT/ACT, and the expression “ACT/365 (fixed)” is sometimes used instead for what we have defined in this book as ACT/365.

Converting between different quotation bases

The basis for quoting the rate on the fixed leg of a swap might be annual, or semi-annual, quarterly, monthly etc. As we have seen above, it may also be on a bond basis or a money market basis. We need to be able to convert between these different bases.

In this context, the market uses the expression “money market basis” to mean ACT/360. The term “bond basis” is used to mean ACT/365, ACT/ACT or 30/360. Over a non-leap year, these three are equivalent over a whole year (because $\frac{365}{365} = \frac{360}{360} = 1$).

Example 8.3

Convert a USD interest rate of 10.3% SABB (semi-annual bond basis) to the AMM (annual money market) equivalent.

$$\left[\left(1 + \frac{0.103}{2} \right)^2 - 1 \right] = 10.565\% \text{ ABB}$$

$$10.565\% \times \frac{360}{365} = 10.420\% \text{ AMM}$$

Answer: 10.42% AMM

Example 8.4

Convert a DEM interest rate of 6.40% ABB (annual bond basis) to the SAMM (semi-annual money market) equivalent.

(i) $\left[(1.064)^{\frac{1}{4}} - 1\right] \times 2 = 6.301\% \text{ SABB}$

(ii) $6.301\% \times \frac{360}{365} = 6.214\% \text{ SAMM}$

Answer: 6.21% SAMM

The conversions between annual and semi-annual in Examples 8.3 and 8.4 are the conversions we saw in the “Financial Arithmetic Basics” chapter, between effective and nominal rates. When we discussed effective rates on a 360-day basis in the “Money Market” chapter however, we saw that it is not possible to make this conversion precisely for rates on a money market basis (ACT/360).

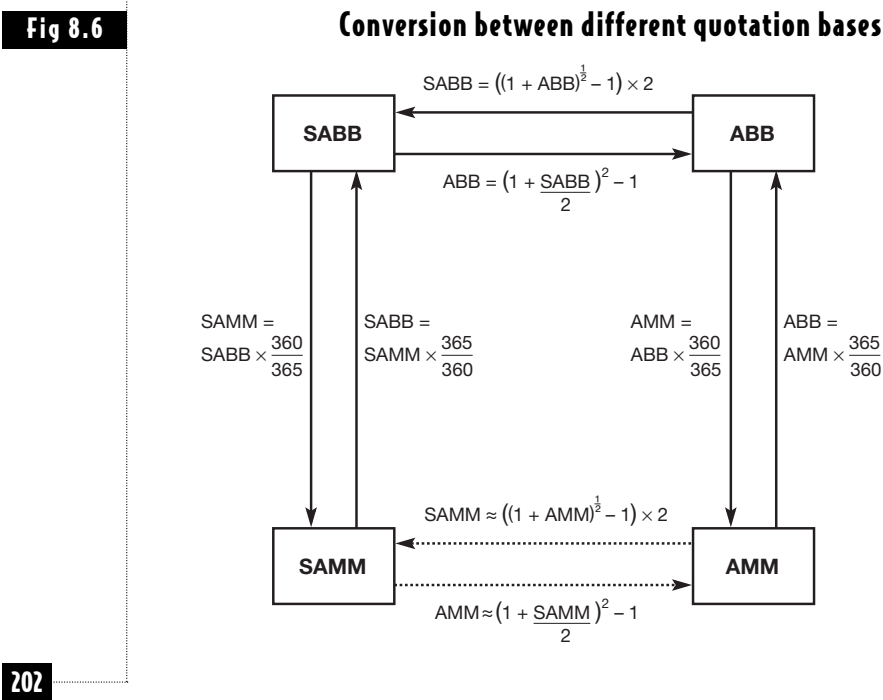
Suppose that in Example 8.4, we try to compound the 6.214% SAMM directly to AMM:

(iii) $\left(1 + \frac{0.06214}{2}\right)^2 - 1 = 6.311\% \text{ AMM}$

If we now finally try to convert back to ABB again, we have:

(iv) $6.311\% \times \frac{365}{360} = 6.399\% \text{ ABB}$

This is almost the same as the 6.40% ABB with which we began in Example 8.4, but not quite. This is because step (iii) above is not valid. Essentially, we can convert between the various bases but completing the square below between SAMM and AMM is only approximate because it would be compounding on a 360-day basis rather than an annual basis:



The following example uses the conversions we have seen above to calculate the approximate net result of an asset swap. We have repeated the example later (see Example 8.11) using exact cashflows and zero-coupon discount factors to calculate a slightly different result.

Example 8.5

An investor purchases a 3-year bond yielding 10.686% (annual bond basis). He transacts a swap to pay 10.3% (annual bond basis) against 6-month LIBOR (semi-annual money market basis). What is the net yield to the investor of this asset swap?



Income from bond:	10.686% (ABB)	=	10.415% (SABB)	=	10.272% (SAMM)
Payment in swap:	10.300% (ABB)	=	10.048% (SABB)	=	9.910% (SAMM)
Receipt in swap:					LIBOR (SAMM)
Net yield:					LIBOR + 0.362% (SAMM)

The investor can therefore achieve an all-in yield on the asset swap of around 36 basis points over LIBOR.

Prices quoted as a spread over government bonds

Swap rates are sometimes quoted as a “spread” over government bond yields – particularly for example in the USD swap market, where there is a good series of government bonds available. In this case, the market is taking the current yield for the government bond of the nearest maturity which is “on the run” – that is, the benchmark bond which has been issued recently and will generally be trading near par. The two-sided swap price quoted is the difference between this yield and the swap rate.

Example 8.6

The 5-year USD swap is quoted as 33 / 37 over treasuries. The current 5-year treasury note yield is 7.43%. What is the swap rate on an AMM (annual money market) basis?

US treasuries always pay semi-annual coupons. The 7.43% yield and spread are therefore SABB. Therefore the swap rate is:

$$(7.43 + 0.33) \% / (7.43 + 0.37) \% = 7.76 \% / 7.80 \% \text{ SABB}$$

$$\left[\left(1 + \frac{0.0776}{2} \right)^2 - 1 \right] \times \frac{360}{365} = 7.802 \%$$

$$\left[\left(1 + \frac{0.0780}{2} \right)^2 - 1 \right] \times \frac{360}{365} = 7.843 \%$$

The AMM equivalent is therefore 7.80% / 7.84%

The pricing link between FRAs and interest rate swaps

Since it is possible to fix in advance the cost of borrowing from 3 months to 6 months (an FRA 3 v 6), from 6 months to 9 months, from 9 months to 12 months etc., it must be possible to fix the cost for all these at once, giving a fixed cost for the whole period. This is the same process as when we constructed strips in the chapters on Forward-forwards and FRAs and Futures. There should theoretically be no arbitrage between the result of doing this and the result of a single interest rate swap for the whole period. This is because borrowing on a rolling 3-month basis at LIBOR and at the same time transacting a swap to receive 3-month LIBOR quarterly against a fixed payment, is an alternative to borrowing on a rolling 3-month basis at LIBOR and fixing the cost with a series of rolling FRAs. It is possible to arbitrage between the swap and the futures contracts if they are out of line. For example, if the swap rate is too high, a dealer could transact a swap to receive the fixed rate and pay LIBOR; at the same time, he would sell a strip of futures (or buy a strip of FRAs) to offset this. This therefore gives a method of arriving at a swap price derived from FRA rates (or equivalently, short-term futures prices).

Example 8.7

Suppose the following rates are available for borrowing dollars:

3-month LIBOR	14.0625%	(91 days)
FRA 3 v 6	12.42%	(91 days)
6 v 9	11.57%	(91 days)
9 v 12	11.25%	(92 days)

It is possible to do the following:

- Borrow USD 1 now for 3 months. At end of 3 months, repay:

$$\text{USD} \left(1 + 0.140625 \times \frac{91}{360} \right) = \text{USD } 1.03555$$

- Arrange an FRA 3 v 6 at 12.42 on an amount of USD 1.03555. Considering the FRA settlement as made after 6 months rather than made on a discounted basis after 3 months, this gives a total repayment at the end of 6 months of:

$$\text{USD } 1.03555 \times \left(1 + 0.1242 \times \frac{91}{360} \right) = \text{USD } 1.06806$$

- Similarly, after 6 months, borrow USD 1.06806 for 3 months, fixed at an FRA 6 v 9 cost of 11.57%, and at the end of 9 months repay:

$$\text{USD } 1.06806 \times \left(1 + 0.1157 \times \frac{91}{360} \right) = \text{USD } 1.09929$$

- Similarly, after 9 months, borrow USD 1.09929 for 3 months, fixed at an FRA 9 v 12 cost of 11.25%, and at the end of 12 months repay:

$$\text{USD } 1.09929 \times \left(1 + 0.1125 \times \frac{92}{360} \right) = \text{USD } 1.13090$$

The effect of this is a fixed cost at the end of 12 months of 13.09%. This should therefore be the rate for a 12-month interest rate swap on an annual basis against quarterly LIBOR payments.

The rate of 13.090% is on a bond basis, not a money market basis, even though the FRA rates from which it has been constructed are quoted on a money market basis. This is because “bond basis” represents the cash interest amount paid by the end of 365 days – which is what we have calculated – while “money market basis” represents the cash interest amount paid by the end of 360 days.

To convert from this annual bond basis to an equivalent quarterly bond basis, we could decompound as before to give:

$$\left((1 + 0.13090)^{\frac{1}{4}} - 1\right) \times 4 = 12.49\% \text{ (QBB)}$$

The equivalent quarterly money market swap rate for 1 year against 3-month LIBOR would then be calculated as:

$$12.49\% \times \frac{360}{365} = 12.32\% \text{ (QMM)}$$

Note however that following Example 8.8, we calculate a slightly different result for this.

Example 8.7 gave a 1-year swap rate of 13.090% on a bond basis. We can use the same process to construct swap rates for other maturities: 3 months, 6 months, 9 months, 15 months, 18 months etc. As long as forward-forward prices are available (either as FRAs or futures) for the periods leading up to the swap maturity, we can construct a strip. The result in each case is a zero-coupon swap rate, as the strip process rolls all the interest to an accumulated total at the end maturity.

If we calculate a series of zero-coupon swap rates in this way, we then have a set of discount factors for valuing cashflows. In the chapter on Zero-coupon Rates and Yield Curves, we used zero-coupon discount factors to calculate the theoretical par bond yield. In exactly the same way, we can use the zero-coupon swap rates to calculate a swap rate.

The swap rates so constructed are called “par” swap rates. This refers to the market rate on the fixed side of a swap, which is conventionally quoted so as to be paid or received against LIBOR exactly on the other side. It may be (as in Example 8.11 below) that the final swap structure agreed involves the payment or receipt of an off-market fixed swap rate, in return for an amount which represents LIBOR plus or minus some compensating spread. Such an off-market swap structure may result in more convenient cashflows for one of the parties.

Example 8.8

Suppose we have the following rates (the first four are the same as in Example 8.7). What is the 2-year par swap rate on a quarterly money market basis?

3-month LIBOR	14.0625%	(91 days)
FRA 3 v 6	12.42%	(91 days)
6 v 9	11.57%	(91 days)
9 v 12	11.25%	(92 days)
12 v 15	11.00%	(91 days)
15 v 18	10.90%	(91 days)
18 v 21	10.80%	(91 days)
21 v 24	10.70%	(92 days)

From Example 8.7, we know that if we borrow USD 1 now for 3 months, we repay USD 1.03555 at the end of the period. Therefore the 3-month discount factor is:

$$\frac{1}{1.03555} = 0.9657$$

Using the other results from example 8.7, we have the discount factor for 6 months as:

$$\frac{1}{1.06806} = 0.9363$$

$$\text{and for 9 months as: } \frac{1}{1.09929} = 0.9097$$

$$\text{and for 12 months as: } \frac{1}{1.13090} = 0.8843$$

We can now extend the strip process for the next year:

$$\text{for 15 months: USD } 1.13090 \times \left(1 + 0.1100 \times \frac{91}{360}\right) = \text{USD } 1.16235$$

$$\text{discount factor} = \frac{1}{1.16235} = 0.8603$$

$$\text{for 18 months: USD } 1.16235 \times \left(1 + 0.1090 \times \frac{91}{360}\right) = \text{USD } 1.19438$$

$$\text{discount factor} = \frac{1}{1.19438} = 0.8373$$

$$\text{for 21 months: USD } 1.19438 \times \left(1 + 0.1080 \times \frac{91}{360}\right) = \text{USD } 1.22699$$

$$\text{discount factor} = \frac{1}{1.22699} = 0.8150$$

$$\text{for 24 months: USD } 1.22699 \times \left(1 + 0.1070 \times \frac{92}{360}\right) = \text{USD } 1.26054$$

$$\text{discount factor} = \frac{1}{1.26054} = 0.7933$$

We can now use these discount factors to calculate the 2-year par swap rate, exactly as in the first example in the chapter on Zero-coupon Rates and Yield Curves. If i is the 2-year swap rate on a quarterly money market basis, then we have:

$$\begin{aligned} 1 = & \left(i \times \frac{91}{360} \times 0.9657\right) + \left(i \times \frac{91}{360} \times 0.9363\right) + \left(i \times \frac{91}{360} \times 0.9097\right) \\ & + \left(i \times \frac{92}{360} \times 0.8843\right) + \left(i \times \frac{91}{360} \times 0.8603\right) + \left(i \times \frac{91}{360} \times 0.8373\right) \\ & + \left(i \times \frac{91}{360} \times 0.8150\right) + \left(\left(1 + i \times \frac{92}{360}\right) \times 0.7933\right) \end{aligned}$$

The solution to this is $i = 11.65\%$

The same approach of valuing the cashflows in a par swap can be applied to the simpler case of a 1-year par swap on a quarterly money market basis. If this rate is i , then using the same discount factors as above, we have:

$$1 = \left(i \times \frac{91}{360} \times 0.9657\right) + \left(i \times \frac{91}{360} \times 0.9363\right) + \left(i \times \frac{91}{360} \times 0.9097\right) + \left(\left(1 + i \times \frac{92}{360}\right) \times 0.8843\right)$$

The solution to this is $i = 12.35\%$

This is not the same result as the 12.32% calculated in Example 8.7. The reason is that we have now discounted each cashflow precisely using an appropriate discount rate. In Example 8.7, however, we decompounded from an annual rate to a quarterly rate. This decompounding process is circular – the interest rate used for discounting is the same as the decompounded result itself. The answer we now have of 12.35% is a more accurate answer based on the data available.

Pricing interest rate swaps from futures or FRAs

- for each successive futures maturity, create a strip to generate a discount factor
- use the series of discount factors to calculate the yield of a par swap

**Calculation
summary**

Pricing longer-term swaps

The last two examples provide a pricing method for interest rate swaps for as far forward in maturity as there are futures contracts available. If futures or FRAs are not available up to the maturity of an interest rate swap however, there is not such a precise arbitrage structure to calculate a swap rate. Longer-term swap rates must however be linked to capital market yields. In Example 8.1, we considered how company AAA achieved a lower LIBOR-based borrowing cost than it could otherwise have done, by combining a fixed-rate bond issue with a swap. In that example, where we ignored any fees associated with issuing a bond and any cost of intermediation by a bank, a swap rate of 7.9% would have given company AAA an all-in borrowing cost of $(8.0\% - 7.9\% + \text{LIBOR}) = \text{LIBOR} + 0.1\%$ – which is exactly the same as the cost achieved through a straightforward LIBOR-based borrowing. It was therefore necessary for the swap rate to be higher than around 7.9% in order for AAA to achieve a lower cost through the liability swap structure (we are ignoring here any differences between annual and semi-annual, or money market and bond basis).

On the other hand, a swap rate of 8.7% would have given company BBB an all-in borrowing cost of $(\text{LIBOR} + 0.8\% + 8.7\% - \text{LIBOR}) = 9.5\%$ – which is exactly the same as the cost achieved through a straightforward fixed-rate borrowing. It was therefore necessary for the swap rate to be less than around 8.7% in order for BBB to achieve a lower cost through the swap structure.

Although the available borrowing costs in Example 8.1 are hypothetical, this shows the type of considerations which generate the swap rate available in the market. As in other markets, it is the rate at which supply and demand balance. Supply and demand will be affected by borrowing and investment rates available to a wide range of market participants, with a range of different opportunities and fee structures.

VALUING SWAPS

Marking-to-market a swap

To value an interest rate swap, we calculate its NPV, exactly as we value other instruments. The most appropriate rates to use for discounting the cashflows to present values are zero-coupon swap yields. A less satisfactory alternative is to use the current par swap rate for the maturity of the swap we are valuing. In the section “The pricing link between FRAs and interest rate swaps”, we created shorter-term zero-coupon swap rates from strips of FRAs. Longer-term zero-coupon swap yields can be created by bootstrapping from the par swap yield curve. This is done in exactly the same way as we constructed zero-coupon bond yields in the chapter on Zero-coupon Rates and Yield Curves. Instead of using a series of existing bonds for the bootstrapping process, we use a series of fictitious bonds, each of which has an initial cost of 100 and a coupon equal to the par swap rate.

One complication is the valuation of the floating-rate side of the swap, as the cashflows are not yet known. One approach is first to calculate, for each period, the forward-forward interest rates consistent with the current yield structure, and then to calculate each floating-rate cashflow assuming this forward-forward rate. Another approach – mathematically equivalent – is to add to the schedule of actual swap cashflows a further set of fictitious cashflows which exactly offsets the unknown amounts. If the additional fictitious cashflows can be arranged so that they have a zero NPV, the NPV that we are trying to calculate will be unaffected. As we have offset the unknown cashflows, we are left with an NPV which we can calculate.

The additional series of fictitious cashflows to be added is effectively a par FRN. Assuming no change in credit risk, an FRN should be priced at par on a coupon date. For example, issuing 100 of an FRN with a 6-monthly coupon of LIBOR is equivalent to borrowing 100 at LIBOR for only six months, and then repeatedly rolling the borrowing over at the end of each six months. It follows that on a coupon date, 100 is the NPV of the future FRN flows. Therefore a series of cashflows consisting of 100 in one direction at the beginning, offset by future FRN flows in the other direction, has a zero NPV. If these cashflows have a zero NPV on a future coupon date, they must also have a zero NPV now. We can therefore add these flows to the swap structure, beginning, at the next swap interest payment date, without changing the total NPV.

Example 8.9

Value the following interest rate swap on 27 March 1998:

Notional amount of swap:	10 million
Start of swap:	21 July 1997
Maturity of swap:	21 July 2000
Receive:	7.4% (annual 30/360)
Pay:	LIBOR (semi-annual ACT/360)
Previous LIBOR fixing:	9.3% from 21 January 1998 to 21 July 1998

The zero-coupon discount factors from 27 March 1998 are:

21 July 1998:	0.9703
21 January 1999:	0.9249
21 July 1999:	0.8825
21 January 2000:	0.8415
21 July 2000:	0.8010

The cashflows are as follows:

<i>Date</i>		<i>Swap</i>
21 July 1998:	+ 10 m × 7.4%	− 10 m × 9.3% × $\frac{181}{360}$
21 Jan 1999:		− 10 m × L_1 × $\frac{184}{360}$
21 July 1999:	+ 10 m × 7.4%	− 10 m × L_2 × $\frac{181}{360}$
21 Jan 2000:		− 10 m × L_3 × $\frac{181}{360}$
21 July 2000:	+ 10 m × 7.4%	− 10 m × L_4 × $\frac{182}{360}$

where: L_1 is LIBOR from 21 July 1998 to 21 January 1999

L_2 is LIBOR from 21 January 1999 to 21 July 1999

L_3 is LIBOR from 21 July 1999 to 21 January 2000

L_4 is LIBOR from 21 January 2000 to 21 July 2000

Method 1

From the discount factors, we can calculate forward-forward rates to substitute for the unknown LIBOR fixings as follows:

$$L_1: \left(\frac{0.9703}{0.9249} - 1 \right) \times \frac{360}{184} = 9.6039\%$$

$$L_2: \left(\frac{0.9249}{0.8825} - 1 \right) \times \frac{360}{181} = 9.5560\%$$

$$L_3: \left(\frac{0.8825}{0.8415} - 1 \right) \times \frac{360}{181} = 9.6907\%$$

$$L_4: \left(\frac{0.8415}{0.8010} - 1 \right) \times \frac{360}{182} = 10.0012\%$$

Using these rates, we have the following cashflows:

Date	Swap	Net cashflows
21 July 1998:	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times 9.3\% \times \frac{181}{360}$	+ 272,417
21 Jan 1999:	$-10\text{ m} \times 9.6039\% \times \frac{184}{360}$	- 490,866
21 July 1999:	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times 9.5560\% \times \frac{181}{360}$	+ 259,546
21 Jan 2000:	$-10\text{ m} \times 9.6907\% \times \frac{181}{360}$	- 487,227
21 July 2000:	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times 10.0012\% \times \frac{182}{360}$	+ 234,384

We can now value these net cashflows using the discount factors, to give:

$$(272,417 \times 0.9703) + (-490,866 \times 0.9249) + (259,546 \times 0.8825) + (-487,227 \times 0.8415) + (234,384 \times 0.8010) = -182,886$$

The swap is therefore showing a mark-to-market valuation of -182,886.

Method 2

Without upsetting the NPV valuation, we can add the cashflows for a fictitious “FRN investment” of 10 million which starts on 21 July 1998, matures on 21 July 2000 and pays LIBOR, because the NPV of these cashflows will be zero on 21 July 1998 (and hence zero on 27 March 1998). The resulting cashflows will then be:

Date	Swap	“FRN”	Net cashflows
21 July 1998:	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times 9.3\% \times \frac{181}{360}$	-10 m	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times 9.3\% \times \frac{181}{360}$ -10 m
21 Jan 1999:	$-10\text{ m} \times L_1 \times \frac{184}{360}$	$+10\text{ m} \times L_1 \times \frac{184}{360}$	
21 July 1999:	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times L_2 \times \frac{181}{360}$	$+10\text{ m} \times L_2 \times \frac{181}{360}$	$+10\text{ m} \times 7.4\%$
21 Jan 2000:	$-10\text{ m} \times L_3 \times \frac{181}{360}$	$+10\text{ m} \times L_3 \times \frac{182}{360}$	
21 July 2000:	$+10\text{ m} \times 7.4\%$ $-10\text{ m} \times L_4 \times \frac{182}{360}$	$+10\text{ m} \times L_4 \times \frac{182}{360}$ $+10\text{ m}$	$+10\text{ m} \times 7.4\%$ $+10\text{ m}$

We can now value these cashflows using the discount factors to give:

$$(-9,727,583 \times 0.9703) + (740,000 \times 0.8825) + (10,740,000 \times 0.8010) = -182,884$$

Reversing a swap transaction

Suppose that a dealer or customer, who has previously entered into a swap transaction, now wishes to close out this position. He can transact another swap in the opposite direction for the remaining term of the existing swap. If he has previously dealt a swap to pay a fixed rate and receive LIBOR, for example, he can now deal instead to receive a fixed rate and pay LIBOR. The LIBOR cashflows will balance but, because the fixed rate on the new swap is unlikely to be the same as the fixed rate on the old swap, there will be a net difference in the fixed amount on each future payment date. This difference may be a net receipt or a net payment.

Rather than put a new swap on the books however, if the counterparty in the new swap is the same as the counterparty in the original swap, it may well be preferable for both parties to settle immediately the difference between the original swap rate and the new swap rate. This would reduce both credit line utilisation and capital adequacy requirements. In order to settle immediately, the potential future cashflows – the difference between the two fixed rates – need to be valued. This is done, as usual, by calculating the NPV of these net cashflows. This NPV is then paid by one party to the other, the original swap is cancelled and no new swap is transacted.

Example 8.10

We have on our books the swap already described in Example 8.9. For value on 21 July 1998, we decide to reverse the swap. The same counterparty quotes a swap rate of 8.25% then for the remaining 2 years. What should the settlement amount be to close out the swap position, using the following discount factors?

21 July 1999:	0.9250
21 July 2000:	0.8530

The LIBOR-based flows on the two swaps offset each other exactly. The remaining flows are then:

<i>Date</i>	<i>Original swap</i>	<i>Reverse swap</i>	<i>Net cashflows</i>
21 July 1999	+10 m × 7.4%	– 10 m × 8.25%	– 85,000
21 July 2000	+10 m × 7.4%	– 10 m × 8.25%	– 85,000

The NPV of the net cashflows is:

$$(-85,000 \times 0.9250) + (-85,000 \times 0.8530) = -151,130$$

We therefore pay the counterparty 151,130 to close out the existing swap.

Constructing an asset swap

The following example considers essentially the same asset swap as Example 8.5. In this case however, we construct a par / par swap – that is, a package with an initial principal amount invested, the same “par” principal amount returned at maturity, and an income stream calculated as a constant spread above or below LIBOR based on this same par amount. This is an asset swap to create a synthetic FRN, priced at par and redeemed at par.

Example 8.11

An investor purchases the following bond and wishes to convert it to a synthetic FRN with an asset swap:

Settlement date:	16 August 1998
Maturity date:	16 August 2001
Coupon:	11.5% (annual)
Price:	102.00
Yield:	10.686%

The 3-year par swap rate from 16 August 1998 is quoted at 10.2% / 10.3% (annual 30/360) against LIBOR (semi-annual, ACT/360). The investor would like a par / par asset swap structure.

The discount factors from 16 August 1998 are given as follows:

16 February 1999:	0.9525
16 August 1999:	0.9080
16 February 2000:	0.8648
16 August 2000:	0.8229
16 February 2001:	0.7835
16 August 2001:	0.7452

Suppose that the principal amount invested is 102, and that the investor transacts a “straightforward” swap based on a par amount of 100. The bond cashflows and swap cashflows would then be as follows:

<i>Date</i>	<i>Bond</i>	<i>Swap</i>
16 Aug 1998:	– 102	
16 Feb 1999:		$+100 \times \text{LIBOR} \times \frac{184}{360}$
16 Aug 1999:	+ 11.5	$- 100 \times 10.3\%$ $+100 \times \text{LIBOR} \times \frac{181}{360}$
16 Feb 2000:		$+100 \times \text{LIBOR} \times \frac{184}{360}$
16 Aug 2000:	+ 11.5	$- 100 \times 10.3\%$ $+100 \times \text{LIBOR} \times \frac{182}{360}$
16 Feb 2001:		$+100 \times \text{LIBOR} \times \frac{184}{360}$
16 Aug 2001:	+ 100 + 11.5	$- 100 \times 10.3\%$ $+100 \times \text{LIBOR} \times \frac{181}{360}$

These cashflows will clearly not provide a “clean” result in terms of a spread relative to LIBOR based on a principal of 100, for two reasons. First, the difference between the 11.5% coupon on the bond and the 10.3% swap rate is an annual cashflow, but LIBOR is semi-annual. Second, the principal invested is 102 rather than 100. These differences have an NPV of:

$$(11.5 - 100 \times 10.3\%) \times 0.9080 + (11.5 - 100 \times 10.3\%) \times 0.8229 \\ + (11.5 - 100 \times 10.3\%) \times 0.7452 + (100 - 102) = 0.9713$$

What interest rate i (ACT/360, semi-annual) is necessary so that a series of cashflows at i based on a principal of 100 would have the same NPV?

$$(100 \times i \times \frac{184}{360} \times .9525) + (100 \times i \times \frac{181}{360} \times .9080) + (100 \times i \times \frac{184}{360} \times .8648) \\ + (100 \times i \times \frac{182}{360} \times .8229) + (100 \times i \times \frac{184}{360} \times .7835) + (100 \times i \times \frac{181}{360} \times .7452) \\ = 0.9713$$

The solution to this is: $i = 0.0038$

We can therefore replace the following fixed cashflows arising from a par swap:
annual 30/360 swap outflows of $(100 \times 10.3\%)$
plus an initial “odd” investment of 2,

by the following fixed cashflows – effectively an off-market swap:
annual 30/360 swap outflows of $(100 \times 11.5\%)$
plus semi-annual ACT/360 inflows of $(100 \times 0.38\%)$.

This 0.38% can then be added to the semi-annual swap inflows of $(100 \times \text{LIBOR})$ which we already have. We have therefore replaced the par swap by one with the same NPV as follows:

Receive: LIBOR + 0.38% (ACT/360, semi-annual)
 Pay: 11.5% (30/360, annual)

The net effect is therefore a par / par asset swap giving the investor LIBOR plus 38 basis points.

It is worth noting that the result of the last example is around 2 basis points worse than the approximate result calculated in Example 8.5. This is because in Example 8.5, the bond yield and swap rate are converted to semi-annual rates by decompounding at their own respective semi-annual rates. In Example 8.11, all cashflows are discounted by the discount factors.

- To value a swap, calculate the NPV of the cashflows, preferably using zero-coupon swap yields or the equivalent discount factors.
- To value floating-rate cashflows, superimpose offsetting floating-rate cashflows known to have an NPV of zero – effectively an FRN.
- A swap at current rates has an NPV of zero.
- If a current swap involves an off-market fixed rate, this is compensated by an adjustment to the other side of the swap, or by a one-off payment, so as to maintain the NPV at zero.

**Calculation
summary**

HEDGING AN INTEREST RATE SWAP

As with every instrument, a dealer may find he has a position which he does not want – either because he has changed his mind about the direction of interest rates, or because he has made a two-way price and been taken up on it. He is therefore exposed to movements in the swap rate and will want to hedge himself. This he can do in any of several ways:

- Deal an exactly offsetting swap.
- Deal in a different instrument which will give him offsetting flows. Suppose for example that under the swap, the dealer is paying out a fixed interest rate for 5 years. As a hedge, he might buy a 5-year bond – or a bond with a modified duration the same as that of a bond with coupons equal to the fixed swap rate. This will give him a fixed income

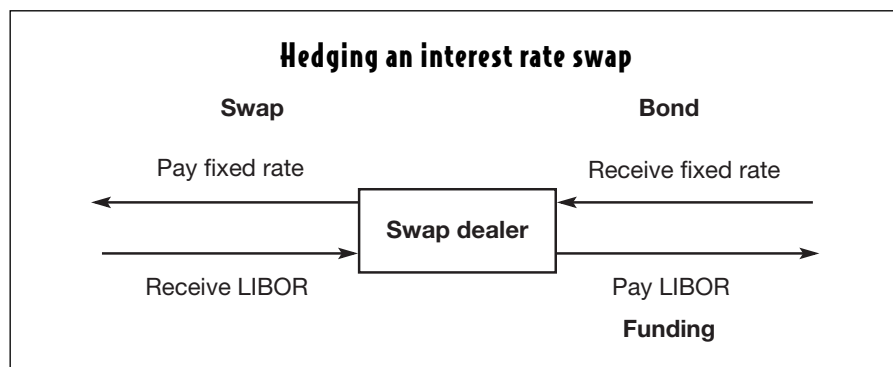


Fig 8.7

to offset the fixed payment on the swap. In order to buy the bond, he can fund himself at LIBOR or a rate linked to LIBOR.

In this way, he is hedged for as long as he holds the bond. If the dealer does subsequently deal a second swap which offsets the first one, but 5-year rates have fallen, he will make a loss. However, the bond will have increased in value correspondingly. Clearly the hedge is not perfect, as the swap dealer is exposed during this period to the risk that the swap market does not move exactly in line with the bond he has used as a hedge – a basis risk.

- (c) Deal in bond futures or options. These may provide a liquid hedge, but the hedge could be less perfect, as the specifications of the futures or options contracts may imply a rather different maturity from the swap, so that the hedge value does not respond to yield changes in the same way as the swap.

AMORTISING AND FORWARD-START SWAPS

Amortising swap

In practice, a company often needs an interest rate swap on a borrowing which is amortising rather than one which has a bullet maturity – that is, the principal is repaid in instalments over the life of the borrowing rather than all at maturity. The borrowing may also be going to start at some time in the future, or already have begun so that the next interest payment is at some period ahead.

Consider a 2-year borrowing of 200 beginning now, with floating rate interest payments each 6 months and amortisations of 50 each 6 months. A single interest rate swap for 200 over 2 years would not match the borrowing profile, while 4 separate swaps of 50 each at different rates (one for 6 months, one for 1 year, one for eighteen months and one for 2 years) would result in irregular interest flows. We therefore wish to create a single swap rate for the amortising structure.

Suppose that the swap rates (semi-annual bond basis against 6-month LIBOR) are as follows:

6 months	6.00% (this is equivalent to current 6 month LIBOR)
1 year	6.50%
18 months	7.00%
2 years	7.50%

We can calculate from these rates a set of zero-coupon discount factors by bootstrapping in the same way as we did in Chapter 6. These can be calculated to be:

6 months:	0.9709
1 year:	0.9380
18 months:	0.9016
2 years:	0.8623

If the single amortising swap rate is i , the cashflows on the borrowing plus swap would be as follows:

<i>Time</i>	<i>Cashflow</i>
Now:	+200
6 months:	$-50 - \left(200 \times \frac{i}{2}\right)$
1 year:	$-50 - \left(150 \times \frac{i}{2}\right)$
18 months:	$-50 - \left(100 \times \frac{i}{2}\right)$
2 years:	$-50 - \left(50 \times \frac{i}{2}\right)$

If the single rate i is consistent with the swap yield curve, the NPV of these flows will be zero:

$$\begin{aligned} &\left(-50 - \left(200 \times \frac{i}{2}\right)\right) \times 0.9709 + \left(-50 - \left(150 \times \frac{i}{2}\right)\right) \times 0.9380 \\ &+ \left(-50 - \left(100 \times \frac{i}{2}\right)\right) \times 0.9016 + \left(-50 - \left(50 \times \frac{i}{2}\right)\right) \times 0.8623 + 200 = 0 \end{aligned}$$

The solution for this is $i = 6.99\%$

Forward-start swap

Consider now an 18-month borrowing of 150 which will not start for 6 months and which will amortise at the rate of 50 each 6 months again. It is possible to calculate the appropriate swap rate for this forward-start borrowing in exactly the same way. In this case, the cashflows are as follows:

<i>Time</i>	<i>Cashflow</i>
6 months:	+150
1 year:	$-50 - \left(150 \times \frac{i}{2}\right)$
18 months:	$-50 - \left(100 \times \frac{i}{2}\right)$
2 years:	$-50 - \left(50 \times \frac{i}{2}\right)$

Again, the NPV of these flows will be zero:

$$\begin{aligned} &\left(-50 - \left(150 \times \frac{i}{2}\right)\right) \times 0.9380 + \left(-50 - \left(100 \times \frac{i}{2}\right)\right) \times 0.9016 \\ &+ \left(-50 - \left(50 \times \frac{i}{2}\right)\right) \times 0.8623 + 150 \times 0.9709 = 0 \end{aligned}$$

The solution for this is $i = 7.69\%$

The forward-start amortising swap rate is therefore 7.69% (semi-annual bond basis).

**Calculation
summary**

The current swap rate for a swap based on an irregular or forward-start notional principal is again the rate which gives the swap an NPV of zero.

Delayed-start swap

A swap may be transacted to start only a short time in the future, in which case the analysis in the last section can not conveniently be used. An alternative approach is to consider that the dealer will hedge himself in the same way that he might when transacting a swap for immediate value – by buying or selling a bond. In this case, however, the bond hedge will be immediate but the swap will not. The dealer will therefore incur a positive or negative net cost of carry – for example, the difference between the income on the bond and the funding cost of buying the bond, if the hedge is a bond purchase. This will be incurred for the period until the swap begins, at which time the hedge can be reversed and the dealer can cover his position with an offsetting swap.

An adjustment should therefore be made to the fixed rate quoted for the delayed-start swap. The NPV of this adjustment over the life of the swap should be equal to the net cost of carry for the hedging period.

CURRENCY SWAPS

Currency swaps involve an exchange of cashflows largely analogous to those in an interest rate swap, but in two different currencies. Thus one company might establish – or have already established – a borrowing in one currency, which it wishes to convert into a borrowing in another currency. One example would be where the existing borrowing, and the effective borrowing arrangement after the swap, are both fixed rate – a fixed-fixed currency swap. Currency swaps can also be fixed-floating or floating-floating. The transaction is the conversion of a stream of cashflows in one currency into a stream of cashflows in another currency.

It is important to note that, unlike an interest rate swap, a currency swap which is based on a borrowing or an asset generally involves exchange of the principal amount as well as the interest amounts. In a single-currency interest rate swap, nothing would be achieved by this – each party would simply be required to pay and receive the same principal amount, which would net to zero. In a currency swap however, the value of the principal amount at maturity depends on the exchange rate. If this is not included in the swap, the all-in result will not be known.

In order to exchange one cashflow stream for another, the two streams must have the same value at the time of the transaction. The appropriate value of each stream, as with other market instruments, is its NPV. To equate the two NPVs which are in different currencies, we use the current spot exchange rate.

**Calculation
summary**

To value cashflows in a different currency,
convert the resulting NPV at the spot exchange rate.

Example 8.12

You purchase a 5-year USD bond with an annual coupon of 6.7%, at a price of 95.00. You convert this investment to a synthetic FRF investment on a par amount, with an asset swap. The swap rate you achieve is 7.85% for USD against 8.35% for FRF (both annual bond basis). The current exchange rate is 6.00. What all-in FRF yield do you achieve?



The future USD cashflows are as follows for each USD 100 face value:

Year 1	+	6.70
Year 2	+	6.70
Year 3	+	6.70
Year 4	+	6.70
Year 5	+	106.70

Discounting at a rate of 7.85%, the NPV of these flows is USD 95.390. At an exchange rate of 6.00, this is equivalent to FRF 572.341.

For each USD 100 face value of the bond, you invested USD 95.00. This is equivalent to FRF 570. You therefore wish to receive a series of FRF cashflows, such that there is a regular FRF cashflow in years 1 to 5, with an additional cashflow of FRF 570 in year 5. The NPV of these cashflows, discounting at 8.35% must be FRF 572.341. This can be solved using the TVM function on an HP calculator to give the regular cashflow as FRF 48.187. Other profiles for the FRF cashflows could be chosen which also have an NPV of FRF 572.341. This profile, however, provides the par structure at which we are aiming.

The yield on the asset swap is therefore the yield derived from an investment of FRF 570, a principal amount of FRF 570 returned at the end of 5 years and an income stream of FRF 48.187 per year. This can be seen to be a yield of:

$$\frac{48.187}{570} = 8.454\%.$$

Date	Bond USD	Swap USD	FRF	Net cashflows FRF
Now	- 95.00			- 570.000
Year 1	+ 6.70	- 6.70	+ 48.187	+ 48.187
Year 2	+ 6.70	- 6.70	+ 48.187	+ 48.187
Year 3	+ 6.70	- 6.70	+ 48.187	+ 48.187
Year 4	+ 6.70	- 6.70	+ 48.187	+ 48.187
Year 5	+ 106.70	- 106.70	+ 618.187	+ 618.187
NPV =		- 95.39	+ 572.341	

EXERCISES

71. You have a USD 10 million borrowing on which you are paying 8.9% fixed (*annual money-market basis*) and which has exactly 5 years left to run. All the principal will be repaid at maturity.

The current 5-year dollar interest rate swap spread is quoted to you as 80 / 90 over treasuries. The current 5-year treasury yield is 9.0%. (*US treasuries are quoted on a semi-annual bond basis.*)

You believe that interest rates are going to fall, and wish to swap the borrowing from fixed to floating. Without discounting all the cashflows precisely, what will the resulting net LIBOR-related cost of the swapped borrowing be approximately?



72. The 3-month USD cash rate and the futures prices for USD are as follows. The first futures contract period begins exactly 3 months after spot.

3-month cash:	(91 days)	6.25%
futures 3 v 6:	(91 days)	93.41
6 v 9:	(91 days)	92.84
9 v 12:	(92 days)	92.63
12 v 15:	(91 days)	92.38
15 v 18:	(91 days)	92.10

- What are the zero-coupon swap rates (annual equivalent, bond basis) for each quarterly maturity from 3 months up to 18 months, based on these prices?
 - What should the 18-month par swap rate be on a quarterly money market basis?
73. A year ago your sterling-based company issued a USD 100 million 4-year bond with a 10% annual coupon, and converted the proceeds to sterling at 1.80. The sterling/dollar exchange rate is now 1.55 and you wish to protect against any further currency loss by swapping the borrowing into sterling. A counterparty is prepared to pay the outstanding USD cashflows on your bond valued at 9% (annual bond basis), and receive equivalent sterling cashflows at 11%.
- What will your future cashflows be if you enter into such a swap?
 - What would the cashflows be if, instead of using a swap, you used long-dated foreign exchange to convert all your dollar liabilities into sterling? To calculate the forward prices, assume that they are based on the following money-market interest rates:

	USD	GBP
1 year	9.0%	14.0%
2 years	8.5%	12.0%
3 years	8.0%	10.0%

- Which method would you prefer to use to hedge the dollar liabilities?

- 74.** You issue a 5-year, 6.5% annual coupon bond for USD 10 million and swap it into floating-rate CHF. The spot USD/CHF exchange rate is 1.50 and the current swap rate is 6.8% fixed USD against 6-month CHF LIBOR. The swap matches your USD flows exactly and achieves a regular floating-rate cost based on the equivalent CHF amount borrowed with the same amount repaid at maturity.

Show the cashflows involved and calculate the all-in CHF floating-rate cost you achieve, assuming all CHF cashflows can be discounted at 4.5% (annual) and all USD cashflows at 6.8% (annual). Ignore all bond-issuing costs.

- 75.** You have previously entered a currency swap to receive fixed-rate US dollars at 8% (annually, 30/360 basis) based on USD 10 million (with USD 10 million received at maturity) and pay floating-rate Deutschemarks at LIBOR (semi-annually, ACT/360 basis) based on DEM 15 million (with DEM 15 million paid at maturity). The swap terminates on 25 May 1999. It is now February 1998 and the spot USD/DEM exchange rate is 1.6500. The last Deutschemark LIBOR fixing was 5.3% for 25 November 1997. The discount factors to the remaining payment dates are as follows. What is the mark-to-market value of the swap now in dollars?

	<i>USD</i>	<i>DEM</i>
25 May 1998:	0.9850	0.9880
25 November 1998:	0.9580	0.9650
25 May 1999:	0.9300	0.9400

- 76.** You issue a 3-year fixed-rate US dollar bond at 7% (annual) with a bullet maturity. After all costs, you receive 99.00 from the issue. You swap the bond to floating-rate dollars. You arrange the swap so that your net cashflows from the swapped bond issue give you a par amount at the beginning, a regular LIBOR-related cost based on this par amount for 5 years, and the same par amount to be repaid at maturity. The current par swap rate for 3 years is 7.5% (annual, 30/360 basis) against LIBOR (semi-annual, ACT/360). Assuming that this same rate of 7.5% (annual) can be used as a rate of discount throughout, what all-in floating-rate cost can you achieve above or below LIBOR?





“As with insurance premiums, assuming that option sellers can accurately assess the probability of each possible outcome, their total payments out on expiry of a portfolio of options sold should approximate to the premiums received. Option pricing theory therefore depends on assessing these probabilities.”

Options

Overview

The ideas behind option pricing

Pricing models

OTC options vs. exchange-traded options

The Greek letters

Hedging with options

Some “packaged” options

Some trading strategies

Some less straightforward options

Exercises

OVERVIEW

Two sections in this chapter include some mathematical equations which may seem rather more complex than those we have looked at so far – the sections on Black–Scholes and Greek Letters. These have been included for completeness, and do not need to be considered in detail by most readers. We have kept to the aim of this book, which is to be practical rather than to frighten, and have therefore not shown here how to derive these formulas. The interested reader will find thorough mathematical treatments of the subject in some of the books mentioned in the Bibliography.

An option is a contract whereby one party has the right to complete a transaction in the future (with a previously agreed amount, date and price) if he/she so chooses, but is not obliged to do so. The counterparty has no choice: they must transact if the first party wishes and cannot otherwise. For the first party, an option is therefore similar to a forward deal, with the difference that they can subsequently decide whether or not to fulfil the deal. For the second party, an option is similar to a forward deal with the difference that they do not know whether or not they will be required to fulfil it. Clearly, the contract will be fulfilled only if advantageous to the first party and disadvantageous to the second party. In return for this flexibility, the first party must pay a “premium” up-front to compensate the second party for the latter’s additional risk.

For someone using an option as a hedge rather than as a trading instrument, it can be considered as a form of insurance. An insurance policy is not called upon if circumstances are satisfactory. The insured person is willing to pay an insurance premium, however, in order to be able to claim on the insurance policy if circumstances are not satisfactory. An option as a hedge is similar. If an option enables a hedger to buy something at a certain rate but it turns out to be cheaper in the market than that rate, the hedger does not need the option. If, however, it turns out to be more expensive than the option rate, the hedger can “claim” on the option. The hedger thus has “insurance protection” at the option rate, for which a premium is paid.

For the trader selling the option, the situation is similar to that of the insurer – the trader is exposed to the risk of being obliged to deliver at the agreed rate but only being able to cover the position in the market at a much worse rate.

Options are available in a wide range of underlying instruments, including currencies, interest rates, bonds and commodities.

Key Point

An option is a deal for forward delivery, where the buyer of the option chooses whether the transaction will be consummated, and pays a premium for this advantage

Basic terminology

Terminology

The first party described above is the purchaser or **holder** of the option. The second party (the seller of the option who receives the premium) is called the **writer** of the option.

To **exercise** an option is to use it, rather than allow it to **expire** unused at maturity.

With a **European** option, the holder can only exercise the option at expiry. With a 3-month option, for example, the holder can only choose at the end of 3 months whether or not to exercise. With an **American** option, however, the holder can choose to exercise at any time between the purchase of the option and expiry. European and American options are both available everywhere; the terms are technical rather than geographical.

The price agreed in the transaction – the **strike price** or **strike rate** – is not necessarily the same as the forward rate for the same future date, but is chosen to suit the option buyer. If it is more advantageous than the forward rate to the option buyer, the option is referred to as **in-the-money**. If it is less advantageous than the forward rate to the option buyer, it is **out-of-the-money**. If the strike is the same as the forward rate, the option is **at-the-money** (ATM). As the market moves after the option has been written, the option will move in- and out-of-the-money.

A **put** option is an option to sell something. A **call** option is an option to buy something. The “something” which is being bought or sold is referred to as the **underlying**. Thus in a bond option, the bond is the underlying. In a USD/DEM option, the exchange rate is the underlying. In a short-term interest rate option, the underlying is an FRA or an interest rate futures contract.

It is always possible to exercise an in-the-money option at an immediate profit or, in the case of a European option, to lock in a profit immediately by reversing it with a forward deal and exercising it later. The locked-in profit – the difference between the strike price and the current market price – is known as the **intrinsic value** of the option. The intrinsic value of an out-of-the-money option is zero rather than negative. The remaining part of the premium paid for the option above this intrinsic value is known as the **time value**.

THE IDEAS BEHIND OPTION PRICING

The concepts

The pricing of an option depends on probability. In principle, ignoring bid offer spreads, the premium paid to the writer should represent the buyer's

expected profit on the option. The profit arises from the fact that the option buyer is always entitled to exercise an option which expires in-the-money, and simultaneously cover the position in the market at a better price. The buyer will never be obliged to exercise the option at a loss. As with insurance premiums, assuming that option sellers can accurately assess the probability of each possible outcome, the writer's total payments out on expiry of a portfolio of options sold should approximate to the premiums received. Option pricing theory therefore depends on assessing these probabilities and deriving from them an expected outcome, and hence a fair value for the premium.

The factors on which these probabilities depend are as follows:

- **The strike price:** the more advantageous the strike is to the buyer at the time of pricing, the greater the probability of the option being exercised, at a loss to the writer, and hence the greater the option premium.
- **Volatility:** volatility is a measure of how much the price fluctuates. The more volatile the price, the greater the probability that the option will become of value to the buyer at some time. This measurement is formalized in option pricing theory as the annualized standard deviation of the logarithm of relative price movements.
- **The maturity:** the longer the maturity of the option, the greater the probability that it will become of value to the buyer at some time, because the price has a longer time in which to fluctuate.
- **Interest rates:** the premium represents the buyer's expected profit when the option is exercised, but is payable up-front and is therefore discounted to a present value. The rate of discount therefore affects the premium to some extent. The forward price – and hence the relationship between the strike and the forward – is also affected by interest rate movements. Most importantly, in the case of an option on a bond or other interest-rate instrument, the interest rate also directly affects the underlying price.

Because currency options involve two commodities (that is, each currency) rather than one, currency option prices can be expressed in various ways:

- a. As a percentage of the base currency amount.
- b. In terms of the base currency per unit of the base currency.
- c. As a percentage of the variable currency amount.
- d. In terms of the variable currency per unit of the variable currency.
- e. In terms of the base currency per unit of the variable currency.
- f. In terms of the variable currency per unit of the base currency.

Example 9.1

A call option on USD 1 million against DEM (or alternatively, a DEM put option against USD) has a strike of 1.50, with a current spot rate of 1.40. The option premium in absolute terms is USD 10,000. This could be expressed as:

$$(a) \ 1.00\% \left(= \frac{10,000}{1,000,000} \right)$$

(b) 1 US cent per dollar

$$(c) 0.93\% \left(= \frac{10,000 \times 1.40}{1,000,000 \times 1.50} \right)$$

(d) 0.93 pfennigs per Deutschemark

(e) 0.67 cents per Deutschemark ($= 0.93 \div 1.40$)

(f) 1.4 pfennigs per dollar ($= 1 \text{ cent} \times 1.40$)

The most usual methods of quotation are (a) and (f).

Basic statistics

Arithmetic mean and standard deviation

The arithmetic mean (or “mean”) of a series of numbers is the average of the numbers. If we have the following numbers:

83, 87, 82, 89, 88

then the mean of the numbers is:

$$\frac{(83 + 87 + 82 + 89 + 88)}{5} = 85.8$$

The “standard deviation” of the same numbers is a measure of how spread out the numbers are around this mean. If all the numbers are exactly the same, the standard deviation would be zero. If the numbers are very spread out, the standard deviation would be very high. The standard deviation is defined as the square root of the “variance.” The variance in turn is the average of the squared difference between each number and the mean.

With the numbers above, we have the following:

<i>Data</i>	<i>Difference between data and 85.8</i>	<i>(Difference)²</i>
83	-2.8	7.84
87	+1.2	1.44
82	-3.8	14.44
89	+3.2	10.24
88	+2.2	4.84

Mean = 85.8

Total = 38.80

The variance is thus $\frac{38.80}{5} = 7.76$ and the standard deviation is $\sqrt{7.76} = 2.79$.

The symbol μ is sometimes used for the mean, the symbol σ for the standard deviation, and σ^2 for the variance.

Calculation summary

Mean (μ) = sum of all the values divided by the number of values

Variance (σ^2) = mean of (difference from mean)²

When estimating the variance from only a sample of the data rather than all the data, divide by one less than the number of values used

Standard deviation (σ) = $\sqrt{\text{variance}}$

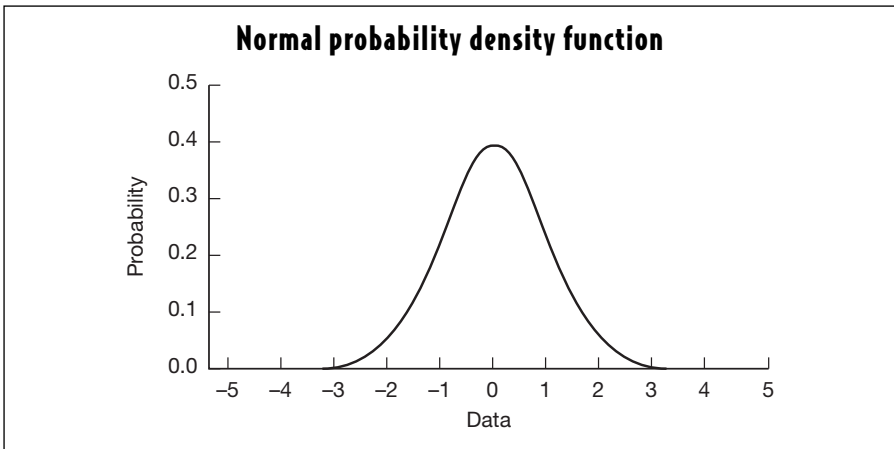
**Calculation
summary**

Probability density

The “probability density” of a series of numbers is a description of how likely any one of them is to occur. Thus the probability density of the results of throwing a die is $\frac{1}{6}$ for each possible result. The probability density of the heights of 100 adult men chosen at random will be relatively high for around 170 cm to 180 cm, and extremely low for less than 150 cm or more than 200 cm. The “shape” of the probability density therefore varies with the type of results being considered.

A particular probability density which is used as an approximate description of many circumstances in life is known as the “normal” probability function (see Figure 9.1):

Fig 9.1



If the probabilities are as shown in Figure 9.1, for example, a number less than -3 or more than $+3$ is extremely unlikely, while a number between say -1 and $+1$ is rather likely.

The normal probability function is not straightforward – the equation for Figure 9.1 is:

$$\text{probability density} = \frac{1}{\sqrt{2\pi} e^{\frac{x^2}{2}}}$$

This function is important here because it is used in option pricing. A standard assumption used for pricing options is that movements in the logarithm of relative prices can be described by this function. By this we mean $\text{LN}\left(\frac{\text{current price}}{\text{previous price}}\right)$. If we are looking at daily price changes, for example, this means that a series of data such as $\text{LN}\left(\frac{\text{today's price}}{\text{yesterday's price}}\right)$ is expected to have a normal probability density.

This relative price change $\frac{\text{current price}}{\text{previous price}}$ is the same as $\left(1 + i \times \frac{\text{days}}{\text{year}}\right)$, where i is the rate of return being earned on an investment in the asset. In Chapter 1, we saw that $\text{LN}\left(1 + i \times \frac{\text{days}}{\text{year}}\right)$ is equal to $r \times \frac{\text{days}}{\text{year}}$ where r is the continuously compounded rate of return. Therefore the quantity $\text{LN}\left(\frac{\text{current price}}{\text{previous price}}\right)$ which we are considering is in fact the continuously compounded return on the asset over the period.

Probability distribution

The “cumulative probability distribution” of a series of numbers is the probability that the result will be no greater than a particular number. Thus the probability distribution for throwing the die is:

- probability $\frac{1}{6}$ that the number thrown will be 1
- probability $\frac{2}{6}$ that the number thrown will be 1 or 2
- probability $\frac{3}{6}$ that the number thrown will be 1, 2 or 3
- probability $\frac{4}{6}$ that the number thrown will be 1, 2, 3 or 4
- probability $\frac{5}{6}$ that the number thrown will be 1, 2, 3, 4 or 5
- probability $\frac{6}{6}$ that the number thrown will be 1, 2, 3, 4, 5 or 6

The normal probability function shown above has the probability distribution shown in Figure 9.2. With this particular probability distribution, for example, there is a probability of around 85% that the outcome will be less than or equal to 1.

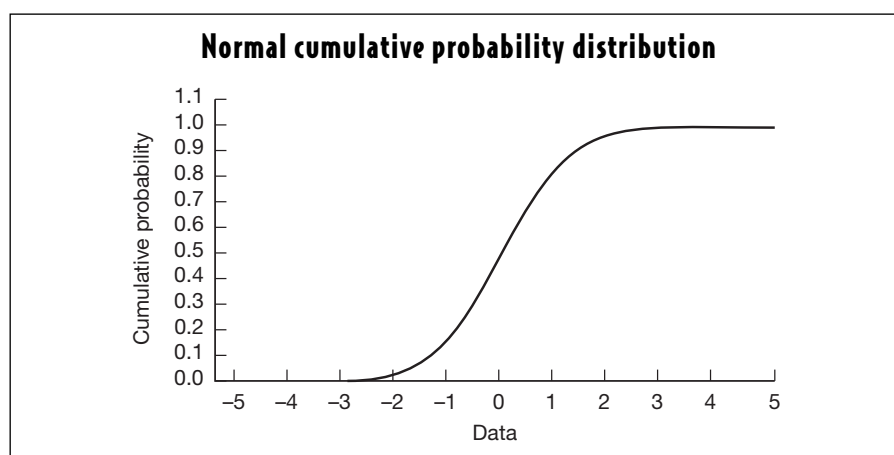


Fig 9.2

Calculating historic volatility

The volatility of an option is defined formally as the annualized standard deviation of the logarithm of relative price movements. As we saw in the previous section, this is the standard deviation of the continuously compounded return over a year.

Example 9.2

Given five daily price data for an exchange rate, the volatility is calculated as follows:

Day	Exchange rate	$LN(\frac{t-8345}{t-8220})$ etc.	Difference from mean	(Difference) ²
1	1.8220			
2	1.8345	0.00684	0.00622	0.000039
3	1.8315	-0.00164	-0.00226	0.000006
4	1.8350	0.00191	0.00129	0.000002
5	1.8265	-0.00464	-0.00526	0.000028
		Mean = 0.00062	Total = 0.000074	

$$\text{Variance} = \frac{\text{sum of (differences)}^2}{(\text{number of data} - 1)} = \frac{0.000074}{3} = 0.000025$$

$$\text{Standard deviation} = \sqrt{\text{variance}} = 0.00497$$

$$\begin{aligned}\text{Volatility} &= \text{standard deviation} \times \sqrt{\text{frequency of data per year}} \\ &= 0.00497 \times \sqrt{252} = 7.9\%\end{aligned}$$

The frequency per year of the data in Example 9.2 assumes that weekends and bank holidays are ignored for daily data. If “dummy” data are included for these days, then the frequency per year is 365. If the data are weekly, the annualized volatility is:

$$0.00497 \times \sqrt{52} = 3.6\%$$

Note that we began with 4 data. Although there are 5 exchange rates, there are only 4 relative price changes. In calculating the variance, we then divided by 3, rather than 4 as suggested in the earlier section on standard deviation. This is because when only a part of the historical data is used rather than all possible data, a better estimate of the true variance is achieved by dividing by (number of data – 1).

What we have calculated is the historic volatility – that is, the volatility of actual recorded prices. When a dealer calculates an option price, he will not in practice use a historic volatility exactly. Instead, he will use a blend of his own forecast for volatility, the current general market estimate of volatility, his own position, and recent actual experience of volatility.

The volatility which is used to calculate an option premium – either the price quoted by a particular dealer or the current general market price – is known as the “implied volatility,” or simply the “implied.” This is because, given the pricing model, the price, and all the other factors, it is possible to work backwards to calculate what volatility is implied in that calculation. Implied volatility therefore means current volatility as used by the market in its pricing.

Calculation summary

$$\begin{aligned}\text{Historic volatility} &= \\ &\text{standard deviation of LN(relative price movement)} \times \\ &\sqrt{\text{frequency of data per year}}\end{aligned}$$

PRICING MODELS

Black–Scholes

The most widely used pricing model for straightforward options was derived by Fischer Black and Myron Scholes and is known as the Black–Scholes formula. This model depends on various assumptions:

- Future relative price changes are independent both of past changes and of the current price.
- Volatility and interest rates both remain constant throughout the life of the option. In practice, volatility and interest rates are not constant throughout the option’s life. In the case of a bond option, for example, this causes significant problems. First, volatility tends towards zero as the

bond approaches maturity, because its price must tend to par. Second, the price of the bond itself is crucially dependent on interest rates, in a way that, say, an exchange rate is not.

- The probability distribution of relative price changes is lognormal. The assumption of a lognormal distribution implies a smaller probability of significant deviations from the mean than is generally the case in practice. This is reflected in how fat or thin the “tails” of the bell-shaped probability curve are and affects the pricing of deep in-the-money and deep out-of-the-money options.
- There are no transaction costs.

Based on these assumptions, the price of a European call option for one unit of an asset which does not pay a dividend is:

Black-Scholes option-pricing formula for a non-dividend-paying asset

$$\text{Call premium} = \text{spot price} \times N(d_1) - \text{strike price} \times N(d_2) \times e^{-rt}$$

$$\begin{aligned} \text{Put premium} &= -\text{spot price} \times N(-d_1) + \text{strike price} \times N(-d_2) \times e^{-rt} \\ &= \text{call premium} + \text{strike price} \times e^{-rt} - \text{spot price} \end{aligned}$$

$$\text{where: } d_1 = \frac{\text{LN}\left(\frac{\text{spot} \times e^{rt}}{\text{strike}}\right) + \frac{\sigma^2 t}{2}}{\sigma \sqrt{t}}$$

$$d_2 = \frac{\text{LN}\left(\frac{\text{spot} \times e^{rt}}{\text{strike}}\right) - \frac{\sigma^2 t}{2}}{\sigma \sqrt{t}}$$

t = the time to expiry of the option expressed as a proportion of a year (365 days)

σ = the annualized volatility

r = the continuously compounded interest rate

$N(d)$ = the standardized normal cumulative probability distribution

Calculation summary

One difficulty with the Black-Scholes formula is that the normal distribution function cannot be calculated precisely as a formula itself. Tables giving values of the function are, however, widely available. Alternatively, it can be very closely approximated – although the approximations are rather messy. One such approximation is:

$$N(d) = 1 - \frac{\frac{0.4361836}{1 + 0.33267d} - \frac{0.1201676}{(1 + 0.33267d)^2} + \frac{0.937298}{(1 + 0.33267d)^3}}{\sqrt{2\pi} e^{\frac{d^2}{2}}} \quad \text{when } d \geq 0$$

$$\text{and } N(d) = 1 - N(-d) \quad \text{when } d < 0$$

In the case of a currency option, the Black-Scholes formula can be rewritten slightly.

**Calculation
summary****Black-Scholes formula for currency option**

$$\text{Call premium} = (\text{forward outright price} \times N(d_1) - \text{strike price} \times N(d_2)) \times e^{-rt}$$

$$\begin{aligned} \text{Put premium} &= (-\text{forward outright price} \times N(-d_1) + \text{strike price} \\ &\quad \times N(-d_2)) \times e^{-rt} \\ &= \text{call premium} + (\text{strike price} - \text{forward price}) \times e^{-rt} \end{aligned}$$

where: the option is a call on a unit of the base currency (that is, a put on the variable currency) and the premium is expressed in units of the variable currency.

$$d_1 = \frac{\text{LN}\left(\frac{\text{forward}}{\text{strike}}\right) + \frac{\sigma^2 t}{2}}{\sigma \sqrt{t}}$$

$$d_2 = \frac{\text{LN}\left(\frac{\text{forward}}{\text{strike}}\right) - \frac{\sigma^2 t}{2}}{\sigma \sqrt{t}}$$

r = the continuously compounded interest rate for the variable currency.

The expressions e^{rt} and e^{-rt} in the formulas above can be replaced by $(1 + i \times t)$ and $\frac{1}{(1 + i \times t)}$ respectively, where i is the simple interest rate for the period rather than the continuously compounded rate.

Example 9.3

What is the cost (expressed as a percentage of the USD amount) of a 91-day FRF call against USD at a strike of 5.60? The spot rate is 5.75, the forward outright is 5.70, volatility is 9.0%, and the FRF 3-month interest rate is 5.0%.

We are calculating the price of a USD put. Using the Black-Scholes formula, with the same notation as above:

$$r = \frac{365}{91} \times \text{LN}\left(1 + 0.05 \times \frac{91}{360}\right) = 0.0504$$

$$d_1 = \frac{\text{LN}\left(\frac{5.70}{5.60}\right) + \frac{0.09^2 \times \frac{91}{365}}{2}}{0.09 \sqrt{\frac{91}{365}}} = 0.4163$$

$$d_2 = \frac{\text{LN}\left(\frac{5.70}{5.60}\right) - \frac{0.09^2 \times \frac{91}{365}}{2}}{0.09 \sqrt{\frac{91}{365}}} = 0.3714$$

$$\begin{aligned} \text{put premium} &= (-5.70 \times N(-0.4163) + 5.60 \times N(-0.3714)) \times e^{-0.0504 \times \frac{91}{365}} \\ &= (-5.70 \times 0.3386 + 5.60 \times 0.3552) \times 0.9875 \\ &= 0.0584 \end{aligned}$$

The cost is therefore 0.0584 FRF for each one dollar underlying the option. As a percentage of the USD amount, this is:

$$\frac{0.0584}{5.75} = 1.02\%$$

Binomial trees

One way of building a model for option pricing is to simplify the assumptions to the possibility that the price of something may move up a certain extent or down a certain extent in a particular time period. Suppose, for example, that the price of a particular asset is now 1, and that the price may rise by a multiplicative factor of 1.010000 or fall by a factor of $\frac{1.0000}{1.0100} = 0.990099$ each month. After two months, the price may have moved along any of the following four routes, with three possible outcomes:

from 1 to 1.010000 and then to $(1.010000 \times 1.010000) = 1.020100$

from 1 to 1.010000 and then to $(1.010000 \times 0.990099) = 1.000000$

from 1 to 0.990099 and then to $(0.990099 \times 1.010000) = 1.000000$

from 1 to 0.990099 and then to $(0.990099 \times 0.990099) = 0.980296$

After three months, there are eight possible routes (up, up, up, or up, up, down, or up, down, up, etc.) and four possible outcomes. A lattice of possible paths for the price, known as a “binomial tree,” can be built up in this way. Depending on the relative probabilities of an up movement or a down movement, we can calculate the expected outcome at the end. From this, we can assess the expected value of an option to buy or sell the asset at the end, at any given strike price. The first step therefore is to find the probabilities of up and down movements.

We can in fact calculate these probabilities if we know the rate of interest which would be earned on a cash deposit – the “risk-free” rate of return. We do this by equating the expected outcome of owning the asset to the known outcome of the deposit, as follows. See below (“Risk free portfolio”) for a justification for equating these two outcomes for this purpose.

Suppose that the probability of a move up in the asset price after one month is p , and that the probability of a move down is $(1 - p)$. Suppose also that the current 1-month interest rate is 6 percent per annum.

Although we do not know what the asset price will be after one month, the expected price can be expressed as:

$$p \times 1.01 + (1 - p) \times 0.990099 = 0.990099 + 0.019901 \times p$$

As the interest rate is 6 percent, if we were to invest 1 in a risk-free deposit for one month, we would expect after one month to receive a total of:

$$1 + 6\% \times \frac{1}{12} = 1.005$$

If this outcome is the same as investing in the asset, we have:

$$0.990099 + 0.019901 \times p = 1.005$$

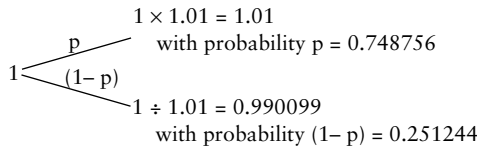
This gives: $p = \frac{1.005 - 0.990099}{0.019901} = 0.748756$

We can thus calculate what is the probability of a move up or down, implied by the current expected rate of return (the interest rate) and the size of the possible movements up and down.

Now suppose that, based on this model, we wish to value a one-month call option on 1 unit of the asset with a strike of 1.004.

There is a probability $p = 0.748756$ that the price will end at 1.01, in which case the option will be worth $1.01 - 1.004 = 0.006$. There is a proba-

bility $(1 - p) = 0.251244$ that the price will move down, in which case the option will expire worthless.



The expected value of the option at maturity is therefore:

$$(0.748756 \times 0.006) + (0.251244 \times 0) = 0.004493$$

The present value of the option is therefore $\frac{0.004493}{(1 + 0.06 \times \frac{1}{12})} = 0.004470$

The price of the option is therefore 0.45%.

Example 9.4

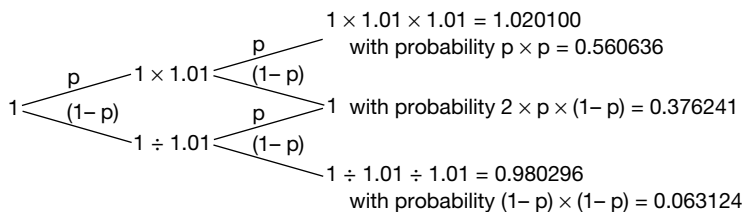
With the same details as above, what is the value of a 2-month call option and a 3-month call option with the same strike of 1.004?

2 months

There is a probability of $p \times p = (0.748756)^2 = 0.560636$ that the price will be $1.01 \times 1.01 = 1.020100$ at maturity, in which case the option will be worth $1.020100 - 1.004 = 0.0161$.

There are two routes along the tree to reach a price of 1 again at the end of two months – either up first then down again, or down first then up again. Therefore there is a probability of $2 \times p \times (1 - p) = 2 \times 0.748756 \times 0.251244 = 0.376241$ that the price will be $1.01 \times 0.990099 = 1$ at maturity, in which case the option will expire worthless.

There is a probability of $(1 - p)^2 = (0.251244)^2 = 0.063124$ that the price will be $0.990099 \times 0.990099 = 0.980296$ at maturity, in which case the option will expire worthless.

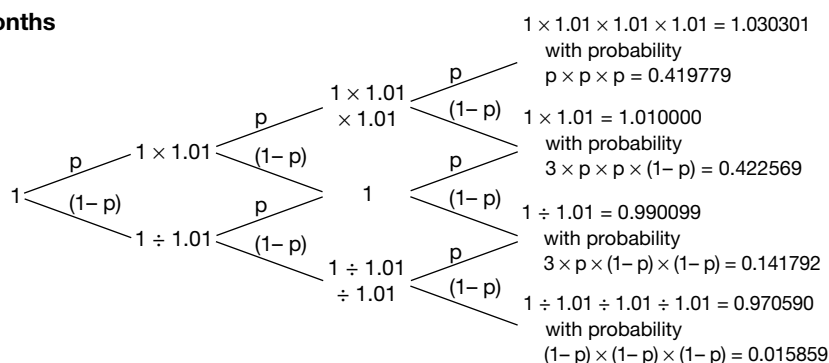


The expected value of this option at maturity is therefore:

$$(0.560636 \times 0.0161) + (0.376241 \times 0) + (0.063124 \times 0) = 0.009026$$

The present value of the option is therefore $\frac{0.009026}{(1 + 0.06 \times \frac{1}{12})^2} = 0.008937$

0.89% is therefore the price of this 2-month option.

3 months

From the tree, we can see that the expected value of the option at maturity is:

$$(0.419779 \times [1.030301 - 1.004]) + (0.422569 \times [1.01 - 1.004]) \\ + (0.141792 \times 0) + (0.015859 \times 0) = 0.013576$$

The option price is therefore the present value of this expected outcome:

$$\frac{0.013576}{\left(1 + 0.06 \times \frac{1}{12}\right)^3} = 1.34\%$$

Risk-free portfolio

It may seem unreasonable, when calculating the probabilities of up and down movements, to equate the expected outcome of investing in the asset with the outcome of investing at a risk-free interest rate. It is possible to justify this, however, by constructing an investment package which does have a known outcome, as follows.

In the example above for a one-month option, we could sell a call option on one unit of the asset for a premium of 0.004470. Suppose that at the same time we buy 0.301492 units of the asset at the current price of 1. The net cost of this investment package would be:

$$0.301492 - 0.004470 = 0.297022$$

If the price is 1.01 at the end of the month, the option would be exercised against us, so that we would make a loss on the option of $1.01 - 1.004 = 0.006$. The price of the asset we purchased would also have risen to 1.01, however, so that the total value of our investment at the end of the month would have become:

$$0.301492 \times 1.01 - 0.006 = 0.298507$$

If the price is 0.990099 at the end of the month, however, the option would not be exercised against us. The value of the asset we purchased would have fallen to 0.990099, so that the total value of our investment would have become:

$$0.301492 \times 0.990099 = 0.298507$$

In this way we have constructed a package – consisting of a short position in one call option and a long position in 0.301492 units of the asset – which has a known outcome equal to 0.298507 whether the price rises or falls. This outcome represents an annual return on our investment, as expected for a risk-free investment, of:

$$\left(\frac{0.298507}{0.297022} - 1 \right) \times \frac{12}{1} = 6\%$$

We now need to consider how to construct such a package in the first place. We need to calculate how much of the asset should be purchased initially in order to make it risk-free. If the amount of asset purchased is A , the two possible outcomes are:

$$\begin{aligned} & A \times 1.01 - 0.006 \quad (\text{if the price rises}) \\ \text{or} \quad & A \times 0.990099 \quad (\text{if the price falls}) \end{aligned}$$

If there is to be no risk, these two outcomes must be equal, so that:

$$A \times 1.01 - 0.006 = A \times 0.990099$$

$$\text{Therefore: } A = \frac{0.006}{1.01 - 0.990099} = 0.301492$$

and the final outcome is therefore:

$$0.301492 \times 0.990099 = 0.298507$$

If the price of the call option on one unit of the asset is C , we now know that our initial investment in this package must be $(0.301492 - C)$. If the rate of return is 6 percent, we know therefore that:

$$(0.301492 - C) \times \left(1 + 0.06 \times \frac{1}{12} \right) = 0.298507$$

This gives the cost of the option, as before, as:

$$0.301492 - \frac{0.298507}{\left(1 + 0.06 \times \frac{1}{12} \right)} = 0.004470$$

Comparison with Black-Scholes

It can be shown that for a given period to the option expiry, if we build a binomial tree which has more and more branches, each of which is shorter and shorter, the result eventually becomes the same as the result of the Black-Scholes formula – as long as the possible up and down price movements are chosen appropriately.

In the example above, what is the volatility of the asset price? We have two possible values for $\text{LN}(\text{relative price movements})$:

$$\begin{aligned} & \text{LN}(1.01) = 0.00995 \text{ with probability } 0.748756 \\ \text{and} \quad & \text{LN}(0.990099) = -0.00995 \text{ with probability } 0.251244 \end{aligned}$$

The mean of these values is:

$$0.00995 \times 0.748756 - 0.00995 \times 0.251244 = 0.00495$$

The variance is therefore:

$$(0.00995 - 0.00495)^2 \times 0.748756 + (-0.00995 - 0.00495)^2 \times 0.251244 = 0.000074$$

The standard deviation is therefore:

$$\sqrt{0.000074} = 0.008631$$

As the data are monthly, the volatility is:

$$0.008631 \times \sqrt{12} = 2.98\%$$

If we now value the three-month option using the Black–Scholes formula, we have:

$$\text{call premium} = 1 \times N(d_1) - \frac{1.004 \times N(d_2)}{\left(1 + 0.06 \times \frac{1}{12}\right)^3}$$

where:

$$d_1 = \frac{\text{LN}\left(\frac{(1 + 0.06 \times \frac{1}{12})^3}{1.004}\right) + \frac{(0.0298)^2 \times \frac{3}{12}}{2}}{0.0298 \times \sqrt{\frac{3}{12}}}$$

$$d_2 = \frac{\text{LN}\left(\frac{(1 + 0.06 \times \frac{1}{12})^3}{1.004}\right) - \frac{(0.0298)^2 \times \frac{3}{12}}{2}}{0.0298 \times \sqrt{\frac{3}{12}}}$$

This gives:

$$\begin{aligned} \text{call premium} &= N(0.7437) - 0.9891 \times N(0.7288) \\ &= 0.7715 - 0.9891 \times 0.7669 = 0.0130 = 1.30\% \end{aligned}$$

This option price of 1.30 percent calculated using the Black–Scholes formula is close to the binomial model's result of 1.34 percent.

In the example above, we assume that we know in advance the possible up and down price movements. In practice, when using a binomial tree to estimate an option price, we need to choose these possible price movements in such a way as to arrive at a result similar to the Black–Scholes model.

In order for the answers from the two models to converge as the number of branches increases, the possible binomial up and down movements in the price must be chosen to suit the parameters assumed by the Black–Scholes model. One possible way of doing this is to choose them as follows:

$$u = e^{\sigma \sqrt{\frac{t}{n}}}$$

$$d = \frac{1}{u}$$

$$p = \frac{(1 + i \times t)^{\frac{1}{n}} - d}{u - d}$$

where: t = time to expiry expressed in years
 n = number of periods in the binomial tree
 σ = volatility
 u = multiplicative up movement
 d = multiplicative down movement
 i = interest rate per annum to expiry

Put / call relationship, synthetic forwards and risk reversal

Suppose that I pay a premium of C to buy a European call option with a strike price of K for an asset which pays no dividends. Suppose that at the same time I receive a premium P to sell a put option on the same asset, also with a strike of K . Third, suppose that I also sell the asset for forward delivery at the current forward price F . If the asset price is above K at expiry, I will exercise my call option at K . If it is below K at maturity, my counterparty will exercise the put option which I have sold to him. Either way, I buy the asset at a price K . However, I also sell the asset at price F , because of the forward deal. I therefore have a profit $(F - K)$. On the basis that “free profits” are not available, this must offset my net payment $(C - P)$. However, $(F - K)$ is received / paid at maturity while $(C - P)$ is paid / received up-front. Therefore:

Calculation summary

Call premium – put premium =
 (forward price – strike price) discounted to a present value

Expressed in terms of the spot price rather than the forward price, this is:

Calculation summary

Call premium – put premium =
 spot price – (strike price discounted to a present value)

If the strike price is set equal to the forward price, $(C - P)$ must be zero. Therefore, with an option struck at the forward price (at-the-money), the put and call premiums are equal. This is the “put / call parity”. This relationship explains the formulas given for the put premium in the section on Black–Scholes.

This relationship is also important because it is related to the creation of synthetic positions. From the above analysis, it can be seen that for any strike price K , it is true that:

sell forward *plus* buy call *plus* sell put = 0

This is the same as saying:

buy forward = buy call *plus* sell put
 or
 sell forward = sell call *plus* buy put

These two relationships show that a synthetic forward deal can be created from two option deals.

Synthetic forwards

Buying a call and selling a put at the same strike creates a synthetic forward purchase – and vice versa

Key Point

The relationship can also be expressed as follows:

buy call = buy put *plus* buy forward
 sell call = sell put *plus* sell forward
 buy put = buy call *plus* sell forward
 sell put = sell call *plus* buy forward

Thus, for example, a trader can either buy a call at a particular strike or, if priced more efficiently, he can buy a put at the same strike and buy forward simultaneously. Viewed from a different standpoint, this is known as “risk reversal.” If, for example, a trader already has a position where he is long of a put, he can reverse this position to become effectively long of a call instead, by buying forward.

Risk reversal

A long or short position in a call can be reversed to the same position in a put by selling or buying forward – and vice versa

Key Point

OTC OPTIONS vs. EXCHANGE-TRADED OPTIONS

The difference between OTC options and those traded on an exchange is parallel to the difference between OTC instruments such as forwards and exchange-traded futures. In the case of an exchange-traded option, the underlying “commodity” may be the corresponding futures contract. Thus on LIFFE, if an interest-rate option is exercised, the option buyer receives a LIFFE interest rate futures contract. On the Philadelphia Currency Options Exchange, on the other hand, most currency options are deliverable into a cash currency exchange. Exchange-traded options may be either European or American. On LIFFE, for example, most options are American, although there is also a European option on the FTSE 100 index. On the IMM, options are American; on the Philadelphia Exchange, there is a range of both American and European currency options.

One significant difference which can arise between OTC and exchange-traded options concerns the premium payment. On the IMM and other

exchanges the option buyer pays a premium up-front as with an OTC option, but pays no variation margin. On LIFFE, however, the premium is effectively paid via the variation margin. The variation margin paid or received each day represents the change in value of the option. If, for example, an option expires worthless at maturity, the variation margin payments made over the life of the option total the change in value from the original premium to zero. There is therefore no premium payable at the beginning.

On LIFFE the exercise of a call (or put) option on a futures position simply causes a long (or short) futures position to be assigned to the option purchaser and a corresponding short (or long) position to be assigned to the option writer. Positions are assigned at the option exercise price and then marked-to-market in the usual way.

Once an option has been exercised, its price is effectively zero. Therefore, the difference between its market price at the time of exercise and zero is paid as settlement margin. Thus the full option premium will then have been paid partly as variation margin during the life of the option and partly as settlement margin.

Strike prices for OTC options can be set at any level agreed between the two parties – generally at the buyer's request. Exchanges, however, set a series of strikes, spaced at regular intervals, which are extended as the underlying price moves up and down.

The tick values on exchange-traded options are not always straightforward. US T-bonds and T-bond futures, for example, are priced in multiples of $\frac{1}{32}$. Options on T-bonds, however, are priced in multiples of $\frac{1}{64}$.

THE GREEK LETTERS

In principle, an option writer could sell options without hedging his position. If the premiums received accurately reflect the expected payouts at expiry, there is theoretically no profit or loss on average. This is analogous to an insurance company not reinsuring its business. In practice, however, the risk that any one option may move sharply in-the-money makes this too dangerous. In order to manage a portfolio of options, therefore, the option dealer must know how the value of the options he has sold and bought will vary with changes in the various factors affecting their price, so that he can hedge the options.

Delta

An option's delta (Δ) measures how the option's value (which is the same as its current premium) varies with changes in the underlying price:

$$\Delta = \frac{\text{change in option's value}}{\text{change in underlying's value}}$$

Mathematically, the delta is the partial derivative of the option premium with respect to the underlying, $\frac{\partial C}{\partial S}$ or $\frac{\partial P}{\partial S}$ (where C is the call premium, P is

the put premium and S is the price of the underlying). Based on the Black–Scholes formula given earlier, and with the same notation, the delta can be shown to be $N(d_1)$ for a call and $-N(-d_1)$ for a put. If an option has a delta of 0.7 (or 70%), for example, a \$100 increase in the value of the underlying will cause a \$70 increase in the value of the option.

For a call option which is deep out-of-the-money, the premium will increase very little as the underlying improves – essentially the option will remain worth almost zero. For an option deep in-the-money, an improvement in the underlying will be reflected completely in the call premium. The delta is therefore close to zero for deep out-of-the-money call options, 0.5 at-the-money, and close to 1 for deep in-the-money call options. For put options, delta is close to zero deep out-of-the-money, -0.5 at-the-money, and close to -1 deep in-the-money.

Consider, for example, a call option on an asset with a strike price of 99. When the current price is 99, the option will have a certain premium value C . If the current price rises to 99.5, the option will have a higher value because it could be exercised for a 0.5 profit if the current price remains at 99.5. However, there is still a probability of approximately 50 percent that the price will fall and the option will expire worthless. The premium increase is therefore only approximately 50 percent of the underlying increase.

When an option trader wishes to hedge an option he has written, he has several choices:

- Buy an exactly matching option.
- Buy or sell the underlying. In this case, the trader will buy or sell enough of the underlying so that if the price changes he will make a profit or loss which exactly offsets the loss or profit on the option position. In the example above, he would buy the underlying asset to the extent of 50 percent of the option amount. In this way, if the price rises from 99 to, say, 100, he will make a profit of 1 on half the amount of the option. This would offset a loss on the option position of 0.5 on the whole amount. In general, the amount of the hedge is equal to:

delta \times the notional amount of the option.

This is known as “delta hedging” and demonstrates the importance of knowing the delta.

- Buy or sell another instrument with the same (but opposite) value for (delta \times notional amount), so that again any change in the underlying price gives rise to a change in the hedge value which exactly offsets the change in the option value. In the example above, such a hedge might be the purchase of an option with a different strike price – say, a larger amount of an option on the same asset which is slightly out-of-the-money (and hence has a smaller delta).

If a trader is short of a call (as in this example) or long of a put, he has a negative delta and needs to buy the underlying in order to hedge. If he is long of a call or short of a put, he has a positive delta and needs to sell the underlying in order to hedge.

Key Point

Delta hedging an option can be achieved by buying or selling the correct amount of the underlying so that any change in the option's P & L is offset by a change in the underlying position's P & L

Gamma

One problem with delta hedging an option or portfolio of options is that the delta itself changes as the underlying price changes, so that although a portfolio may be hedged, or “delta neutral” at one moment, it may not be so the next moment. An option's gamma (Γ) measures how much the delta changes with changes in the underlying price:

$$\Gamma = \frac{\text{change in delta}}{\text{change in price}}$$

Mathematically, this is the second partial derivative of the premium with respect to the underlying price, $\frac{\partial^2 C}{\partial S^2}$ or $\frac{\partial^2 P}{\partial S^2}$. Based on the Black-Scholes formula, Γ can be shown to be $\frac{1}{S\sigma\sqrt{2\pi t}e^{\frac{d_1^2}{2}}}$ for a call or a put.

As already discussed, the delta does not change rapidly when an option is deep out-of-the-money (the delta remains close to zero) or when an option is deep in-the-money (the delta remains close to 1 or -1), so that gamma is very small. When an option is close to the money, however, the delta changes rapidly, and the gamma of a call is at its greatest slightly out-of-the-money. Gamma is positive for long option positions, both calls and puts, and negative for short calls and short puts.

Ideally, a trader who wishes to be fully hedged would like to be gamma-neutral – that is, to have a portfolio of options where the delta does not change at all.

Suppose, for example, that an option portfolio is currently delta-neutral, but has a portfolio gamma (= gamma \times portfolio size) of -60. A particular option which could be used to hedge this has a delta of 0.5 and a gamma of 0.6. The gamma of the portfolio could be reduced to zero by adding a long position of $\frac{-60}{0.6} = 100$ units of the option. However, the delta of the portfolio would now be $100 \times 0.5 = 50$. It is therefore necessary to superimpose on this, for example, a further hedge of a short position of 50 in the underlying – to reduce the delta back to zero. This will not affect the portfolio's gamma, because the underlying has a delta of 1 but a gamma of zero. The portfolio would still need to be hedged dynamically – because the gamma and the delta will change as the underlying moves – but it would be less vulnerable.

Vega

An option's vega (or epsilon (ϵ), eta (η), lambda (λ) or kappa (κ)) measures how much an option's value changes with changes in the volatility of the underlying:

$$\text{vega} = \frac{\text{change in option's value}}{\text{change in volatility}}$$

Mathematically, this is the partial derivative of the option premium with respect to volatility, $\frac{\partial C}{\partial \sigma}$ or $\frac{\partial P}{\partial \sigma}$. Based on the Black-Scholes formula, this is $\frac{S\sqrt{\frac{t}{2\pi}}}{e^{\frac{d_1^2}{2}}}$ for a call or a put.

Vega is at its highest when an option is at-the-money and falls as the market and strike prices diverge. Options closer to expiration have a lower vega than those with more time to run. Positions with positive vega will generally have positive gamma. To be long vega (to have a positive vega) is achieved by purchasing either put or call options.

Theta

An option's theta (Θ) measures how much an option's value changes with changes in the time to maturity:

$$\text{theta } (\Theta) = - \frac{\text{change in option's value}}{\text{change in time}}$$

Mathematically, this is $-\frac{\partial C}{\partial t}$ or $-\frac{\partial P}{\partial t}$. Based on the Black-Scholes formula, this is

$$-\frac{S\sigma}{2\sqrt{2\pi t}} e^{-\frac{d_1^2}{2}} - Kre^{-rt}N(d_2) \text{ for a call or } -\frac{S\sigma}{2\sqrt{2\pi t}} e^{-\frac{d_1^2}{2}} + Kre^{-rt}N(-d_2) \text{ for a}$$

put, where K is the strike price.

Theta is negative for a long option position and positive for a short option position. The more the market and strike prices diverge, the less effect time has on an option's price and hence the smaller the theta. Positive theta is generally associated with negative gamma and vice versa.

Rho

An option's rho (ρ) measures how much an option's value changes with changes in interest rates:

$$\text{rho } (\rho) = \frac{\text{change in option's value}}{\text{change in interest rate}}$$

Mathematically, this is $\frac{\partial C}{\partial r}$ or $\frac{\partial P}{\partial r}$. Based on the Black-Scholes formula, this is $Kte^{-rt}N(d_2)$ for a call or $-Kte^{-rt}N(-d_2)$ for a put.

Rho tends to increase with maturity.

**Calculation
summary****Based on the Black-Scholes formula**

Delta (Δ) = $N(d_1)$ for a call
or $-N(-d_1)$ for a put

$$\text{Gamma } (\Gamma) = \frac{1}{S\sigma\sqrt{2\pi t} e^{\frac{d_1^2}{2}}}$$

$$\text{Vega} = \frac{S\sqrt{\frac{t}{2\pi}}}{e^{\frac{d_1^2}{2}}}$$

$$\text{Theta } (\Theta) = -\frac{S\sigma}{2\sqrt{2\pi t}} e^{-\frac{d_1^2}{2}} - Kre^{-rt}N(d_2) \text{ for a call}$$

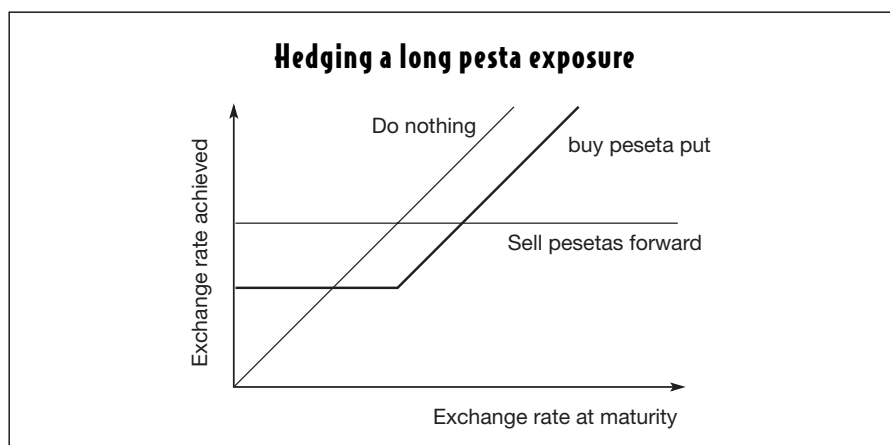
$$\text{or } -\frac{S\sigma}{2\sqrt{2\pi t}} e^{-\frac{d_1^2}{2}} + Kre^{-rt}N(-d_2) \text{ for a put}$$

Rho (ρ) = $Kte^{-rt}N(d_2)$ for a call
or $-Kte^{-rt}N(-d_2)$ for a put

where: S = spot price
 K = strike price

HEDGING WITH OPTIONS**Comparison with forwards**

A company with a long peseta position which it wishes to hedge against guilders has three basic choices. It can do nothing and remain unhedged, it can sell the pesetas forward, or it can buy a peseta put option. In general, the option will never provide the best outcome because of the premium cost: if the peseta falls, the company would be better selling forward; if the peseta rises, the company would be better remaining unhedged. On the other hand, the option provides the safest overall result because it protects against a peseta fall while preserving opportunity gain if the peseta rises. Essentially, if the company firmly believes the peseta will fall, it should sell forward; if the company firmly believes the peseta will rise, it should do nothing; if the company believes the peseta will rise but cannot afford to be wrong, it should buy a peseta put option. The outcomes of the three possibilities are as follows. Figure 9.3 shows the effective net outcome (in terms of the exchange rate achieved net of the option premium cost) for a hedger with an underlying position. It is important to note that, unlike the subsequent figures in this chapter, it is *not* a profit / loss profile on a “naked” position with no underlying exposure.

Fig 9.3

Interest rate guarantees

A parallel situation arises in interest rate hedging. A company with a short-term borrowing rollover in the future also has three basic choices. It can remain unhedged, buy an FRA or buy or sell an option on an FRA (an “interest rate guarantee” or IRG). A currency option is an option to buy an agreed amount of one currency against another currency at an agreed exchange rate on an agreed date. An IRG is the interest rate equivalent of this.

An IRG is effectively an option to buy or sell an agreed amount of an FRA in one particular currency at an agreed rate for an agreed maturity and on an agreed delivery date. An interest rate guarantee can therefore fix the maximum cost on a future borrowing (or the minimum return on a future deposit). The IRG does not entail an actual borrowing or deposit. Like a futures contract, it is a “contract for differences”: the difference between the strike rate and the actual rate at settlement is paid or received.

An IRG can be either a “borrower’s option” or a “lender’s option.” This terminology may be safer than thinking in terms of “call” or “put” because a call on an FRA is equivalent to a put on an interest rate futures contract.

Caps and floors

A cap or ceiling is a series of IRGs (borrower’s options) generally at the same strike, purchased together to secure a series of borrowing rollovers. Suppose, for example, that a borrower has a 5-year loan which he rolls over each 3 months at the 3-month LIBOR then current. He can buy a 5-year cap which will put a maximum cost on each of the rollovers. Whenever the rollover rate exceeds the cap strike rate, he receives the difference. Whenever the cap strike rate exceeds the rollover rate, nothing is paid or received.

A floor is similarly a series of IRGs (lender’s options) purchased to secure a series of deposits, by putting a minimum return on each rollover.

SOME “PACKAGED” OPTIONS

Various OTC option-based products are offered by banks to their customers, some of which can be constructed from straightforward options. The common products available are as follows.

Range forward (or collar, cylinder, tunnel or corridor)

A straightforward option provides a fixed worst-case level at which the customer can deal, but allows him to deal at the market rate if this turns out better. A range forward allows the customer to deal at a better market rate only up to a certain level. Beyond that level, the customer must deal at another fixed best-case level. In return for this reduced opportunity, the customer pays a lower premium for the option. Indeed the premium can be zero (a zero-cost option) or even negative.

Such an arrangement can be constructed by buying one option (for example, a USD put option) and selling another (in this case a USD call option). The premium earned from the second option offsets the premium paid on the first option, either partly or (in the case of a zero-cost option) completely. The obligation to deal at the strike rate of the second option, if the counterparty wishes, determines the best-case level. Setting the two strike rates determines the net premium. Alternatively, setting the size of the net premium and one of the strike rates determines the other strike rate.

There is generally a technical difference between a range forward and a collar. With a range forward, the customer is usually obliged to deal with the bank. If neither of the range limits is reached (i.e. neither option is exercised) at expiry, the customer must deal at the spot rate with the bank. With a collar, the customer is not obliged to deal with any particular bank if neither option is exercised. The term “range forward” usually applies to a deal with a single future date, but in general a collar can be applied to any underlying instrument. It is often used, for example, to describe the simultaneous purchase of a cap and sale of a floor.

Break forward (or forward with optional exit)

A break forward is a forward deal at a fixed rate (the worst-case level) with another “break” level at which the customer may reverse the forward deal if he/she chooses. For example, a break forward to sell USD at 1.52 with a break at 1.55 obliges the customer to sell USD at 1.52 but allows him/her to buy USD back at 1.55 if he/she chooses. If the USD strengthens to 1.61, for example, the customer may buy the USD back again at 1.55 and sell them in the market at 1.61 – an all-in effective rate of 1.58 ($= 1.52 - 1.55 + 1.61$).

A break forward has exactly the same profit / loss profile as a straightforward option, because it is in fact a straightforward option with deferred payment of the premium. The fixed rate and break rates are set as follows. Suppose that the forward rate for selling USD against DEM is currently 1.56

and the customer asks for a fixed rate of 1.52 as above. The bank calculates what strike rate would be necessary on a USD put option so that the strike rate less the future value of the premium is 1.52. This strike rate then becomes the break rate.

Calculating the necessary strike rate can be done conveniently using the put / call parity. As before, if C is the call premium and P is the put premium:

$$C - P = (\text{forward} - \text{strike}) \text{ discounted to a present value}$$

or

$$\text{strike} - P^* = \text{forward} - C^* \text{ where } P^* \text{ and } C^* \text{ are the future values of } P \text{ and } C$$

If the fixed rate set in the transaction is 1.52, this must represent the true strike rate adjusted for the deferred put premium. In other words:

$$\text{strike} - P^* = 1.52$$

From the put / call relationship, it follows that:

$$\text{forward} - C^* = 1.52$$

Since the forward rate is 1.56, it follows that:

$$C^* = 1.56 - 1.52 = 0.04$$

The call premium is therefore the present value of 0.04. This then determines the true strike rate behind the deal, which is used as the break level.

Participation forward

A participation forward provides a worst-case level in the same way as an option. If the customer wishes not to deal at this level because the market level is better, he/she shares the benefit with the bank. For example, a participation forward to sell USD at 1.50 with a participation rate of 70 percent provides a worst-case level of 1.50. If the market rate is 1.60, the customer receives 70 percent of the 0.10 benefit – i.e. 0.07 – and sells USD at 1.57. There is no premium to pay for a participation forward, but there is a potential 30 percent loss of advantage compared with a straight-forward option.

A participation forward can be constructed by buying an option (in this case a USD put option) and selling a smaller amount of an opposite option (in this case a USD call option) at the same strike rate.

Setting the strike rates of the two options determines the two premiums. In order for the total net premium to be zero, this then sets the amount of the second option which must be sold, and hence the participation rate. Alternatively, setting the participation rate will determine the strike rate for the options which will result in a net zero premium.

SOME TRADING STRATEGIES

Calls and puts

The most basic trading strategies are the purchase or sale of a call or a put. The purchase of either gives rise to limited potential loss (the premium paid) and almost unlimited potential profit. The sale of either gives the reverse. Assuming that the price of the underlying can only fall as far as zero, the potential profit from holding a put (or the loss from writing a put) is not actually unlimited. The profit / loss profile can be illustrated as shown in Figures 9.4 – 9.7.

Fig 9.4

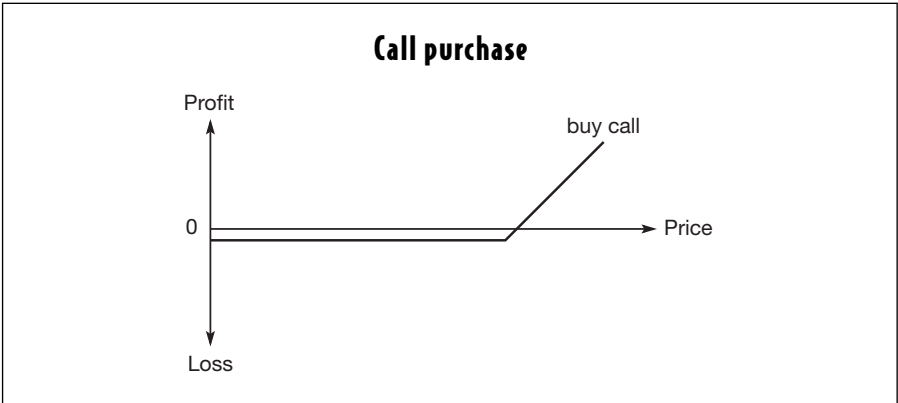
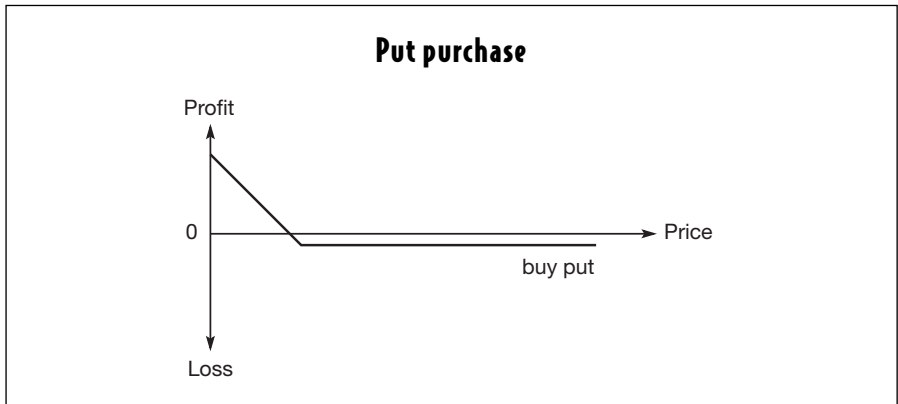
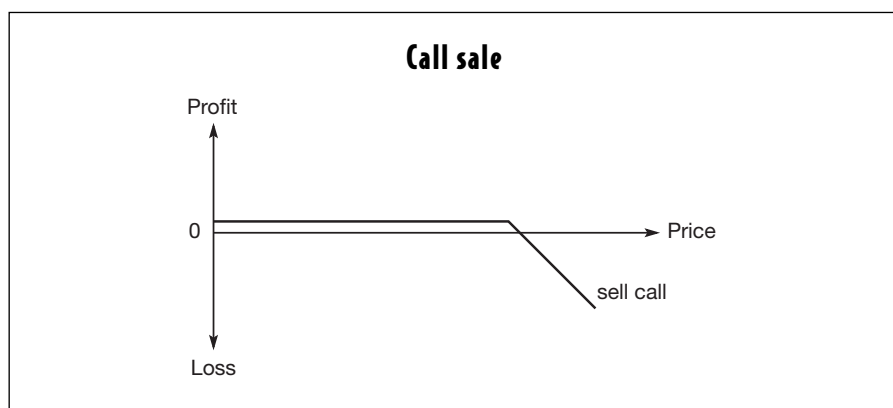
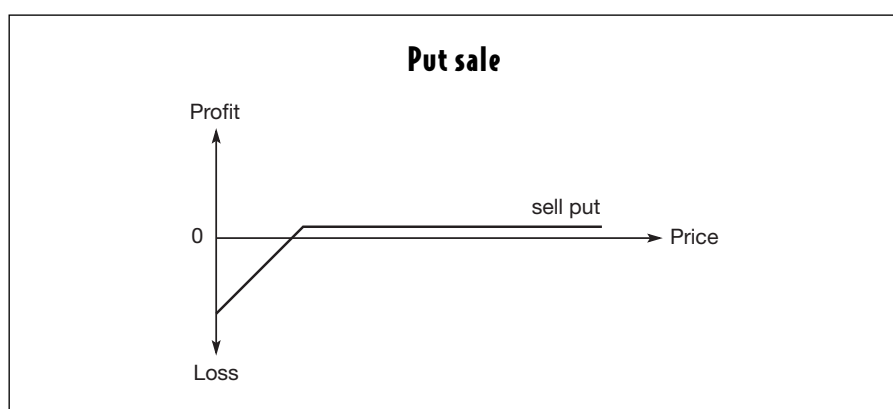


Fig 9.5



**Fig 9.6****Fig 9.7**

Covered calls and puts

To sell a call or put as above without any underlying exposure is to write a “naked” option. A “covered” call or put arises when the writer has an offsetting position in the underlying – a long position in the underlying to offset the selling of a call, or a short position in the underlying to offset the selling of a put.

Sale of a covered call can, for example, be used by a fund manager to increase income by receiving option premium. It would be used for a security he is willing to sell only if the underlying goes up sufficiently for the option to be exercised. Generally, covered call writers would undertake the strategy only if they thought implied volatility was too high. The lower the volatility, the less the covered call writer gains in return for giving up potential profit in the underlying. It provides protection against potential loss only to the extent that the option premium offsets a market downturn.

A covered put can be used by a fund manager who is holding cash because he has shorted securities, to increase income by receiving option premium. Covered put writing can also be used as a way of target buying: if an investor has a target price at which he wants to buy, he can set the strike price of the option at that level and receive option premium meanwhile to increase the

yield of the asset. Investors may also sell covered puts if markets have fallen rapidly but seem to have bottomed, because of the high volatility typically received on the option in such market conditions.

Fig 9.8

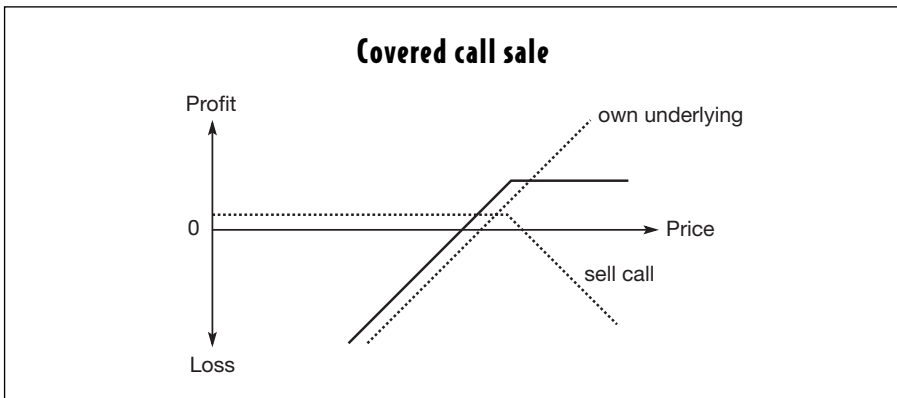
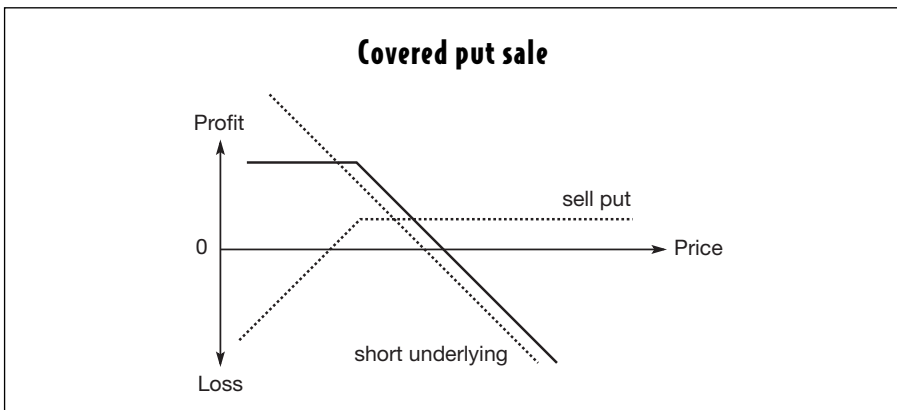
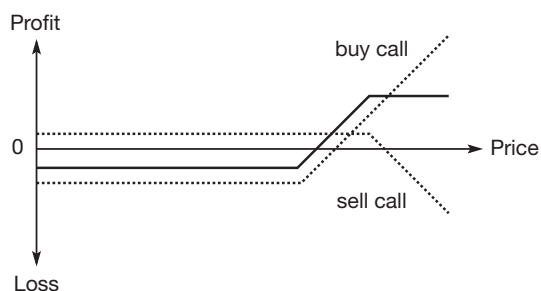
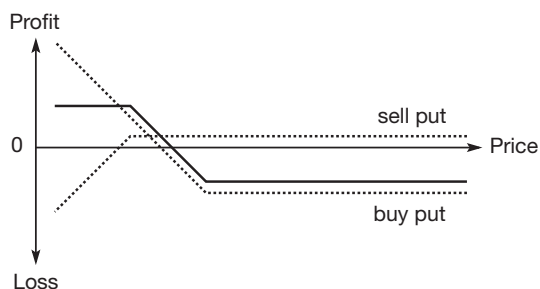
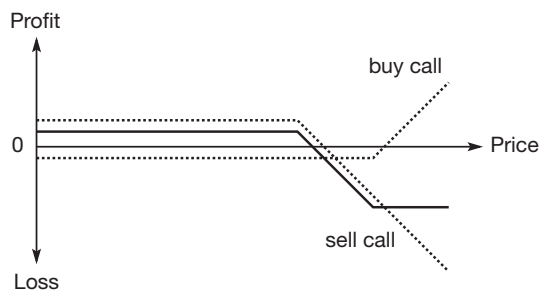


Fig 9.9



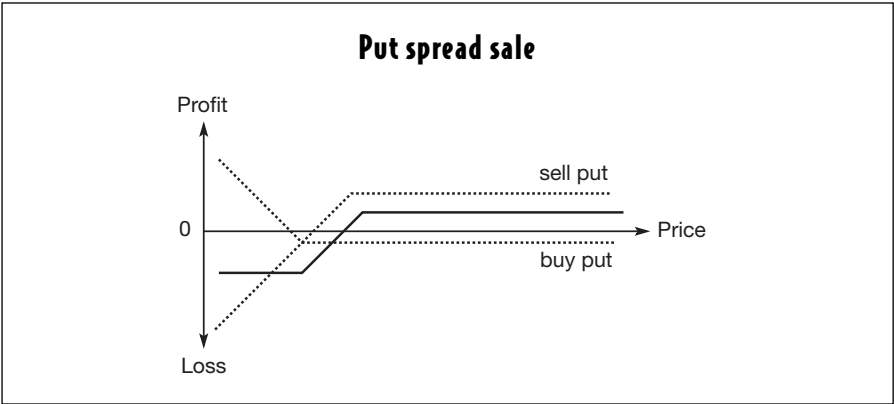
Spread

Spreads involve the simultaneous purchase and sale of two different calls, or of two different puts. A long call spread is the purchase of a call at one strike price, offset by the simultaneous sale of a call at another strike price less in-the-money (or more out-of-the-money) than the first. This limits the potential gain if the underlying goes up, but the premium received from selling the second call partly finances the purchase of the first call. A call spread may also be advantageous if the purchaser thinks there is only limited upside in the underlying. A put spread is similarly the simultaneous purchase of a put at one strike price, offset by the simultaneous sale of a put less in-the-money (or more out-of-the-money) than the first – used if the purchaser thinks there is limited downside for the underlying. To short a spread is the reverse. (See Figures 9.10–9.13.)

Call spread purchase**Fig 9.10****Put spread purchase****Fig 9.11****Call spread sale****Fig 9.12**

A bull spread is either the purchase of a call spread or the sale of a put spread. A bear spread is either the sale of a call spread or the purchase of a put spread.

Fig 9.13



A calendar spread is the purchase of a call (or put) and the simultaneous sale of a call (or put) with the same strike price but a different maturity. For example, if one-month volatility is high and one-year volatility low, a trader might buy one-year options and sell one-month options, thereby selling short-term volatility and buying long-term volatility. If short-term volatility falls relative to long-term volatility, the strategy can be reversed at a profit.

Spreads constructed from options with the same maturity but different strikes are sometimes known as “vertical” spreads, while calendar spreads are “horizontal” spreads. A spread constructed from both different strikes and different maturities is a “diagonal” spread.

Straddle

To go long of a straddle is to buy both a put and a call at the same strike price. In return for paying two premiums, the buyer benefits if the underlying moves far enough in either direction. It is a trade which expects increased volatility. The seller of a straddle assumes unlimited risk in both directions but receives a double premium and benefits if volatility is low. (See Figures 9.14 and 9.15.)

Fig 9.14

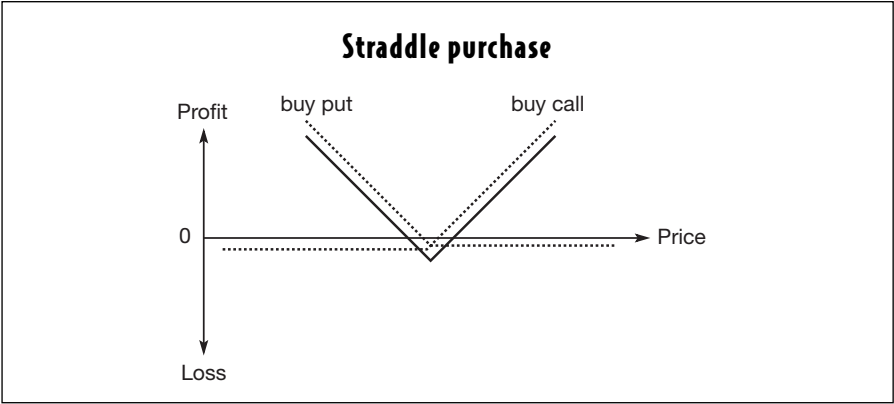
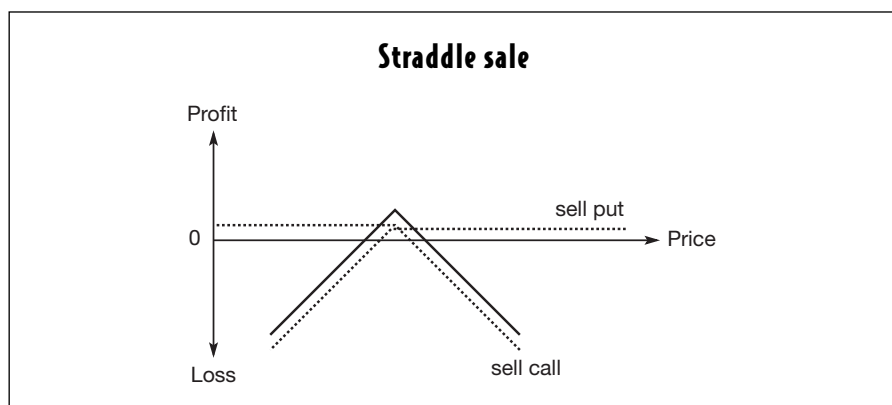
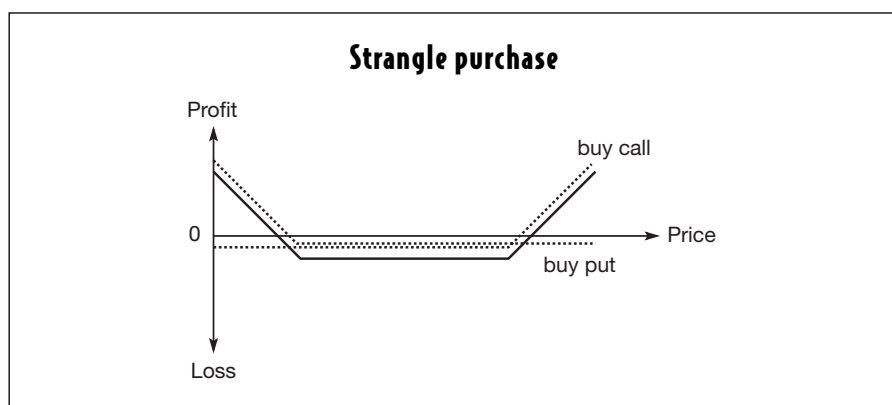
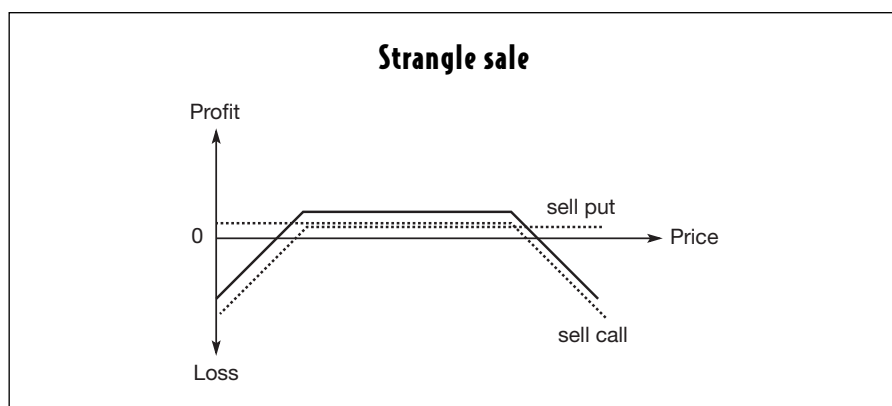


Fig 9.15

Strangle

A strangle is similar to a straddle but the premiums are reduced by setting the two strike prices apart – generally each strike will be out-of-the-money. Profits are only generated on a long strangle position if the underlying moves significantly. (See Figures 9.16 and 9.17)

Fig 9.16**Fig 9.17**

With combination strategies such as straddles and strangles, a “top” combination is one such as a straddle sale or a strangle sale which has a top limit to its profitability and a “bottom” combination is the reverse, such as a straddle purchase or a strangle purchase.

SOME LESS STRAIGHTFORWARD OPTIONS

We conclude, by way of illustration, with a selection of a few of the less straightforward option types available.

Average rate option (or Asian option)

This is a cash-settled option, paying the buyer the difference (if positive) between the strike and the average of the underlying over an agreed period. The volatility of the underlying's average is less than the volatility of the underlying itself, so that the option cost is reduced.

Average strike option

Also cheaper than a straightforward option, this sets the strike to be the average of the underlying over a period, which is then compared with the actual underlying rate at expiry.

Barrier option

A barrier option is one which is either activated (a “knock-in” option) or cancelled (a “knock-out” option) if the underlying reaches a certain trigger level during the option's life. For example, an “up-and-in” option becomes active if the underlying rate moves up to a certain agreed level; if that level is not reached, the option never becomes active, regardless of the strike rate. “Up-and-out”, “down-and-in” and “down-and-out” options are defined analogously. The circumstances in which the writer will be required to pay out on a barrier option are more restricted, so the option is cheaper.

Binary option (or digital option)

A binary option has an “all or nothing” profit / loss profile. If the option expires in-the-money, the buyer receives a fixed payout regardless of how far the underlying has moved beyond the strike rate. A “one touch” binary option pays out if the strike is reached at any time during the option's life.

Compound option

An option on an option.

Contingent option

This is an option where the buyer pays no premium unless the option expires in-the-money. If it does expire in-the-money, however, the buyer must then pay the premium. The cost is higher than for a straightforward option.

Quanto option (or guaranteed exchange rate option)

A quanto option is one where the underlying is denominated in one currency but payable in another at a fixed exchange rate.

Swaption

A swaption is an option on a swap. Exercising the option delivers the agreed swap from the time of exercise onwards.



EXERCISES

- 77.** What is the estimated annualized volatility of the USD/DEM exchange rate, based on the following daily data, assuming the usual lognormal probability distribution for relative price changes and 252 days in a year?

Day 1	1.6320
Day 2	1.6410
Day 3	1.6350
Day 4	1.6390
Day 5	1.6280
Day 6	1.6300
Day 7	1.6250
Day 8	1.6200
Day 9	1.6280
Day 10	1.6200

- 78.** A 6-month (182 days) FRF call option against USD at a strike of 5.6000 costs 1.5% of the USD amount. What should a FRF put cost at the same strike rate?

USD/FRF spot:	5.7550
USD/FRF 6-month outright:	5.7000
USD 6-month LIBOR:	7%
FRF 6-month LIBOR:	5%

- 79.** Construct a three-step binomial tree to calculate a price for a 3-month put option on an asset at a strike of 101. The current price is 100. At each step, the price either rises or falls by a factor of 2% (that is either multiplied by 1.02 or divided by 1.02). The risk-free interest rate is 12% per annum.

Practice ACL Exam, Hints and Answers

A Complete Practice ACI Exam



The following pages contain a full exam, laid out similarly to the Financial Calculations exam currently set as part of the ACI series of exams leading to fellowship of the ACI.

If you are serious about taking the ACI's exam, we recommend that you work through this practice version, and would like to make the following suggestions.

Familiarize yourself with the material throughout the book first and work through the examples and worked answers to the exercises.

Try to work through the practice exam under 'exam conditions.' The exam lasts 2½ hours and is in two parts:

- **Part 1: 30 minutes; around twelve short questions; 20 points available**
- **Part 2: 2 hours; four exercises each containing several questions; 80 points available**

Use of a Hewlett Packard calculator – preferably a HP17BII or HP19BII – in the exam is expected.

Chapter 11 includes answers to the exam.

Good luck!

ACI FINANCIAL CALCULATIONS EXAM

- A total of 100 points is available. The pass mark is 50 points; a distinction is awarded for 80 points or more.
- All dates are shown using the European date convention; that is, 12 April 1999 would be written as 12/04/99.
- Programmable calculators such as a Hewlett Packard HP17BII or HP19BII are allowed and recommended. Candidates are allowed to have their own programs stored in the calculator. In Part 2 of the exam, *intermediate calculations must be shown and correct answers without intermediate calculations will earn no points at all.*

PART 1 30 MINUTES

20 POINTS AVAILABLE

Please answer ALL the questions.

1. You buy a 20-year annuity with a semi-annual yield of 10.5% (bond basis). How much must you invest in the annuity now to receive 50,000 each six months?
2. Give a formula, on a coupon date, for the value of a bond which pays a 7% annual coupon and has 3 years left to maturity.
3. You deposit 10 million for 5 years. It accumulates interest at 7% (bond basis) paid semi-annually for 2 years, then 7.5% paid quarterly for the next 3 years. The interest is automatically added to the capital at each payment date. What is the total accumulated value at the end of 5 years?
4. Calculate the 3 v 9 forward-forward rate, given the following rates quoted on an ACT/360 basis. Show the formula as well as the result.
 3 months (92 days): 9.00%
 9 months (275 days): 9.00%
 How would you deposit your money, if you believe that the 6-month deposit rate available to you in 3 months' time will be 8.90%?
5. A three-month (91 days) DEM call option against GBP costs 1.0% of the GBP amount at a strike of 2.60. What should a DEM put cost at the same strike?
 GBP/DEM spot: 2.6760
 3-month outright: 2.6500
 3-month GBP LIBOR: 8% (ACT/365)
 3-month DEM LIBOR: 4% (ACT/360)
6. A 1-year interest rate is quoted as 8.35% (ACT/365) with all the interest paid at maturity. What would the equivalent quotation be with interest paid (a) semi-annually; (b) daily?
7. What is the price on a coupon date of a 15-year bond, with a semi-annual coupon of 10%, yielding 10.2%?

8. You place £1 million on deposit for 1 year at 6.2% (ACT/365). What total value will you have accumulated at the end of the year if the interest is paid quarterly and can be reinvested at 6.0% also paid quarterly?
9. The rate for a 57-day deposit is quoted as 8.5% (ACT/360). What is the effective rate on an ACT/365 basis?
10. Which of the following provides the best return for an investor considering a 182-day investment?
 - a. 7.50% yield quoted on an ACT/365 basis
 - b. 7.42% yield quoted on an ACT/360 basis
 - c. 7.21% discount rate quoted on an ACT/365 basis
 - d. 7.18% discount rate quoted on an ACT/360 basis
11. What is the 12-month interest rate (ACT/360 basis) implied by the following rates (all ACT/360)?

3 months (91 days):	6.5%
FRA 3 v 6 (92 days):	6.6%
FRA 6 v 9 (91 days):	6.8%
FRA 9 v 12 (91 days):	7.0%
12. Are the following true or false?
 - a. If a bond's yield is exactly equal to its coupon, the price of the bond must be 100.
 - b. The cheapest to deliver bond for a futures contract is the bond with the lowest implied repo rate.
 - c. The longer a bond's duration, the lower its volatility.
 - d. The present value of a future cashflow cannot be greater than the cashflow itself.
 - e. A 1-year deposit is better for the depositor if the rate is 8.60% paid annually than if it is 8.45% paid semi-annually.
 - f. If a bond is trading at a price of 99, the market yield is lower than the bond's coupon.
 - g. The intrinsic value of the DEM call option in question (5) is 0.05 DEM per GBP 1.

PART 2 2 HOURS

80 POINTS AVAILABLE

ALL FOUR EXERCISES SHOULD BE ANSWERED.

EXERCISE NO. 1

25 POINTS AVAILABLE

INTERMEDIATE CALCULATIONS MUST BE SHOWN

You have the following bond:

Settlement date:	15 June 1998
Previous coupon date:	18 September 1997

Coupon:	8.0% annually
Maturity date:	18 September 2003
Accrued interest calculation basis:	ACT/ACT
Price / yield calculation basis:	30/360

- Calculate the bond's yield for settlement on 15 June 1998 if the clean price is 107.50. Show the formula for the bond's price before calculating the yield.
- If you purchase the bond on 15 June 1998 at a price of 107.50, and sell it on 21 August 1998 at 106.40, what is the simple rate of return on your investment over that period on an ACT/360 basis? What is the effective rate of return on an ACT/365 basis?
- If the above bond is the CTD for the bond futures contract and the cash bond settlement date is 15 May 1998, what is the theoretical futures price based on the following?

Delivery date of futures contract:	19 June 1998
Short-term money market interest rate:	5% (ACT/360)
Clean price of CTD bond:	108.00
Previous coupon payment:	18 September 1997
Conversion factor:	1.1000

- The actual futures price is quoted in the market at 97.73. Discuss what arbitrage opportunity this may create.

EXERCISE NO. 2

15 POINTS AVAILABLE

INTERMEDIATE CALCULATIONS MUST BE SHOWN

- What are duration and modified duration? How are they related?
- You own a portfolio consisting of the following two bonds:
 - zero coupon 3-year bond, price 71.18, nominal amount 30 million
 - 20% coupon 4-year bond, price 114.27, nominal amount 10 million

Which has the shorter duration?

- What are the modified durations of the two bonds?
- If all market yields rise by 10 basis points, what is your approximate profit or loss?
- You wish to hedge your portfolio in the short term, against instantaneous movements in the yield curve, by selling bond futures contracts. What information do you need to calculate how many futures contracts to sell?

EXERCISE NO. 3

20 POINTS AVAILABLE

INTERMEDIATE CALCULATIONS MUST BE SHOWN

Maturity	Coupon	Price
2 years	8% (annual)	104.50
3 years	5.5% (annual)	98.70

Based on the above and a 1-year yield of 5.00% (bond basis), calculate the following:

- a. The 2-year and 3-year zero-coupon yields and discount factors.
- b. The 1-year v 2-year and 2-year v 3-year forward-forward rates.
- c. The theoretical yield to maturity of a 3-year 12% annual coupon bond.
- d. The 2-year and 3-year par yields.

EXERCISE NO. 4

20 POINTS AVAILABLE

INTERMEDIATE CALCULATIONS MUST BE SHOWN

- a. You have the following interest rate swap on your book:

Notional amount: 50 million

You pay 6-month LIBOR (ACT/360 basis)

You receive a fixed rate of 10.00% (annual payments, 30/360 basis)

The swap started on 2 June 1997 and ends on 2 June 2000

The last interest rate fixing, for 2 December 1998, was 8.5% for 6-month LIBOR. The spot value date now is 15 April 1999. What is the mark-to-market value of the swap for value on that date, based on the following discount factors?

Spot to 2 June 1999: 0.9885

Spot to 2 December 1999: 0.9459

Spot to 2 June 2000: 0.9064

- b. You buy a 3-year 9% annual coupon bond at a price of 103.50, which you wish to swap to a floating-rate asset with a par initial investment amount and a regular LIBOR-related income based on this par amount. The current 3-year par swap rate is 7.5%. Based on the following discount factors, what LIBOR-related floating rate should you be able to achieve on this asset swap? LIBOR is on an ACT/360 basis.

6 months: 0.9650

1 year: 0.9300

1½ years: 0.8970

2 years: 0.8650

2½ years: 0.8350

3 years: 0.8050

Hints and Answers to Exercises and Practice Exam

Hints on exercises

Answers to exercises

Answers to practice exam

HINTS ON EXERCISES

1. $\text{Future value} = \text{present value} \times \left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)$
2. $\text{Present value} = \frac{\text{future value}}{\left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)}$
3. $\text{Yield} = \left(\frac{\text{future value}}{\text{present value}} - 1\right) \times \frac{\text{year}}{\text{days}}$
4. $\text{Future value} = \text{present value} \times (1 + \text{yield})^N$
5. Is a present value generally greater or smaller than a future value?
6. $\text{Future value} = \text{present value} \times (1 + \text{yield})^N$
 $\text{Interest} = \text{future value} - \text{principal}$
7. $\text{Present value} = \frac{\text{future value}}{(1 + \text{yield})^N}$
8. $\text{Future value} = \text{present value} \times (1 + \text{yield})^N$
9. $\text{Yield} = \left(\frac{\text{future value}}{\text{present value}}\right)^{\frac{1}{N}} - 1$
10. $\text{Future value} =$

$$\text{present value} \times \left(1 + \frac{\text{first yield}}{\text{frequency}}\right)^{\text{number of periods}} \times \left(1 + \frac{\text{second yield}}{\text{frequency}}\right)^{\text{number of periods}}$$
11. $\text{Future value} = \text{present value} \times \left(1 + \frac{\text{yield}}{\text{frequency}}\right)^{\text{number of periods}}$

With reinvestment at a different rate, consider each cashflow separately, reinvested to maturity.
12. Either use the TVM keys on the HP, or calculate the present value of each cashflow and add them together:

$$\text{present value} = \frac{\text{future value}}{(1 + \text{yield})^N}$$
13. Consider the problem in terms of monthly periods rather than years. On this basis, how many periods are there and what is the interest rate for each period? Then use the TVM keys on the HP.

$$14. \text{Effective rate} = \left(1 + \frac{\text{semi-annual rate}}{2}\right)^2 - 1$$

$$15. \text{Rate} = \left[(1 + \text{annual rate})^{\frac{1}{\text{frequency}}} - 1\right] \times \text{frequency}$$

$$16. \text{Future value} = \text{present value} \times e^{r \times \frac{\text{days}}{\text{year}}}$$

$$\text{effective rate} = e^r - 1$$

$$\text{continuously compounded rate} = \text{LN}(1 + i)$$

$$17. \text{Effective rate} = \left(1 + i \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}}$$

$$\text{daily equivalent rate} = \left[\left(1 + i \times \frac{\text{days}}{\text{year}}\right)^{\frac{1}{\text{days}}} - 1\right] \times \text{year}$$

$$\text{discount factor} = \frac{1}{\left(1 + i \times \frac{\text{days}}{\text{year}}\right)}$$

$$18. \text{First known rate} +$$

$$\text{difference between known rates} \times \frac{\text{days from first date to interpolated date}}{\text{days between known dates}}$$

$$19. \text{What is the period between cashflows? What is the interest rate for this period? Are there any zero cashflows?}$$

The NPV can be calculated either by using the HP cashflow function, or from first principles:

$$\text{present value} = \frac{\text{future value}}{(1 + \text{yield})^N}$$

$$20. \text{Use the IRR function on the HP and then consider for what period the result is expressed.}$$

$$21. \text{Proceeds} = \text{face value} \times \left(1 + \text{coupon rate} \times \frac{\text{days}}{\text{year}}\right)$$

$$22. \text{Price} = \text{present value} = \frac{\text{future value}}{\left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)}$$

$$\text{simple yield} = \left(\frac{\text{future cashflow}}{\text{present cashflow}} - 1\right) \times \frac{\text{year}}{\text{days held}}$$

$$\text{effective yield} = \left(\frac{\text{future cashflow}}{\text{present cashflow}}\right)^{\frac{365}{\text{days}}} - 1$$

$$23. \text{ Overall return} = \left[\frac{\left(1 + \text{yield on purchase} \times \frac{\text{days on purchase}}{\text{year}}\right)}{\left(1 + \text{yield on sale} \times \frac{\text{days on sale}}{\text{year}}\right)} - 1 \right] \times \frac{\text{year}}{\text{days held}}$$

Therefore: yield on sale =

$$\left[\frac{\left(1 + \text{yield on purchase} \times \frac{\text{days on purchase}}{\text{year}}\right)}{\left(1 + \text{overall return} \times \frac{\text{days held}}{\text{year}}\right)} - 1 \right] \times \frac{\text{year}}{\text{days on sale}}$$

$$24. \text{ Rate on ACT/365 basis} = \text{rate on ACT/360 basis} \times \frac{365}{360}$$

$$\text{effective yield (ACT/365 basis)} = \left(1 + \text{interest rate} \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1$$

$$\text{effective yield on ACT/360 basis} = \text{effective yield} \times \frac{360}{365}$$

25. What is the day/year count?

$$\text{price} = \text{present value} = \frac{\text{future value}}{\left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)}$$

$$26. \text{ Discount rate} = \frac{\text{rate of true yield}}{\left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)}$$

$$\text{discount amount} = \text{principal} \times \text{discount rate} \times \frac{\text{days}}{\text{year}}$$

$$27. \text{ Rate of true yield} = \frac{\text{discount rate}}{\left(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}}\right)}$$

$$\text{amount paid} = \text{principal} \times \left(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}}\right)$$

28. Discount amount = face value – amount paid

$$\text{discount rate} = \frac{\text{discount amount}}{\text{face value}} \times \frac{\text{year}}{\text{days}}$$

29. What is the day/year count?

$$a. \text{ Amount paid} = \text{principal} \times \left(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}}\right)$$

$$b. \text{ Rate of true yield} = \frac{\text{discount rate}}{\left(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}}\right)}$$

Then convert to 365-day basis

$$30. \text{ Yield} = \left(\frac{\text{future cashflow}}{\text{present cashflow}} - 1 \right) \times \frac{\text{year}}{\text{days held}}$$

$$31. \text{Yield} = \left(\frac{\text{future cashflow}}{\text{present cashflow}} - 1 \right) \times \frac{\text{year}}{\text{days held}}$$

32. What is the day/year count in each case?

Is the quote a yield or a discount rate in each case?

33. Convert all rates to the same basis in order to compare them – for example, true yield, on a 365-day basis.

34. Taking account of non-working days as appropriate, what was the last coupon date, and what are the remaining coupon dates?

How many days are there between these dates and what are the exact coupon payments?

Discount each cashflow back to the previous date, using exact day counts, add the actual cashflows for that date, and so on back to the settlement date.

$$35. \text{Forward-forward rate} = \left[\frac{\left(1 + \text{longer rate} \times \frac{\text{days}}{\text{year}} \right)}{\left(1 + \text{shorter rate} \times \frac{\text{days}}{\text{year}} \right)} - 1 \right] \times \left(\frac{\text{year}}{\text{days difference}} \right)$$

If the above is based on middle rates rather than offered rates, you should add around 0.06% to benchmark against LIBOR.

36. a. For the FRA, are you a borrower (protecting against the risk of higher rates) or an investor (protecting against the risk of lower rates)?

b. The price-taker always gets the worse price.

c. Consider the cashflows – exactly what amount will you be rolling over?

$$d. \text{FRA settlement amount} = \text{principal} \times \frac{(\text{FRA rate} - \text{LIBOR}) \times \frac{\text{days}}{\text{year}}}{\left(1 + \text{LIBOR} \times \frac{\text{days}}{\text{year}} \right)}$$

e. Consider all the exact cashflows and timings and on which side of the market you will be dealing.

37. Remember that the profit / loss relates to a 3-month period rather than a whole year.

38. Create a strip.

39. Calculate the implied 3 v 9 and 6 v 12 rates.

Interpolate for 3 v 7 and 6 v 10.

Interpolate further for 4 v 8.

The hedge follows the same construction.

40. First, calculate the various cash forward-forward rates (3 v 6, 6 v 9, 3 v 9)
Then, compare the various combinations possible:
- forward-forward, FRA or futures for 3 v 6
 - forward-forward, FRA or futures for 6 v 9
 - forward-forward or FRA for 3 v 9

41. Use the TVM function of the HP.

42. What are all the cashflows from the bond?

Clean price = NPV using the yield.

Clean price = dirty price because there is no accrued coupon.

$$\text{Current yield} = \frac{\text{coupon rate}}{\text{clean price} / 100}$$

$$\text{Simple yield to maturity} = \frac{\text{coupon rate} + \left(\frac{\text{redemption amount} - \text{clean price}}{\text{years to maturity}} \right)}{\text{clean price} / 100}$$

$$\text{Duration} = \frac{\sum (\text{present value of cashflow} \times \text{time to cashflow})}{\text{dirty price}}$$

43. This question is complicated by the fact that the calculation bases for accrued coupon and price are different.
- Calculate the clean price and the accrued interest assuming that both are calculated on an ACT/ACT basis.
 - Add together to give the correct dirty price.
 - Recalculate the accrued interest on the correct 30/360 basis.
 - Subtract this from the dirty price to give the correct clean price.

Or, using the bond price formula rather than the functions built into the HP calculator:

$$\text{Dirty price} = \frac{100}{\left(1 + \frac{i}{n}\right)^w} \left[\frac{R}{n} \times \frac{\left(1 - \frac{1}{\left(1 + \frac{i}{n}\right)^N}\right)}{\left(1 - \frac{1}{\left(1 + \frac{i}{n}\right)}\right)} + \frac{1}{\left(1 + \frac{i}{n}\right)^{N-1}} \right]$$

$$\text{Clean price} = \text{dirty price} - \text{accrued coupon.}$$

44. If you use the HP calculator's built-in bond functions, it is again necessary to make an adjustment for the fact that the price / yield calculation is on a 30/360 basis but the accrued coupon is on an ACT/365 basis, as follows:
- Calculate the correct accrued interest.
 - Add to the clean price to give the correct dirty price.
 - Calculate the accrued coupon as if it were on a 30/360 basis.
 - Subtract from the dirty price to give an adjusted clean price.

45. What is the fraction of a period to the next quasi-coupon date?

$$\text{Price} = \frac{100}{\left(1 + \frac{\text{yield}}{2}\right)^{(\text{number of periods to maturity})}}$$

46. True yield on bond year basis = $\frac{\text{discount rate}}{(1 - \text{discount rate} \times \frac{\text{days}}{360})} \times \frac{365}{360}$

47. yield = $\frac{-\frac{\text{days}}{365} + \left(\left(\frac{\text{days}}{365}\right)^2 \times 2 \times \left(\frac{\text{days}}{365} - \frac{1}{2}\right) \times \left(\frac{1}{(1 - D \times \frac{\text{days}}{360})} - 1\right)\right)^{\frac{1}{2}}}{\left(\frac{\text{days}}{365} - \frac{1}{2}\right)}$

48. This needs the same equation as the previous question. However, you need to manipulate the equation into the form “D = ...” rather than “i = ...”.

49. The HP calculator bond function cannot be used for a bond with stepped coupons. The easiest method is to work from first principles, as follows:

- Discount the final cashflow to a value one year earlier.
- Add the coupon cashflow paid then and discount back a further year.
- Repeat the process back to the first remaining coupon.
- Discount back to settlement date (what is the $\frac{\text{day}}{\text{year}}$ basis?) to give the current dirty price.
- Subtract the accrued interest to give the clean price.

50. Because bond price / yield formulas generally assume a redemption amount of 100, one approach is to scale down every cashflow by the same factor to correspond to a redemption amount of 100.

$$\text{Simple return} = \left(\frac{\text{sale proceeds}}{\text{amount invested}} - 1 \right) \times \frac{\text{year}}{\text{days}}$$

$$\text{Effective return} = \left(\frac{\text{sale proceeds}}{\text{amount invested}} \right)^{\frac{365}{\text{days}}} - 1$$

51. For each one:

- Are coupons paid annually or semi-annually?
- What was the last coupon date?
- What is the day/year basis?

52. Compare the cost of funding with the current yield. The difference between the futures price and the bond price should compensate for this.

53. Either build up the price from the arbitrage mechanism:

- Buy the bond.
- Borrow to finance the bond purchase.

- At delivery of the futures contract, repay the financing plus interest.
- Deliver the bond in return for payment plus accrued.

Or

Theoretical futures price =

$$\frac{\left([\text{bond price} + \text{accrued coupon now}] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] \right) - (\text{accrued coupon at delivery of futures})}{\text{conversion factor}}$$

$$54. \quad \text{Implied repo rate} = \left[\frac{(\text{futures price} \times \text{conversion factor}) + (\text{accrued coupon at delivery of futures})}{(\text{bond price} + \text{accrued coupon now})} - 1 \right] \times \frac{\text{year}}{\text{days}}$$

55. Assume the cash-and-carry arbitrage is:

- Buy the cash CTD bond now.
- Fund this purchase by repoing the bond.
- Sell the bond futures contract.
- Deliver the bond at maturity of the futures contract.

Accrued coupon for CTD bond = ?

Cost of buying CTD bond per DEM 100 nominal = ?

Total borrowing (principal + interest) to be repaid at the end = ?

Anticipated receipt from selling futures contract and delivering bond per DEM 100 nominal = ?

Profit per DEM 100 nominal = ?

Size of DEM bond futures contract = ?

Face value of bond purchased in the arbitrage = ?

Therefore profit per futures contract = ?

56. For each bond, calculate the yield, the modified duration, the accrued interest, the dirty price, then the total value.

$$\text{modified duration of portfolio} \approx \frac{\sum (\text{modified duration} \times \text{value})}{\text{portfolio value}}$$

$$\text{change in value} \approx - \text{value} \times \text{change in yield} \times \text{modified duration}$$

57. For each bond, calculate the yield, the modified duration, the accrued interest and the dirty price.

Hedge ratio =

$$\frac{\text{notional amount of futures contract required to hedge a position in bond A}}{\text{face value of bond A}} = \frac{\text{dirty price of bond A}}{\text{dirty price of CTD bond}} \times \frac{\text{modified duration of bond A}}{\text{modified duration of CTD bond}} \times \frac{\text{conversion factor for CTD bond}}{\left(1 + i \times \frac{\text{days}}{\text{year}}\right)}$$

where i = short-term funding rate

58. Bootstrap to create the zero-coupon yields and discount factors.

$$1 \text{ year v } 2 \text{ year forward-forward} = \frac{1\text{-year discount factor}}{2\text{-year discount factor}} - 1 \text{ etc.}$$

59. a. Create the zero-coupon yields from strips of the 1-year rate and forward-forwards.

Calculate the discount factors.

The par yield is the coupon of a bond such that the NPV of the bond's cashflows (using the discount factors) is par.

b. Calculate the NPV of the cashflows using the discount factors.
Use the TVM function of the HP to calculate the yield.

60. Bootstrap to create the 18-month and 24-month discount factors

$$\text{forward-forward rate} = \left[\frac{18\text{-month discount factor}}{24\text{-month discount factor}} - 1 \right] \times \frac{\text{year}}{\text{days}}$$

61. What are the day/year bases?

$$\text{forward swap} = \text{spot} \times \frac{\left(\text{variable currency rate} \times \frac{\text{days}}{\text{year}} - \text{base currency rate} \times \frac{\text{days}}{\text{year}} \right)}{\left(1 + \text{base currency rate} \times \frac{\text{days}}{\text{year}} \right)}$$

62. a. Indirect rates: divide opposite sides. Customer gets the worse price.

b. One direct rate and one indirect: multiply same sides.

c. Add or subtract the forward points?

d. Add or subtract the forward points?

e. Is sterling worth more forward than spot or vice versa?

f. Similar to (d) and (e).

g. Forward points = outright – spot.

63. a. The value date is after spot (add or subtract forward points?).
 b. The value date is before spot (add or subtract forward points?).
 c. The customer gets the worse price.
 d. Similar to (c).
 e. Take the difference between opposite sides.
 f. Should the price be bigger or smaller than the one-month swap? Take care with +/- signs.
64. a. Calculate NOK and FRF outright against the USD.
 Calculate cross-rate spots and outright.
 Cross-rate swap = outright – spot.
 Forward-forward price is difference between opposite sides.
 b. For a swap from before spot to after spot, add the same sides.
65. a. You need to interpolate between the 1-month and the 2-month dates. Then the method is similar to 64(a).
 b. Similar to 64(b) but combine the O/N price as well as the T/N.
 c. Again, combine the O/N and T/N.
66. a. The big figure on the offer side of USD/ITL spot is 1634. The 6-month USD/ITL forward swap price means 22.37 lire / 22.87 lire.
 b. Assume that the expected changes happen, and calculate the effect on the forward outright price, using middle prices for the comparison.
67. Either consider the exact cashflows:
- Initial DEM investment = ?
 - Total proceeds at maturity of CP = ?
 - Buy and sell USD (sell and buy DEM) spot against 3 months

Or use the formula for covered interest arbitrage:

variable currency rate =

$$\left[\left(1 + \text{base currency rate} \times \frac{\text{days}}{\text{base year}} \right) \times \frac{\text{outright}}{\text{spot}} - 1 \right] \times \frac{\text{variable year}}{\text{days}}$$

68. Action now

- (i) Arrange FRA on a notional 3 v 9 borrowing of USD.
- (ii) Convert this notional loan from USD to SEK via a foreign exchange swap.

Action in 3 months' time

- (iii) Assume a borrowing of SEK.
- (iv) Convert this borrowing to a USD borrowing to match (i), via a foreign exchange swap.

Settlement at the end of 9 months

- (i) Receive deferred FRA settlement.

69. Write out all the resulting cashflows, then calculate present values.
70. a. Value the DEM cashflows in USD using current outright. Recalculate the outright after the spot rate movement, then revalue the DEM cashflows.
- b. Calculate the NPV of these net future cashflows using an appropriate USD interest rate for each period.
- Calculate the NPV of the original DEM positions using an appropriate DEM interest rate for each period; the appropriate spot hedge should offset this NPV.
71. On what basis is the swap quoted? Convert the absolute swap rate and the bond coupon both to semi-annual money market. The difference represents the approximate sub-LIBOR spread.
72. a. Convert the futures prices to implied interest rates. Then for each maturity, create a zero-coupon rate from a strip of 3-month rates. Is this on a bond basis or a money market basis? Calculate the effective annual equivalent.
- b. Calculate the par rate in the same way that you would calculate a par bond yield – the coupon and yield are the same for an instrument priced at par. The quarterly coupon cashflows need to be calculated on a quarterly money market basis.
73. a. The USD swap cashflows match the remaining bond cashflows (including the principal). What is the NPV in USD of these cashflows? Convert to GBP at spot. The GBP side of the swap must have an NPV equal to this. The GBP swap cashflows can therefore be 11% per annum plus principal, based on this NPV amount.
- b. Long-dated forward outright = $\text{spot} \times \frac{(1 + \text{variable interest rate})^N}{(1 + \text{base interest rate})^N}$
- Convert the USD bond cashflows to GBP at the forward rates.
- c. What is the NPV of the alternative GBP cashflow streams, using various rates of discount to calculate the NPV?
74. Convert the USD cash raised from the bond issue to CHF and create CHF swap cashflows (floating-rate interest payments plus principal) on this amount. Calculate the NPV of the USD swap cashflows (which match all the bond cashflows) and convert to CHF at spot; the NPV of the CHF swap cashflows needs to equal this amount.
- Therefore adjust the CHF interest payments by a regular amount which brings the NPV of all the CHF cashflows to this amount.
75. Write out the cashflows in each currency, remembering to include the final principal payments. Calculate the NPV of the USD cashflows using the USD discount factors.

Some of the DEM cashflows are unknown because they depend on future LIBOR fixings. Eliminate these by adding an appropriate FRN structure beginning on 25 May 1998. Alternatively, calculate forward-forward rates for the future LIBOR fixings. Then calculate the NPV of the net DEM cashflows using the DEM discount factors. Convert this NPV to USD and net against the USD NPV already calculated.

76. Write out all the cashflows arising from issuing USD100 face value of the bond. Add the cashflows arising from a swap based on a notional amount of USD100. Consider the net result as a structure of 100 borrowed, 100 repaid at maturity and $(100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360})$ each six months plus some irregular cashflows.

Calculate the NPV of these irregular cashflows.

Offset them by matching cashflows and replace by a series of regular six-monthly cashflows with the same NPV.

What spread above or below LIBOR is necessary to generate these regular six-monthly cashflows?

77. From the ten price data, calculate nine daily relative price changes ($\frac{\text{day 2 price}}{\text{day 1 price}}$ etc.).

Take the natural logarithm of each relative price change.

Calculate the standard deviation of these nine logarithms, as usual:

- Calculate the mean.
- Calculate the differences from the mean.
- Square the differences.
- Add the squares and divide by eight (one less than the number of data) to give the variance.
- The standard deviation is the square root of the variance.

The annualized volatility is:

$$\text{standard deviation} \times \sqrt{\text{number of data observations per year}}$$

78. Convert the FRF call option premium to a cost in FRF.
Use the put / call relationship expressed in units of the variable currency:

$$\begin{aligned} &\text{premium for call on base currency} - \text{premium for put on base currency} \\ &= \text{present value of (forward} - \text{strike)} \end{aligned}$$

Convert the result back to percentage of the USD amount.

79. Calculate the probability (p) of an up movement in the price and of a down movement (1 - p):

The expected outcome after 1 month is $[p \times \text{increased price} + (1 - p) \times \text{decreased price}]$. This should equal the result of depositing 100 for 1 month at the risk-free interest rate.

Construct a tree showing all the possible outcomes at the end of three months.

Calculate the probability of arriving at each of these outcomes by combining the probabilities along each route along the tree.

Which of these outcomes would make the option in-the-money at expiry?

What would the profit be for each outcome?

Multiply each profit by the probability of it happening, and add to give the expected value of the option.

The PV of this expected value is the option's premium.

ANSWERS TO EXERCISES

1. Future value = $£43 \times \left(1 + 0.075 \times \frac{120}{365}\right) = £44.06$

.075 ENTER 120 \times 365 \div 1 + 43 \times

2. Present value = $\frac{£89}{\left(1 + 0.101 \times \frac{93}{365}\right)} = £86.77$

.101 ENTER 93 \times 365 \div 1 + 89 \square $x \geq y \div$

3. Yield = $\left(\frac{83.64}{83.00} - 1\right) \times \frac{365}{28} = 10.05\%$

83.64 ENTER 83 \div 1 - 365 \times 28 \div

4. $£36 \times (1 + 0.09)^{10} = £85.23$

1.09 ENTER 10 \square \wedge 36 \times

5. Choose **DEM1,000 now** unless interest rates are negative. With positive rates, DEM1,000 now must be worth more than a smaller amount in the future.

6. $£342 \times (1 + 0.06)^5 = £457.67$
 $£457.67 - £342 = £115.67$

1.06 ENTER 5 \square \wedge 342 \times 342 -

7. DEM $\frac{98}{(1 + 0.11)^5} = \text{DEM } 58.16$

1.11 ENTER 5 \square \wedge 98 \square $x \geq y \div$

8. $£1,000 \times (1 + 0.054)^4 = £1,234.13$

1.054 ENTER 4 $\square \wedge$ 1,000 \times

OR

FIN TVM

4 N

1,000 +/- PV

0 PMT

5.4 I% YR

FV

$$9. \left(\frac{1,360.86}{1,000} \right)^{\frac{1}{7}} - 1 = 4.50\%$$

1,360.86 ENTER 1,000 \div

7 $\square 1/x \square \wedge$ 1 -

OR

FIN TVM

7 N

1,000 +/- PV

0 PMT

1360.86 FV

I%YR

$$10. \pounds 1,000,000 \times \left(1 + \frac{0.06}{4} \right)^{20} \times \left(1 + \frac{0.065}{2} \right)^{10} = \pounds 1,854,476.99$$

.06 ENTER 4 \div 1 + 20 $\square \wedge$

.065 ENTER 2 \div 1 + 10 $\square \wedge \times$

1,000,000 \times

$$11. \pounds 1,000,000 \times \left(1 + \frac{0.085}{4} \right)^4 = \pounds 1,087,747.96$$

.085 ENTER 4 \div 1 + 4 $\square \wedge$ 1,000,000 \times

With reinvestment at 8.0%:

First interest payment plus reinvestment: $\pounds 1,000,000 \times \frac{0.085}{4} \times \left(1 + \frac{0.08}{4} \right)^3$

Second interest payment plus reinvestment: $\pounds 1,000,000 \times \frac{0.085}{4} \times \left(1 + \frac{0.08}{4} \right)^2$

Third interest payment plus reinvestment: $\pounds 1,000,000 \times \frac{0.085}{4} \times \left(1 + \frac{0.08}{4} \right)$

Fourth interest payment plus reinvestment: $\pounds 1,000,000 \times \frac{0.085}{4}$

Principal amount:

£1,000,000

Total:

£1,087,584.17

```
.08 ENTER 4 ÷ 1 + 3  $\square$   $\wedge$ 
.08 ENTER 4 ÷ 1 + 2  $\square$   $\wedge$  +
.08 ENTER 4 ÷ 1 + +
1,000,000  $\times$  .085  $\times$  4 ÷
.085 ENTER 4 ÷ 1 + 1,000,000  $\times$  +
```

12. £32,088.29

```
FIN TVM
10 N
9 I%YR
5,000 PMT
0 FV
PV
```

13. There are 300 payment periods. The interest for each period is $\frac{7.25\%}{12}$.

The regular payment must be £650.53.

```
FIN TVM
300 N
7.25 ENTER 12 ÷ I%YR
90,000 PV
0 FV
PMT
```

$$14. \left(1 + \frac{0.114}{2}\right)^2 - 1 = 11.72\%$$

```
.114 ENTER 2 ÷ 1 + 2  $\square$   $\wedge$  1 -
```

$$15. [(1 + 0.12)^{\frac{1}{4}} - 1] \times 4 = 11.49\%$$

$$[(1 + 0.12)^{\frac{1}{12}} - 1] \times 12 = 11.39\%$$

```
1.12 ENTER 4  $\square$   $\frac{1}{x}$   $\square$   $\wedge$  1 - 4  $\times$ 
1.12 ENTER 12  $\square$   $\frac{1}{x}$   $\square$   $\wedge$  1 - 12  $\times$ 
```

$$16. £1,000,000 \times e^{0.07} = £1,072,508.18$$

Effective rate is 7.2508% per annum

```
 $\square$  MATH LOGS .07 EXP 1,000,000  $\times$ 
```

$\text{LN}(1.09) = 8.6178\%$ is the continuously compounded rate.

□ MATH LOGS 1.09 LN

$$17. \text{Effective rate} = \left(1 + 0.065 \times \frac{138}{365}\right)^{\frac{365}{138}} - 1 = 6.6321\%$$

$$\text{Daily equivalent rate} = \left[\left(1 + 0.065 \times \frac{138}{365}\right)^{\frac{1}{138}} - 1\right] \times 365 = 6.4220\%$$

$$\text{Discount factor} = \frac{1}{\left(1 + 0.065 \times \frac{138}{365}\right)} = 0.9760$$

.065 ENTER 138 × 365 ÷ 1 + 365 ENTER 138 ÷ □ ^ 1 -
 .065 ENTER 138 × 365 ÷ 1 + 138 □ 1/x □ ^ 1 - 365 ×
 .065 ENTER 138 × 365 ÷ 1 + □ 1/x □

$$18. 5.2\% + (5.4\% - 5.2\%) \times \frac{(41 - 30)}{(60 - 30)} = 5.2733\%$$

41 ENTER 30 - 60 ENTER 30 - ÷
 5.4 ENTER 5.2 - ×
 5.2 +

19. Because the cashflows are 6-monthly, the effective annual interest rate must be converted to an equivalent rate for this 6-monthly period, rather than an annual rate:

$$(1.10)^{\frac{1}{2}} - 1 = 4.88\%$$

The NPV can then be calculated either by using the HP cashflow function (remembering that the 30-month cashflow is zero), or from first principles as:

$$-\$105 - \frac{\$47}{(1.0488)} - \frac{\$47}{(1.0488)^2} - \frac{\$47}{(1.0488)^3} - \frac{\$93}{(1.0488)^4} + \frac{\$450}{(1.0488)^6} = \$27.95$$

FIN CFLO GET *NEW

[If necessary to clear previous data, type "CLEAR DATA" followed by "YES"]

105 +/- INPUT

47 +/- INPUT 3 INPUT

93 +/- INPUT INPUT

0 INPUT INPUT

450 INPUT INPUT

CALC

1.1 ENTER $\square \sqrt{x}$ 1 - 100 \times I%

NPV

OR

1.1 ENTER $\square \sqrt{x}$ STO 1

6 $\square \wedge$ 450 $\square x \geq y \div$

93 RCL 1 4 $\square \wedge \div -$

47 RCL 1 3 $\square \wedge \div -$

47 RCL 1 2 $\square \wedge \div -$

47 RCL 1 $\div -$

105 -

20. 7.047% is the IRR on the basis of the semi-annual periods. This is then annualized:

$$(1 + 0.07047)^2 - 1 = 14.59\%$$

Following on from the previous calculation using the HP cashflow function:

IRR%

100 \div 1 + 2 $\square \wedge$ 1 -

21. $\pounds 1,000,000 \times \left(1 + 0.11 \times \frac{181}{365}\right) = \pounds 1,054,547.95$

.11 ENTER 181 \times 365 \div 1 + 1,000,000 \times

22. $\frac{\pounds 1,000,000 \times \left(1 + 0.11 \times \frac{181}{365}\right)}{\left(1 + 0.10 \times \frac{134}{365}\right)} = \pounds 1,017,204.02 \text{ price}$

.11 ENTER 181 \times 365 \div 1 + 1,000,000 \times

.1 ENTER 134 \times 365 \div 1 + \div

$$\left[\frac{\left(1 + 0.10 \times \frac{134}{365}\right)}{\left(1 + 0.095 \times \frac{71}{365}\right)} - 1 \right] \times \frac{365}{63} = 10.37\% \text{ simple return}$$

.1 ENTER 134 \times 365 \div 1 +

.095 ENTER 71 \times 365 \div 1 + \div

1 - 365 \times 63 \div

$$\left[\frac{\left(1 + 0.10 \times \frac{134}{365}\right)}{\left(1 + 0.095 \times \frac{71}{365}\right)} \right]^{\frac{365}{63}} - 1 = 10.83\% \text{ effective return}$$

.1 ENTER $134 \times 365 \div 1 +$
 .095 ENTER $71 \times 365 \div 1 + \div$
 365 ENTER $63 \div \square \wedge 1 -$

$$23. \left[\frac{\left(1 + 0.10 \times \frac{134}{365}\right)}{\left(1 + i \times \frac{71}{365}\right)} - 1 \right] \times \frac{365}{63} = 10.00\%$$

$$\text{Therefore } i = \left[\frac{\left(1 + 0.10 \times \frac{134}{365}\right)}{\left(1 + 0.10 \times \frac{63}{365}\right)} - 1 \right] \times \frac{365}{71} = 9.83\%$$

.1 ENTER $134 \times 365 \div 1 +$
 .1 ENTER $63 \times 365 \div 1 + \div$
 1 - 365 $\times 71 \div$

$$24. 11.5\% \times \frac{365}{360} = 11.66\%$$

$$\left(1 + 0.115 \times \frac{91}{360}\right)^{\frac{365}{91}} - 1 = 12.18\%$$

$$12.18\% \times \frac{360}{365} = 12.01\%$$

11.5 ENTER $365 \times 360 \div$
 .115 ENTER $91 \times 360 \div 1 + 365$ ENTER $91 \div \square \wedge 1 -$
 360 $\times 365 \div$

25. Day/year count is ACT/365 basis – that is, $\frac{62}{365}$ in this case.

$$\text{Purchase price} = \frac{\pounds 2,000,000}{\left(1 + 0.082 \times \frac{62}{365}\right)} = \pounds 1,972,525.16$$

.082 ENTER $62 \times 365 \div 1 + 2,000,000 \square x \geq y \div$

$$26. \frac{9.5\%}{\left(1 + 0.095 \times \frac{60}{365}\right)} = 9.35\%$$

$$9.35\% \times \pounds 1,000,000 \times \frac{60}{365} = \pounds 15,369.86$$

.095 ENTER $60 \times 365 \div 1 + .095 \square x \geq y \div$
 .0935 ENTER $1,000,000 \times 60 \times 365 \div$

$$27. \frac{9.5\%}{\left(1 - 0.095 \times \frac{60}{365}\right)} = 9.65\%$$

$$9.5\% \times £1,000,000 \times \frac{60}{365} = £15,616.44$$

$$\text{Amount paid} = £1,000,000 - £15,616.44 = £984,383.56$$

.095 ENTER 60 × 365 ÷ 1 - +/- .095 \square x ≥ y ÷
 .095 ENTER 60 × 365 ÷ 1 - +/- 1,000,000 ×

$$28. \left(\frac{1,000,000 - 975,000}{1,000,000} \right) \times \frac{365}{60} = 15.21\%$$

1,000,000 ENTER 975,000 - 1,000,000 ÷ 365 × 60 ÷

$$29. \text{a. Amount paid} = \$1,000,000 \times \left(1 - 0.065 \times \frac{91}{360}\right) = \$983,569.44$$

$$\text{b. Yield} = \frac{6.5\%}{\left(1 - 0.065 \times \frac{91}{360}\right)} \times \frac{365}{360} = 6.70\%$$

.065 ENTER 91 × 360 ÷ 1 - +/- 1,000,000 ×
 .065 ENTER 91 × 360 ÷ 1 - +/- 6.5 \square x ≥ y ÷ 365 × 360 ÷

$$30. \left[\frac{\left(1 - 0.067 \times \frac{112}{360}\right)}{\left(1 - 0.070 \times \frac{176}{360}\right)} - 1 \right] \times \frac{365}{64} = 7.90\%$$

.067 ENTER 112 × 360 ÷ 1 -
 .07 ENTER 176 × 360 ÷ 1 - ÷
 1 - 365 × 64 ÷

$$31. \left[\frac{\left(1 - 0.075 \times \frac{172}{360}\right)}{\left(1 - 0.070 \times \frac{176}{360}\right)} - 1 \right] \times \frac{365}{4} = -15.22\%$$

.075 ENTER 172 × 360 ÷ 1 -
 .07 ENTER 176 × 360 ÷ 1 - ÷
 1 - 365 × 4 ÷

32. US

$$\text{US\$}1,000,000 \times \left(1 - 0.05 \times \frac{91}{360}\right) = \text{US\$}987,361.11$$

UK

$$£1,000,000 \times \left(1 - 0.05 \times \frac{91}{365}\right) = £987,534.25$$

Belgium

$$\frac{BEF1,000,000}{\left(1 + 0.05 \times \frac{91}{365}\right)} = BEF987,687.73$$

France

$$\frac{FRF1,000,000}{\left(1 + 0.05 \times \frac{91}{360}\right)} = FRF987,518.86$$

.05 ENTER 91 × 360 ÷ 1 - +/- 1,000,000 ×
 .05 ENTER 91 × 365 ÷ 1 - +/- 1,000,000 ×
 .05 ENTER 91 × 365 ÷ 1 + 1,000,000 □ $x \geq y$ ÷
 .05 ENTER 91 × 360 ÷ 1 + 1,000,000 □ $x \geq y$ ÷

33. Convert all rates, for example to true yield on a 365-day basis to compare:

30-day T-bill (£) 8¼% discount rate

$$\text{yield} = \frac{8.25\%}{\left[1 - 0.0825 \times \frac{30}{365}\right]} = 8.3063\%$$

.0825 ENTER 30 × 365 ÷ 1 - +/- 8.25 □ $x \geq y$ ÷

30-day UK CP (£) 8.1875% yield

30-day ECP (£) 8.125% yield

30-day US T-bill 8.3125% discount rate

$$\begin{aligned} \text{yield} &= \frac{8.3125\%}{\left[1 - 0.083125 \times \frac{30}{360}\right]} = 8.3705\% \text{ on 360-day basis} \\ &= 8.3705\% \times \frac{365}{360} = 8.4867\% \text{ on 365-day basis} \end{aligned}$$

.083125 ENTER 30 × 360 ÷ 1 - +/- 8.3125 □ $x \geq y$ ÷ 365 × 360 ÷

30-day interbank deposit (£) 8.25% yield

30-day US CP 8.5% discount rate on 360-day basis

$$\text{yield} = \frac{8.5\%}{\left[1 - 0.085 \times \frac{30}{360}\right]} = 8.5606\% \text{ on 360-day basis}$$

$$= 8.5606\% \times \frac{365}{360} = 8.6795\% \text{ on 365-day basis}$$

.085 ENTER 30 × 360 ÷ 1 - +/- 8.5 □ $x \geq y \div 365 \times 360 \div$

30-day US\$ CD 8.625% yield on ACT/360 basis

$$= 8.625\% \times \frac{365}{360} = 8.7448\% \text{ on 365-day basis}$$

30-day French T-bill 8.5% yield on ACT/360 basis

$$= 8.5\% \times \frac{365}{360} = 8.6181\% \text{ on 365-day basis}$$

Therefore in descending order:

US\$ CD	8.7448%
US CP	8.6795%
French T-bill	8.6181%
US T-bill	8.4867%
UK T-bill (£)	8.3063%
Interbank deposit (£)	8.25%
UK CP	8.1875%
ECP (£)	8.125%

34. The last coupon date was: (0) 26 March 1999
 The remaining coupon dates are: (1) 27 September 1999 (26 September is a Sunday)
 (2) 27 March 2000 (26 March is a Sunday)
 (3) 26 September 2000
 (4) 26 March 2001

with the same notation as before,

F	=	1,000,000
R	=	0.075
year	=	360
d_{p1}	=	130
d_{01}	=	185
d_{12}	=	182
d_{23}	=	183
d_{34}	=	181
i	=	0.08

$$A_1 = \left(1 + 0.08 \times \frac{130}{360} \right) = 1.028889$$

$$A_2 = 1.028889 \times \left(1 + 0.08 \times \frac{182}{360} \right) = 1.070502$$

$$A_3 = 1.070502 \times \left(1 + 0.08 \times \frac{183}{360} \right) = 1.114035$$

$$\begin{aligned}
 A_4 &= 1.114035 \times \left(1 + 0.08 \times \frac{181}{360} \right) = 1.1158844 \\
 P &= F \times \left[\frac{R}{\text{year}} \times \left(\frac{d_{01}}{A_1} + \frac{d_{12}}{A_2} + \frac{d_{23}}{A_3} + \frac{d_{34}}{A_4} \right) + \frac{1}{A_4} \right] \\
 &= \$1,000,000 \\
 &\quad \times \left[\frac{0.075}{360} \times \left(\frac{185}{1.028889} + \frac{182}{1.070502} + \frac{183}{1.114035} + \frac{181}{1.158844} \right) + \frac{1}{1.158844} \right] \\
 &= \$1,002,569.65
 \end{aligned}$$

```

.08 ENTER 130 × 360 ÷ 1 + STO1
.08 ENTER 182 × 360 ÷ 1 + × STO2
.08 ENTER 183 × 360 ÷ 1 + × STO3
.08 ENTER 181 × 360 ÷ 1 + × STO4
185 ENTER RCL1 ÷
182 ENTER RCL2 ÷ +
183 ENTER RCL3 ÷ +
181 ENTER RCL4 ÷ +
.075 × 360 ÷
RCL4 □ 1/x ÷ +
1,000,000 ×

```

35. Using middle-market rates:

$$\left(\frac{1 + 0.1006 \times \frac{273}{360}}{1 + 0.09935 \times \frac{91}{360}} - 1 \right) \times \frac{360}{(273 - 91)} = 9.87\%$$

As we have used middle rates, we need to add approximately 0.06% to give a rate which can be settled against LIBOR:

$$9.87\% + 0.06\% = 9.93\% \text{ middle FRA}$$

```

.1006 ENTER 273 × 360 ÷ 1 +
.09935 ENTER 91 × 360 ÷ 1 ÷
1 - 360 × 273 ENTER 91 - ÷

```

An only slightly different result can be calculated by using the offer side of the 3-month and 6-month rates instead of middle rates, and then not adding 0.06%.

36. a. Sell the FRA to cover the deposit which will need to be rolled over

b. 3 v 6 FRA at 7.10%

c. Ideally, cover the maturing amount of the 3-month deposit:

$$\text{DEM 5 million} \times \left(1 + 0.0675 \times \frac{91}{360} \right) = \text{DEM 5,085,312.50}$$

$$\text{d. } \frac{\text{DEM 5,085,312.50} \times (7.10\% - 6.90\%) \times \frac{92}{360}}{\left(1 + 0.069 \times \frac{92}{360} \right)}$$

$$= \text{DEM 2,554.12 received by you (the FRA seller)}$$

.071 ENTER .069 - 92 × 360 ÷ 5,085,312.50 ×
 .069 ENTER 92 × 360 ÷ 1 + ÷

- e. Roll over the maturing deposit plus FRA settlement amount at LIBID to give:

$$\text{DEM } (5,085,312.50 + 2,554.12) \times \left(1 + 0.0685 \times \frac{92}{360}\right)$$

$$= \text{DEM } 5,176,932.55$$

$$\text{Total borrowing repayment} = \text{DEM } 5 \text{ million} \times \left(1 + 0.07 \times \frac{183}{360}\right)$$

$$= \text{DEM } 5,177,916.67$$

$$\text{Net loss} = \text{DEM } 984.12$$

2,554.12 ENTER 5,085,312.5 + .0685 ENTER 92 × 360 ÷ 1 + ×
 .07 ENTER 183 × 360 ÷ 1 + 5,000,000 × -

37. Selling a futures contract implies **expecting interest rates to rise**

$$\text{Profit / loss} = \text{size of contract} \times \frac{\text{change in price}}{100} \times \frac{1}{4}$$

$$= \text{USD1 million} \times \frac{(94.20 - 94.35)}{100} \times \frac{1}{4} = \text{USD } 375 \text{ loss}$$

38. Create a strip from the implied forward interest rates for each 3-month period:

$$\left[\left(1 + 0.0455 \times \frac{92}{360}\right) \times \left(1 + 0.048 \times \frac{91}{360}\right) \times \left(1 + 0.0495 \times \frac{90}{360}\right) - 1 \right] \times \frac{360}{273}$$

$$= 4.82\%$$

.0455 ENTER 92 × 360 ÷ 1 +
 .048 ENTER 91 × 360 ÷ 1 + ×
 .0495 ENTER 90 × 360 ÷ 1 + ×
 1 - 360 × 273 ÷

39. The FRA rates implied by the futures prices are:

3 v 6 (92 days): 4.55%
 6 v 9 (91 days): 4.80%
 9 v 12 (90 days): 4.95%

$$\text{The implied 3 v 9 rate is } \left[\left(1 + 0.0455 \times \frac{92}{360}\right) \times \left(1 + 0.048 \times \frac{91}{360}\right) - 1 \right] \times \frac{360}{183}$$

$$= 4.7021\%$$

.0455 ENTER 92 × 360 ÷ 1 + .048 ENTER 91 × 360 ÷ 1 + ×
 1 - 360 × 183 ÷

The implied 6 v 12 rate is $\left[\left(1 + 0.048 \times \frac{91}{360} \right) \times \left(1 + 0.0495 \times \frac{90}{360} \right) - 1 \right] \times \frac{360}{181}$
 $= 4.9044\%$

.048 ENTER 91 × 360 ÷ 1 + .0495 ENTER 90 × 360 ÷ 1 + ×
 1 - 360 × 181 ÷

Interpolation gives:

$$3 \text{ v } 7 \text{ rate} = 3 \text{ v } 6 \text{ rate} + (3 \text{ v } 9 \text{ rate} - 3 \text{ v } 6 \text{ rate}) \times \frac{30}{91} = 4.6001\%$$

$$6 \text{ v } 10 \text{ rate} = 6 \text{ v } 9 \text{ rate} + (6 \text{ v } 12 \text{ rate} - 6 \text{ v } 9 \text{ rate}) \times \frac{31}{90} = 4.8360\%$$

Further interpolation gives:

$$4 \text{ v } 8 \text{ rate} = 3 \text{ v } 7 \text{ rate} + (6 \text{ v } 10 \text{ rate} - 3 \text{ v } 7 \text{ rate}) \times \frac{30}{92} = 4.68\%$$

For the hedge, use effectively the same construction by interpolation, but in “round amounts” because futures contracts can be traded only in standardized sizes:

$$\begin{aligned} \text{FRA } 4 \text{ v } 8 \text{ is equivalent to } & \frac{2}{3}(\text{FRA } 3 \text{ v } 7) + \frac{1}{3}(\text{FRA } 6 \text{ v } 10) \\ &= \frac{2}{3} \left(\frac{2}{3}(\text{FRA } 3 \text{ v } 6) + \frac{1}{3}(\text{FRA } 3 \text{ v } 9) \right) + \frac{1}{3} \left(\frac{2}{3}(\text{FRA } 6 \text{ v } 9) + \frac{1}{3}(\text{FRA } 6 \text{ v } 12) \right) \\ &= \frac{4}{9}(\text{FRA } 3 \text{ v } 6) + \frac{2}{9}(\text{FRA } 3 \text{ v } 6 + \text{FRA } 6 \text{ v } 9) + \frac{2}{9}(\text{FRA } 6 \text{ v } 9) + \frac{1}{9}(\text{FRA } 6 \text{ v } 9 + \text{FRA } 9 \text{ v } 12) \\ &= \frac{6}{9}(\text{FRA } 3 \text{ v } 6) + \frac{5}{9}(\text{FRA } 6 \text{ v } 9) + \frac{1}{9}(\text{FRA } 9 \text{ v } 12) \end{aligned}$$

Therefore for each USD9 million FRA sold to the customer, you sell 6 June futures, 5 September futures and 1 December futures. When the June futures contracts expire, replace them by 6 September futures. This will create a basis risk, hedged by selling a further 6 September futures and buying 6 December futures, giving a net position then of short 17 September futures and long 5 December futures.

40. Forward-forward borrowing costs created from the cash market are as follows:

$$3 \text{ v } 6: \left[\frac{\left(1 + 0.0475 \times \frac{183}{360} \right)}{\left(1 + 0.045 \times \frac{92}{360} \right)} - 1 \right] \times \frac{360}{91} = 4.9549\%$$

$$6 \text{ v } 9: \left[\frac{\left(1 + 0.049 \times \frac{273}{360} \right)}{\left(1 + 0.046 \times \frac{183}{360} \right)} - 1 \right] \times \frac{360}{90} = 5.3841\%$$

$$3 \text{ v } 9: \left[\frac{\left(1 + 0.049 \times \frac{273}{360} \right)}{\left(1 + 0.045 \times \frac{92}{360} \right)} - 1 \right] \times \frac{360}{181} = 5.0453\%$$

.0475 ENTER 183 × 360 ÷ 1 + .045 ENTER 92 × 360 ÷ 1 + ÷ 1 - 360 × 91 ÷	(3 v 6)
.049 ENTER 273 × 360 ÷ 1 + .046 ENTER 183 × 360 ÷ 1 + ÷ 1 - 360 × 90 ÷	(6 v 9)
.049 ENTER 273 × 360 ÷ 1 + .045 ENTER 92 × 360 ÷ 1 + ÷ 1 - 360 × 181 ÷	(3 v 9)

3 v 6 period

Forward-forward: 4.9459%

FRA: 4.96%

September futures (100 - 95.03): 4.97%

} forward-forward is cheapest

6 v 9 period

Forward-forward: 5.3841%

FRA: 5.00%

December futures (100 - 95.05): 4.95%

} December futures is cheapest

3 v 9 period

Forward-forward: 5.0453%

FRA: 5.07%

} forward-forward is cheaper

Combining the 3 v 6 forward-forward and December futures gives a cost of:

$$\left[\left(1 + 0.049459 \times \frac{91}{360} \right) \times \left(1 + 0.0495 \times \frac{90}{360} \right) - 1 \right] \times \frac{360}{181} = 4.98\%$$

.049459 ENTER 91 × 360 ÷ 1 + .0495 ENTER 90 × 360 ÷ 1 + × 1 - 360 × 181 ÷
--

This is cheaper than the 3 v 9 forward-forward. Therefore the cheapest cover is provided by borrowing for 6 months at 4.75% and depositing for 3 months at 4.50% to create a 3 v 6 forward-forward, and selling 10 December futures at 95.05. (This ignores any balance sheet costs of the forward-forward.)

41. Using HP calculator: 7.23%

FIN TVM
15 N
102.45 +/- PV
7.5 PMT
100 FV
I%YR

42. Cashflows remaining are FRF 8 million after 1 year, FRF 8 million after 2 years and FRF 108 million after 3 years. Discount to NPV at 7.0%:

$$\begin{aligned} \text{Cost} &= \text{FRF } \frac{8,000,000}{(1 + 0.07)} + \frac{8,000,000}{(1 + 0.07)^2} + \frac{108,000,000}{(1 + 0.07)^3} \\ &= \text{FRF } 102,624,316.04 \end{aligned}$$

8,000,000 ENTER 1.07 ÷
 1.07 ENTER 2 $\square \wedge$ 8,000,000 $\square x \geq y \div +$
 1.07 ENTER 3 $\square \wedge$ 108,000,000 $\square x \geq y \div +$

$$\text{Current yield} = \frac{8}{102.62} = 7.80\%$$

$$\text{Simple yield to maturity} = \frac{8 + \frac{(100 - 102.62)}{3}}{102.62} = 6.94\%$$

8 ENTER 102.62 ÷ (current yield)
 100 ENTER 102.62 - 3 ÷ 8 + 102.62 ÷ (simple yield to maturity)

$$\text{Duration} = \frac{\frac{8}{1.07} \times 1 + \frac{8}{(1.07)^2} \times 2 + \frac{108}{(1.07)^3} \times 3}{102.62} \text{ years} = 2.79 \text{ years}$$

8 ENTER 1.07 ÷
 1.07 ENTER 2 $\square \wedge$ 8 $\square x \geq y \div 2 \times +$
 1.07 ENTER 3 $\square \wedge$ 108 $\square x \geq y \div 3 \times +$
 102.62 ÷

43. Clean price =

$$\frac{3.4}{\left(1 + \frac{0.074}{2}\right)^{\frac{134}{181}}} + \frac{3.4}{\left(1 + \frac{0.074}{2}\right)^{1 + \frac{134}{181}}} + \frac{3.4}{\left(1 + \frac{0.074}{2}\right)^{2 + \frac{134}{181}}} + \dots + \frac{103.4}{\left(1 + \frac{0.074}{2}\right)^{14 + \frac{134}{181}}} - \left(6.8 \times \frac{46}{360}\right)$$

The price is **96.64**. The accrued coupon amount is **43,444.44**.

This question is complicated by the fact that the calculation bases for accrued coupon and price are different. To make the calculation using the HP it is therefore necessary to:

- Calculate the clean price (96.6261) and the accrued interest (0.8829) using HP, assuming that both are calculated on an ACT/ACT basis.
 $96.621 + 0.8829 = 97.5090$
- Add together to give the correct dirty price.
- Re-calculate the accrued interest on the correct 30/360 basis.
 $6.8 \times \frac{46}{360} = 0.8689$
- Subtract this from the dirty price to give the correct clean price.
 $97.5090 - 0.8689 = 96.64$

FIN BOND	
TYPE A/A SEMI EXIT	
14.111997 SETT	
28.032005 MAT	
6.8 CPN%	
MORE 7.4 YLD%	
PRICE	(Clean price assuming ACT/ACT accrued)
ACCRU	(Accrued coupon assuming ACT/ACT)
+	(Actual dirty price)

MORE TYPE 360 EXIT	
MORE ACCRU	(Actual accrued coupon)
-	(Actual clean price)
ACCRU 100 ÷ 5,000,000 ×	(Amount of accrued coupon)

Or, using the bond price formula rather than the functions built into the HP calculator:

$$\text{Dirty price} = \frac{100}{\left(1 + \frac{0.074}{2}\right)^{\frac{134}{181}}} \left[\frac{0.068}{2} \times \frac{\left(1 - \frac{1}{\left(1 + \frac{0.074}{2}\right)^{15}}\right)}{\left(1 - \frac{1}{\left(1 + \frac{0.074}{2}\right)}\right)} + \frac{1}{\left(1 + \frac{0.074}{2}\right)^{14}} \right]$$

$$= 97.5090$$

$$\text{Accrued coupon amount} = 5,000,000 \times 0.068 \times \frac{46}{360} = 43,444.44$$

TIME CALC	
28.091997 DATE1	
14.111997 DATE2 360D	(30/360 days accrued coupon)
28.031998 DATE2 DAYS	(ACT/ACT days in coupon period)
14.111997 DATE1 DAYS	(ACT/ACT days to next coupon)
.074 ENTER 2 ÷ 1 + 14 □ ∧ □ 1/x	
.068 ENTER 2 ÷ +	
.068 ENTER .074 ÷ +	
.074 ENTER 2 ÷ 1 + 14 □ ∧ .074 x .068 □ x ≥ y ÷ -	
100 ×	
.074 ENTER 2 ÷ 1 +	
134 ENTER 181 ÷ □ ∧ ÷	(Dirty price)
6.8 ENTER 46 x 360 ÷	(Accrued coupon)
-	(Clean price)

44. The yield is 7.7246%. If you use the HP calculator's bond functions, it is again necessary to make an adjustment for the fact that the price / yield calculation is on a 30/360 basis but the accrued coupon is on an ACT/365 basis as follows:

- Calculate the correct accrued interest: $8.3 \times \frac{129}{365} = 2.933425$
- Add to the clean price to give the correct dirty price:
 $102.48 + 2.933425 = 105.413425$

- Calculate the accrued coupon as if it were on a 30/360 basis:

$$8.3 \times \frac{127}{360} = 2.928056$$

- Subtract from the dirty price to give an adjusted clean price:

$$105.413425 - 2.928056 = 102.485369$$

FIN BOND	
TYPE 360 ANN EXIT	
20.101997 SETT	
13.062003 MAT	
8.3 CPN%	
EXIT EXIT TIME CALC	
13.061997 DATE1	
20.101997 DATE2 DAYS	(ACT/365 days accrued coupon)
365 ÷ 8.3 x	(Accrued coupon)
102.48 +	(Dirty price]
EXIT EXIT FIN BOND	
MORE ACCRU	(Accrued coupon on 30/360 basis)
-	(Clean price assuming 30/360 accrued)
PRICE	
YLD%	

45. The last quasi-coupon date was 27 March 1998
 The next quasi-coupon date is 27 September 1998
 The quasi-coupon period is 184 days
 The fraction of a period (ACT/ACT) from settlement to 27 September 1998 is $\frac{69}{184}$.

$$\text{Therefore } 65.48 = \frac{100}{\left(1 + \frac{i}{2}\right)^{\left(\frac{69}{184} + 15\right)}}$$

$$\text{Therefore } i = \left[\left(\frac{100}{65.48} \right)^{\frac{1}{\left(\frac{69}{184} + 15\right)}} - 1 \right] \times 2 = 5.5845\%$$

100 ENTER 65.48 ÷
 69 ENTER 184 ÷ 15 + □ 1/x □ ^
 1 - 2 ×

OR

FIN BOND
 TYPE A/A SEMI EXIT

20.071998 SETT
27.032006 MAT
0 CPN%
MORE 65.48 PRICE
YLD%

$$46. \frac{8.0\%}{1 - 0.08 \times \frac{97}{360}} \times \frac{365}{360} = 8.29\%$$

.08 ENTER 97 × 360 ÷ 1 - +/- 8 □ $x \geq y$ ÷ 365 × 360 ÷

47. With the same notation as before, the number of days from purchase to maturity is 205:

$$i = \frac{-\frac{205}{365} + \left(\left(\frac{205}{365} \right)^2 + 2 \left(\frac{205}{365} - \frac{1}{2} \right) \times \left(\frac{1}{\left(1 - 0.08 \times \frac{205}{360} \right)} - 1 \right) \right)^{\frac{1}{2}}}{\left(\frac{205}{365} - \frac{1}{2} \right)} = 8.46\%$$

.08 ENTER 205 × 360 ÷ 1 - +/- □ $1/x$ 1 -
205 ENTER 365 ÷ .5 - × 2 ×
205 ENTER 365 ÷ □ x^2 + □ \sqrt{x}
205 ENTER 365 ÷ -
205 ENTER 365 ÷ .5 - ÷

48. The same equation as in the previous question can be manipulated to give:

$$\begin{aligned} D &= \frac{360}{\text{days}} \left(1 - \frac{2}{i^2 \times \left(\frac{\text{days}}{365} - \frac{1}{2} \right) + 2i \times \frac{\text{days}}{365} + 2} \right) \\ &= \frac{360}{205} \left(1 - \frac{2}{(0.09)^2 \times \left(\frac{205}{365} - \frac{1}{2} \right) + 2 \times 0.09 \times \frac{205}{365} + 2} \right) \\ &= 8.49\% \end{aligned}$$

205 ENTER 365 ÷ .5 - .09 × .09 ×
2 ENTER .09 × 205 × 365 ÷ +
2 +
2 □ $x \geq y$ ÷ 1 - +/-
360 × 205 ÷

49. The HP bond calculation function cannot be used for a bond with stepped coupons. The easiest method is to work from first principles, as follows:

- Discount the final cashflow of 105.75 to a value one year earlier:

$$\frac{105.75}{1.0524} = 100.4846$$

- Add the coupon cashflow of 5.50 paid then and discount back a further year:

$$\frac{100.4846 + 5.50}{1.0524} = 100.7075$$

- Repeat the process back to 3 March 1998:

$$\frac{100.7075 + 5.25}{1.0524} = 100.6818$$

$$\frac{100.6818 + 5.00}{1.0524} = 100.4198$$

$$\frac{100.4198 + 4.75}{1.0524} = 99.9333$$

$$99.9333 + 4.5 = 104.4333$$

- Discount the value of 104.4333 back to settlement date (112 days on a 30/360 basis) to give the current dirty price:

$$\frac{104.4333}{(1.0524)^{\frac{112}{360}}} = 102.7870$$

- Subtract the accrued interest (248 days on a 30/360 basis) to give the clean price:

$$102.7870 - 4.5 \times \frac{248}{360} = 99.69$$

105.75 ENTER 1.0524 ÷	
5.5 + 1.0524 ÷	
5.25 + 1.0524 ÷	
5 + 1.0524 ÷	
4.75 + 1.0524 ÷	
4.5 +	
1.0524 ENTER 112 ENTER 360 ÷ □ ∧ ÷	(Dirty price)
248 ENTER 360 ÷ 4.5 × -	(Clean price)

50. Because bond price / yield formulas generally assume a redemption amount of 100, a straightforward method is to scale down every cashflow by the same factor to correspond to a redemption amount of 100. Thus for each $(\frac{100}{1.10})$ nominal amount of bond, the price paid is $(\frac{98}{1.10})$, the

coupons paid are $(\frac{3.3}{1.10})$ and the redemption payment is 100. This gives a yield of **4.39%**.

Using the HP bond function, however, it is possible to achieve the answer more simply by entering 110 as the “call value.”

FIN BOND TYPE 360 ANN EXIT
8.121997 SETT
20.092007 MAT
3.3 ENTER 1.1 ÷ CPN%
98 ENTER 1.1 ÷ MORE PRICE
YLD%

OR

FIN BOND TYPE 360 ANN EXIT
8.121997 SETT
20.092007 MAT
3.3 CPN%
110 CALL
MORE 98 PRICE
YLD%

(Remember afterwards to reset the call value to 100 for future calculations!)

$$\text{All-in initial cost} = 98 + 3.3 \times \frac{78}{360} = 98.715$$

$$\text{All-in sale proceeds} = 98.50 + 3.3 \times \frac{85}{360} = 99.279$$

$$\text{Simple rate of return (ACT/360)} = \left(\frac{99.279}{98.715} - 1 \right) \times \frac{360}{7} = 29.39\%$$

$$\text{Effective rate of return (ACT/365)} = \left(\frac{99.279}{98.715} \right)^{\frac{365}{7}} - 1 = 34.60\%$$

3.3 ENTER 85 x 360 ÷ 98.5 +
3.3 ENTER 78 x 360 ÷ 98 + ÷ STO 1
1 - 360 x 7 ÷
RCL1 365 ENTER 7 ÷ □ ^ 1 -

(Simple rate)
(Effective rate)

51. a. Last coupon 7 June 1997

$$\text{ACT/365 basis: } \frac{51}{365} \times 7.5 = 1.047945$$

b. Last coupon 15 February 1997

Next coupon 15 August 1997 (181-day coupon period)

$$\text{ACT/ACT basis: } \frac{163}{362} \times 5.625 = 2.532804$$

c. Last coupon 26 October 1996

$$30/360 \text{ basis: } \frac{272}{360} \times 6.25 = 4.722222$$

d. Last coupon 25 October 1996

Next coupon 25 October 1997 (365-day coupon period)

$$\text{ACT/ACT basis: } \frac{276}{365} \times 7.25 = 5.482192$$

e. Last coupon 20 March 1997

$$\text{ACT/365 basis: } \frac{130}{365} \times 3.00 = 1.068493$$

f. Last coupon 15 November 1996

$$30/360 \text{ basis: } \frac{253}{360} \times 7.00 = 4.919444$$

g. Last coupon 28 October 1996

Next coupon 28 October 1997 (365-day coupon period)

$$\text{ACT/ACT basis: } \frac{273}{365} \times 8.80 = 6.581918$$

h. Last coupon 1 February 1997

$$30/360 \text{ basis plus 1 extra day: } \frac{178}{360} \times 9.50 = 4.697222$$

52. The futures price would be **less than 100**. Between now and delivery of the futures contract, the purchaser of the futures contract is earning a lower yield on the money market than the coupon he would earn by buying the bond in the cash market. The price is therefore lower to compensate.

53. Payment for the bond purchased by the futures seller to hedge himself is made on 25 April. Coupon on the purchase of the bond is accrued for 112 days. Therefore:

$$\text{Accrued coupon now} = 7.375 \times \frac{112}{360} = 2.294444$$

Delivery of the bond to the futures buyer would require payment to the futures seller on 10 September. The futures seller must therefore fund his position from 25 April to 10 September (138 actual days) and coupon on the bond on 10 September will be accrued for 247 days. Therefore:

$$\text{Accrued coupon then} = 7.375 \times \frac{247}{360} = 5.060069$$

Theoretical futures price =

$$\frac{(106.13 + 2.294444) \times \left(1 + 0.0335 \times \frac{138}{360}\right) - 5.060069}{1.1247} = 93.14$$

7.375 ENTER 112 x 360 ÷ (Accrued coupon now)
 106.13 +
 .0335 ENTER 138 x 360 ÷ 1 + x
 7.375 ENTER 247 x 360 ÷ (Accrued coupon in September)
 - 1.1247 ÷

54. Implied repo rate = $\left[\frac{(93.10 \times 1.1247) + 5.060069}{(106.13 + 2.294444)} - 1 \right] \times \frac{360}{138} = 3.24\%$

93.1 ENTER 1.1247 x 5.060069 +
 106.13 ENTER 2.294444 + ÷
 1 - 360 x 138 ÷

55. Assume to start with that the implied repo rate is higher than the actual current repo rate.

Cost of buying CTD bond per DEM 100 nominal is (clean price + accrued coupon) = DEM (102.71 + 3.599) = DEM 106.309

Total borrowing (principal + interest) to be repaid at the end

$$= \text{DEM } 106.309 \times \left(1 + 0.068 \times \frac{24}{360}\right) = \text{DEM } 106.790934$$

Anticipated receipt from selling futures contract and delivering bond per DEM 100 nominal = (futures price × conversion factor) + accrued coupon

$$= \text{DEM } (85.31 \times 1.2030) + 4.191 = \text{DEM } 106.818930$$

Profit = DEM (106.818930 - 106.790934) = 0.027996 per DEM 100 nominal

Size of DEM bond futures contract is DEM 250,000 nominal

Therefore face value of bond purchased against each futures contract is

$$\text{DEM } \frac{250,000}{1.2030} = \text{DEM } 207,814$$

Therefore profit per futures contract

$$= \text{DEM } 0.027996 \times \frac{207,814}{100} = \text{DEM } 58.18$$

56.	<i>Bond A</i>	<i>Bond B</i>	<i>Bond C</i>
Yield (from HP):	7.92%	7.17%	7.52%
Duration:	4.41	2.31	3.61
Modified duration = $\frac{\text{duration}}{(1+\text{yield})}$:	4.09	2.16	3.36
Accrued coupon (from HP):	0.50	4.80	5.00
Clean price:	88.50	111.00	94.70
Dirty price (= clean price + accrued):	89.00	115.80	99.70
Face value:	10 million	5 million	15 million
Total value (= face value $\times \frac{\text{dirty price}}{100}$):	8.9 million	5.79 million	14.955 million

$$\text{modified duration of portfolio} \approx \frac{\sum (\text{modified duration} \times \text{value})}{\text{portfolio value}}$$

$$= \frac{4.09 \times 8.9 + 2.16 \times 5.79 + 3.36 \times 14.955}{8.9 + 5.79 + 14.955} = 3.34$$

$$\begin{aligned} \text{Change in value} &\approx -\text{change in yield} \times \text{modified duration} \times \text{total value} \\ &= -0.001 \times 3.34 \times 29.645 \text{ million} = -99,014 \end{aligned}$$

57.	<i>Bond A</i>	<i>Bond B</i>
Yield (from HP):	7.92%	8.92%
Duration:	4.41	7.56
Modified duration = $\frac{\text{duration}}{(1+\text{yield})}$:	4.09	6.94
Accrued coupon (from HP):	0.50	3.50
Clean price:	88.50	107.50
Dirty price (= clean price + accrued):	89.00	111.00

For an increase of say 1 basis point in yield, the change in value of bond A is:

$$\begin{aligned} &-0.0001 \times \text{modified duration of bond A} \times \text{face value of bond A} \\ &\times \frac{\text{dirty price of bond A}}{100} \\ &= -0.0001 \times 4.09 \times 10 \text{ million} \times 0.89 = -3,640 \end{aligned}$$

For an increase of 1 basis point in yield, the change in value of bond B is:

$$\begin{aligned} &-0.0001 \times \text{modified duration of bond B} \times \text{face value of bond B} \\ &\times \frac{\text{dirty price of bond B}}{100} \end{aligned}$$

$$= -0.0001 \times 6.94 \times \text{face value} \times 1.1100 = -0.0007703 \times \text{face value}$$

Therefore face value of bond B required for a hedge is $\frac{3,640}{0.0007703}$

$$= 4,725,186$$

The ratio for using futures to hedge a position in the CTD bond is:

$$\frac{\text{conversion factor}}{(1 + \text{funding cost} \times \frac{\text{days}}{\text{year}})} = \frac{1.2754}{(1 + 0.10 \times \frac{32}{360})} = 1.2642$$

Therefore notional value of futures required to hedge 10 million in bond A is:

$$4,725,186 \times 1.2642 = 5,973,580$$

As each contract has a notional size of 100,000, you need **60 contracts**.

58. Bootstrap to create 2-year zero-coupon yield:

Year			Net flows
0	-97.700	$+\frac{9}{1.1}$	-89.518
1	+9.000	-9.000	
2	+109.000		+109.000

2-year zero-coupon yield is $\left(\frac{109}{89.518}\right)^{\frac{1}{2}} - 1 = 10.35\%$

2-year discount factor is $\frac{89.518}{109} = 0.8213$

Bootstrap to create 3-year zero-coupon yield:

Year				Net flows
0	-90.900	$+\frac{7}{1.1}$	$+(7 \times 0.8213)$	-78.787
1	+7.000	-7.000		
2	+7.000		-7.000	
3	+107.000			+107.000

3-year zero-coupon yield is $\left(\frac{107}{78.787}\right)^{\frac{1}{3}} - 1 = 10.74\%$

3-year discount factor is $\frac{78.787}{107} = 0.7363$

Bootstrap to create 4-year zero-coupon yield:

Year					Net flows
0	-99.400	$+\frac{11}{1.1}$	$+(11 \times 0.8213)$	$+(11 \times 0.7363)$	-72.266
1	+11.000	-11.000			
2	+11.000		-11.000		
3	+11.000			-11.000	
4	+111.000				+111.000

4-year zero-coupon yield is $\left(\frac{111}{72.266}\right)^{\frac{1}{4}} - 1 = 11.33\%$

4-year discount factor is $\frac{72.266}{111} = 0.6510$

1-year discount factor is $\frac{1}{1.10} = 0.9091$

1 year v 2 year forward-forward = $\frac{1\text{-year discount factor}}{2\text{-year discount factor}} - 1$
 $= \frac{0.9091}{0.8213} - 1 = 10.69\%$

2 year v 3 year forward-forward = $\frac{0.8213}{0.7363} - 1 = 11.54\%$

3 year v 4 year forward-forward = $\frac{0.7363}{0.6510} - 1 = 13.10\%$

59.

a Strip to create zero-coupon yields:

2-year: $(1.08 \times 1.0824)^{\frac{1}{2}} - 1 = 8.12\%$

3-year: $(1.08 \times 1.0824 \times 1.09)^{\frac{1}{3}} - 1 = 8.41\%$

4-year: $(1.08 \times 1.0824 \times 1.09 \times 1.095)^{\frac{1}{4}} - 1 = 8.68\%$

Discount factors are:

1-year: $\frac{1}{1.08} = 0.9259$

2-year: $\frac{1}{(1.0812)^2} = 0.8554$

3-year: $\frac{1}{(1.0841)^3} = 0.7849$

4-year: $\frac{1}{(1.0868)^4} = 0.7168$

If i is the 2-year par yield, then:

$i \times 0.9259 + (1 + i) \times 0.8554 = 1$

Therefore $i = \frac{1 - 0.8554}{0.9259 + 0.8554} = 8.12\%$

Similarly, 3-year par yield = $\frac{1 - 0.7849}{0.9259 + 0.8554 + 0.7849} = 8.38\%$

4-year par yield = $\frac{1 - 0.7168}{0.9259 + 0.8554 + 0.7849 + 0.7168}$
= 8.63%

- b. Discounting each cashflow at a zero-coupon yield, the price of the 4-year bond is:

$(12 \times 0.9259) + (12 \times 0.8554) + (12 \times 0.7849) + (112 \times 0.7168) = 111.076$

Using the TVM function of the calculator, this gives the yield to maturity as 8.61%.

60. Bootstrap using middle rates to create 18-month discount factor:

Time				Net flows
0	-100	$\frac{+8.85 \times \frac{183}{360}}{(1 + 0.0875 \times \frac{365}{360})}$	$\frac{+8.85 \times \frac{182}{360}}{(1 + 0.0865 \times \frac{182}{360})}$	-91.581
6	$+8.85 \times \frac{182}{360}$		$-8.85 \times \frac{182}{360}$	
12	$+8.85 \times \frac{183}{360}$	$-8.85 \times \frac{183}{360}$		
18	$+100 + 8.85 \times \frac{183}{360}$			+104.499

18-month discount factor is $\frac{91.581}{104.499} = 0.8764$

Bootstrap to create 24-month discount factor:

Time					Net flows
0	-100	$+8.95 \times \frac{183}{360} \times 0.8764$	$\frac{+8.95 \times \frac{183}{360}}{(1 + 0.0875 \times \frac{365}{360})}$	$\frac{+8.95 \times \frac{182}{360}}{(1 + 0.0865 \times \frac{182}{360})}$	-87.499
6	$+8.95 \times \frac{182}{360}$			$-8.95 \times \frac{182}{360}$	
12	$+8.95 \times \frac{183}{360}$		$-8.95 \times \frac{183}{360}$		
18	$+8.95 \times \frac{183}{360}$	$-8.95 \times \frac{183}{360}$			
24	$+100 + 8.95 \times \frac{182}{360}$				+104.525

24-month discount factor is $\frac{87.499}{104.525} = 0.8371$

$18 \text{ v } 24 \text{ FRA} = \left[\frac{\frac{1}{0.8371}}{\frac{1}{0.8764}} - 1 \right] \times \frac{360}{183} = 9.24\%$

Assuming FRA is benchmarked against LIBOR, add .05% (half the bid-offer spread) to this calculation: 9.29%

61. Remember that Eurosterling is on a 365-day basis and EuroDeutsche marks are on a 360-day basis.

$$\text{Middle swap price} = 2.5585 \times \frac{(0.09 \times \frac{365}{360} - 0.13 \times \frac{365}{365})}{(1 + 0.13 \times \frac{365}{365})}$$

$$= -0.0877$$

= 877 points Deutschemark premium

$$\begin{aligned} &.09 \text{ ENTER } 365 \times 360 \div .13 - \\ &1.13 \div 2.5585 \times \end{aligned}$$

62. a. $5.1020 \div 1.5145 = 3.3688$
 $5.1040 \div 1.5140 = 3.3712$
 Spot DEM/FRF is **3.3688 / 3.3712**.
 Customer buys FRF at **3.3688**.

- b. $1.9490 \times 5.1020 = 9.9438$
 $1.9500 \times 5.1040 = 9.9528$
 Spot GBP/FRF is **9.9438 / 9.9528**.
 Customer sells GBP at **9.9438**.

- c. $5.1020 / 5.1040$
 $246 / 259$

5.1266 / 5.1299 USD/FRF 3 months forward outright.

- d. $1.9490 / 1.9500$
 $268 / 265$

1.9222 / 1.9235 GBP/USD 3 months forward outright.

- e. $1.9222 \times 5.1266 = 9.8544$
 $1.9235 \times 5.1299 = 9.8674$
 3 months forward outright GBP/FRF is **9.8544 / 9.8674**
GBP interest rates are higher than FRF rates because sterling is worth fewer FRF forward than spot.

- f. $1.5140 / 1.5145$
 $29 / 32$

1.5169 / 1.5177 USD/DEM 3 months forward outright
 $5.1266 \div 1.5177 = 3.3779$
 $5.1299 \div 1.5169 = 3.3818$
 3 months forward outright DEM/FRF is **3.3779 / 3.3818**
FRF interest rates are higher than DEM rates because the DEM is worth more FRF forward than spot.

- g. Outright **3.3779 / 3.3818**
 Spot $3.3688 / 3.3712$

91 / 106 DEM/FRF 3 months forward swap.

63. a. $\begin{array}{r} 5.1020 \\ 18 \end{array} / \begin{array}{r} 5.1040 \\ 20 \end{array}$ Spot
S/W
 $\underline{5.1038 / 5.1060}$ USD/FRF 1 week outright.
- b. $\begin{array}{r} 5.1020 \\ 2.3 \end{array} / \begin{array}{r} 5.1040 \\ 2.9 \end{array}$ Spot
T/N
 $\underline{5.10171 / 5.10377}$ USD/FRF tomorrow outright.
Before spot, use the opposite side of the forward swap price.
- c. $\begin{array}{r} 5.1020 \\ 2.3 \\ 2.0 \end{array} / \begin{array}{r} 5.1040 \\ 2.9 \\ 2.5 \end{array}$ Spot
T/N
O/N
 $\underline{5.10146 / 5.10357}$ USD/FRF today outright.
Customer buys FRF value today at **5.10146**.
- d. $\begin{array}{r} 1.9490 \\ 3.5 \\ 10.6 \end{array} / \begin{array}{r} 1.9500 \\ 3.3 \\ 10.1 \end{array}$ Spot
T/N
O/N
 $\underline{1.95034 / 1.95141}$ GBP/USD today outright.
Customer buys GBP value today at **1.95141**.
- e. $\begin{array}{r} 96 / 94 \\ 23 / 22 \end{array}$ 1-month swap
1-week swap
 $\underline{74 / 71}$ 1 week against 1 month forward-forward swap GBP/USD.
Customer “buys and sells” GBP at **74**.

The forward-forward price is the difference between opposite sides of the prices. The bank buys the base currency on the left on the far date.
- f. $\begin{array}{r} 96 / 94 \\ 3.5 / 3.3 \end{array}$ 1-month swap
T/N swap
 $\underline{99.5 / 97.3}$ Tomorrow against 1 month forward-forward swap.
Customer “buys and sells” GBP at **99.5**.

64. Calculate the USD/FRF and USD/NOK forward outrights as usual:

USD/FRF		USD/NOK
6.26965 / 6.27175	(tomorrow outright)	6.76195 / 6.76398
6.2385 / 6.2425	(3-month outright)	6.7605 / 6.7655
6.2145 / 6.2205	(6-month outright)	6.7670 / 6.7740

Calculating the cross-rate spot, outrights and swaps as usual gives:

NOK/FRF		
(spot)	0.92689 / 0.92746	
(tomorrow outright)	0.92692 / 0.92751	(T/N swap) 0.5 / 0.3
(3-month outright)	0.9221 / 0.9234	(3-month swap) 48 / 40
(6-month outright)	0.9174 / 0.9192	(6-month swap) 95 / 82

(Remember to “reverse” the T/N swap price)

- a. 3 months v 6 months forward-forward price is:

$$(95 - 40) / (82 - 48) - \text{i.e. } 55 / 34$$

The bank buys the base currency (in this case NOK) on the far date on the left side. You therefore deal on a price of 55.

- b. Tomorrow v 3 months forward-forward price is:

$$(48 + 0.5) / (40 + 0.3) - \text{i.e. } 48.5 / 40.3$$

Your customer wishes to buy NOK on the far date, which is the right side. You therefore deal on a price of 40.3. This can be broken down into the two swaps as follows. First, between tomorrow and spot, he is “selling and buying” the base currency (NOK); as the bank is selling the base currency on the far date, this is the right side (minus 0.3). Second, between spot and 3 months, he is again “selling and buying” the base currency (NOK); as the bank is selling the base currency on the far date, this is again the right side (minus 40 points). The combination is (minus 0.3 points) plus (minus 40 points) = (minus 40.3 points).

65. a. Today is Friday 19 April.

Spot is Tuesday 23 April.

Spot-a-week is Tuesday 30 April.

1 month is Thursday 23 May.

2 months is Monday 24 June (23 June is a Sunday).

Number of days from 23 May to 24 June is 32.

Number of days from 23 May to 3 June is 11.

Therefore price for 3 June is:

1-month price + $(\frac{11}{32} \times \text{difference between 2-month price and 1-month price})$

USD/FRF		GBP/USD
5.2590	(Middle spot)	1.9162
25 / 23	(Swap to 30 April)	11 / 9
133 / 119	(Swap to 3 June)	69 / 62

Your customer is “buying and selling” FRF (in that order). The bank buys the variable currency on the far date on the right side. In the USD/FRF swap prices, you therefore need the right side of “133 / 119” but the left side of “25 / 23”. In the GBP/USD prices, however, the customer is “selling and buying” (in that order) the base currency GBP (because GBP is quoted “direct”). Therefore you again need the right side of “69 / 62” and the left side of “11 / 9”.

The GBP/FRF prices you need are therefore as follows:

(outright value 3 June):

$$(5.2590 - 0.0119) \times (1.9162 - 0.0062) = 10.0220$$

(outright value 30 April):

$$(5.2590 - 0.0025) \times (1.9162 - 0.0011) = 10.0667$$

$$\quad \quad \quad - 0.0447$$

The forward-forward price where the customer can buy FRF value 30 April and sell FRF value 3 June is therefore **447 points GBP discount** – that is 447 points in the customer's favour (because he/she is selling on the *far* date the currency which is worth more in the future).

- b. For short dates you need to combine the swaps, because you are rolling an existing contract from today to tomorrow, from tomorrow to the next day and from then until a week later. The total swap from today to a week after spot is therefore:

O/N	– 0.4 / + 0.1
T/N	– 1.5 / – 1
S/W	– 11 / – 9
	<hr/>
	– 12.9 / – 9.9

Your customer needs to sell GBP / buy USD on the far date. The bank buys the base currency (GBP) on the far date on the left, at **12.9 points GBP discount** – that is, 12.9 points against the customer.

OR

Your customer needs to “buy and sell” GBP (in that order) today against tomorrow. The bank buys the base currency (GBP) on the far date on the left side. The prices are all bigger on the left – GBP is at a discount, worth less in the future. As the customer is selling GBP on the far date, 0.4 points will be against him.

Similarly, he needs to “buy and sell” GBP tomorrow against spot – another 1.5 points against him. He also needs to “buy and sell” GBP spot against 1 week – another 11 points against him. The total swap price will therefore be $0.4 + 1.5 + 11 = 12.9$ points against him.

- c. Spot: 1.9157 / 67
 T/N: 1.5 / 1
 Outright value tomorrow: 1.9158 / 1.91685 (“reverse” the short-date)

O/N: –0.4 / +0.1
 Outright value today: 1.91579 / 1.91689 (“reverse” the short-date)

Deal at **1.91689**

66. a. USD/ITL

Spot:	1633.25 / 1634.25
Swap:	22.37 / 22.87
	<hr/>
Outright:	1655.62 / 1657.12

The bank sells the base currency (USD) on the right side, so the outright price quoted is **1657.12**.

The USD is worth more in the future and the ITL is worth less, so the **ITL is at a discount**.

- b. The current interest rates are consistent with the current swap price. Assume that the expected changes do happen, and calculate the effect on the forward outright price, using middle prices for the comparison.

After the rates have moved, they will be as follows:

Spot rates

$$\left. \begin{array}{l} \text{USD/DEM } 1.6150 \\ \text{DEM/ITL } 1005 \end{array} \right\} \text{USD/ITL} = 1.6150 \times 1005 = 1623.08$$

Interest rates

USD 4.50% / 4.625% (middle: 4.5625%)

ITL 9.00% / 9.25% (middle: 9.125%)

$$\text{Middle swap price} = 1623.08 \times \frac{(0.09125 - 0.045625) \times \frac{180}{360}}{(1 + 0.045625 \times \frac{180}{360})} = 36.20$$

The outright middle rate would therefore be $1623.08 + 36.20 = 1659.28$

This is slightly worse than the current outright middle rate of USD/ITL 1656.37, so it is not worth waiting according to these expectations; the improvement in the spot rate has been more than offset by the movement in the swap rate – even though the swap is for a slightly shorter period.

The movement in the swap price could be approximated as follows:

Interest rate differential widens by 1.75%; period is half a year.

Therefore swap price moves by approximately:

$$\text{spot} \times 0.0175 \times \frac{1}{2} = 14.3$$

The customer is selling the currency which is at a discount (worth less in the future) and the discount is increasing, so this movement in the swap price of approximately 14.3 must be against him/her. He/she must therefore expect a spot movement of at least this much in his/her favour for it to be worthwhile waiting.

67. Invest DEM 15 million for 91 days

Investor buys and sells USD (sells and buys DEM) spot against 3 months at 1.6730 and 1.6557

Investor's cashflows spot:

invest DEM 15 million

sell DEM 15 million and buy USD 8,965,929.47 at 1.6730

invest USD 8,965,929.47 in USD CP

CP yields LIBOR + 4 bp = 8.375% + 0.04% = 8.415%

Total proceeds at maturity of CP

$$= \text{USD } 8,965,929.47 \times \left(1 + 0.08415 \times \frac{91}{360}\right)$$

$$= \text{USD } 9,156,646.00$$

Investor sells USD 9,156,646.00 forward at 1.6557 against DEM 15,160,658.77

Investor's cashflows after 3 months:

receive USD 9,156,646.00 from CP

sell USD 9,156,646.00

buy DEM 15,160,658.77

$$\text{Overall DEM return} = \left(\frac{15,160,658.77}{15,000,000.00} - 1\right) \times \frac{360}{91} = 4.24\%$$

This is effectively **DEM LIBOR minus 1 bp**. Assuming that the investor's alternative would be a deposit at DEM LIBID, the covered interest arbitrage is more attractive.

Note that, in practice, the investor could probably not buy USD commercial paper with a face value of USD 9,156,646 (the total maturity proceeds for CP are the same as its face value); he would instead need to purchase a round amount but this would not affect the rate of return.

Note also that the amounts dealt on each leg of the swap are mismatched. The bank would therefore generally wish to base the prices on the left side of the spot price (rather than the middle) because that is the "correct" side for a forward outright for the mismatch difference.

15,000,000 ENTER 1.6730 ÷	(Amount invested)
.08415 ENTER 91 × 360 ÷ 1 + ×	(Maturity proceeds in USD)
1.6557 ×	(Maturity proceeds in DEM)
15,000,000 ÷ 1 - 360 × 91 ÷	(Rate of return)

Alternatively, using the formula for covered interest arbitrage:

variable currency rate =

$$\left[\left(1 + \text{base currency rate} \times \frac{\text{days}}{\text{base year}}\right) \times \frac{\text{outright}}{\text{spot}} - 1\right] \times \frac{\text{variable year}}{\text{days}}$$

$$= \left[\frac{1.6557}{1.6730} \times \left(1 + 0.08415 \times \frac{91}{360}\right) - 1\right] \times \frac{360}{91} = 4.24\%$$

$$.08415 \text{ ENTER } 91 \times 360 \div 1 + 1.6557 \times 1.6730 \div 1 - 360 \times 91 \div$$

68. Action now

(i) Arrange FRA 3 v 9 on a notional 6-month borrowing

$$\text{of USD } \left(\frac{1,000,000}{6.0600}\right) = \text{USD } 165,016.50$$

Assuming FRA settlement at the end of 9 months (rather than discounted after 3 months as is conventional), the total notional repayment on this borrowing would be:

$$\text{USD } 165,016.50 \times \left(1 + 0.0575 \times \frac{182}{360}\right) = \text{USD } 169,813.44$$

(ii) Convert this notional borrowing from USD to SEK:

- sell USD 165,016.50 / buy SEK 1,000,000.00 (at 6.0600) for value 3 months forward
- buy USD 169,813.44 / sell SEK 1,050,839.53 (at 6.1882) for value 9 months forward

Action in 3 months' time

(iii) Assume borrowing of SEK $(165,016.50 \times 6.2060)$

= SEK 1,024,092.40 for 6 months

Total repayment will be:

$$\text{SEK } 1,024,092.40 \times \left(1 + 0.1062 \times \frac{182}{360}\right) = \text{SEK } 1,079,075.92$$

(iv) Convert this borrowing to a notional USD borrowing to match (i):

- buy USD 165,016.50 / sell SEK 1,024,092.40 (at 6.2060) for value spot
- sell USD 170,022.00 / buy SEK 1,078,857.60 (at 6.3454) for value 6 months forward

(USD 170,022.00 is the total repayment which would be due on a 6-month loan of USD 165,016.50 taken at the rate of 6.00% now prevailing.)

Settlement at the end of 9 months

Receive FRA settlement of $\text{USD } 165,016.50 \times (0.06 - 0.0575) \times \frac{182}{360}$

= USD 208.56

	<i>Total flows</i>		
	<i>SEK</i>		<i>USD</i>
After 3 months:	+ 1,024,092.40	(iii)	
	– 1,024,092.40	(iv)	+ 165,016.50
	+ 1,000,000.00	(ii)	– 165,016.50
	<u>+ 1,000,000.00</u>		<u>–</u>
After 9 months:	– 1,079,075.92	(iii)	
	+ 1,078,857.60	(iv)	– 170,022.00
		(i)	+ 208.56
	– 1,050,839.53	(ii)	+ 169,813.44
	<u>– 1,051,057.85</u>		<u>–</u>

$$\text{Effective cost is } \frac{51,057.85}{1,000,000} \times \frac{360}{(273 - 91)} = 10.10\%$$

Note that, in practice, the USD FRA settlement would be received after 3 months on a discounted basis but could then be invested until 9 months. If this investment were at only 5.87% (LIBID), the final result would be changed very slightly.

69. The USD cashflows net to zero both spot and forward.

	DEM cashflows	
	spot	6 months
(a)	-16,510,000	+16,350,000
(b)	+16,495,000	-16,325,000
Net:	- 15,000	+ 25,000

$$\text{NPV} = -\text{DEM}15,000 + \text{DEM} \frac{25,000}{(1 + 0.045 \times \frac{182}{360})} = \text{DEM}9,443.90$$

70. a. DEM cashflows:

3 months	6 months	12 months
+ 10,164,306	+ 10,000,000	- 10,000,000
		- 10,709,722

Valuation in USD at current forward exchange rates:

+ 5,634,316	+ 5,528,527	- 11,397,756
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New rates after USD/DEM moves from 1.80 to 2.00:

	USD/DEM	USD%	DEM%
Spot:	2.0000		
3 months:	2.0045	6.5	7.4
6 months:	2.0098	6.5	7.5
12 months:	2.0189	7.0	8.0

Valuation in USD at new forward rates:

3 months	6 months	12 months
+ 5,070,744	+ 4,975,619	- 10,257,924

Change in valuation in USD:

- 563,572	- 552,908	+ 1,139,832
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Total net profit = USD 23,352

b. The profits / losses discounted to spot become:

3 months	6 months	12 months
- 554,462	- 535,317	+ 1,064,297

Total net present value of profits / losses: – USD 25,482

It is possible to hedge this exposure by a spot transaction of the net present value of the forward DEM positions:

	3 months	6 months	12 months
Actual DEM cashflows:	+ 10,164,306	+ 10,000,000	– 20,709,722
Discounted to PV:	+ 9,977,668	+ 9,634,685	– 19,155,961
Total NPV:	+ 456,392		

This NPV is the amount of DEM which should be sold to achieve a hedge. Suppose that this had been done spot at 1.80. A move to a spot rate of 2.00 would then have produced a profit on the hedge of:

$$\text{USD} \left(\frac{456,392}{1.80} - \frac{456,392}{2.00} \right) = \text{USD } 25,355$$

Allowing for rounding differences, this would offset the loss shown above.

71. As you will be receiving fixed payments under the swap, the spread over treasuries will be 80 basis points (rather than 90). The receipt will therefore be 9.80% on a semi-annual bond basis.

Your cashflows are therefore:

pay:	8.90% annual money market	≈ 8.71% semi-annual money market
receive:	9.80% semi-annual bond	= 9.67% semi-annual money market
pay:	<u>LIBOR semi-annual money market</u>	
net pay:	<u>LIBOR – 96 basis points</u>	

This answer does not discount the cashflows precisely.

72. a. Convert the futures price to implied forward-forward interest rates and then create strips to calculate the effective (annual equivalent) zero-coupon swap rates for each quarterly period up to 18 months:

18-month rate =

$$\left[\left(1 + 0.0625 \times \frac{91}{360} \right) \times \left(1 + 0.0659 \times \frac{91}{360} \right) \times \left(1 + 0.0716 \times \frac{91}{360} \right) \times \left(1 + 0.0737 \times \frac{92}{360} \right) \times \left(1 + 0.0762 \times \frac{91}{360} \right) \times \left(1 + 0.0790 \times \frac{91}{360} \right) \right]^{\frac{365}{547}} - 1 = 7.4470\%$$

15-month rate:

$$\left[\left(1 + 0.0625 \times \frac{91}{360}\right) \times \left(1 + 0.0659 \times \frac{91}{360}\right) \times \left(1 + 0.0716 \times \frac{91}{360}\right) \times \left(1 + 0.0737 \times \frac{92}{360}\right) \times \left(1 + 0.0762 \times \frac{91}{360}\right) \right]^{\frac{365}{456}} - 1 = 7.2868\%$$

12-month rate =

$$\left[\left(1 + 0.0625 \times \frac{91}{360}\right) \times \left(1 + 0.0659 \times \frac{91}{360}\right) \times \left(1 + 0.0716 \times \frac{91}{360}\right) \times \left(1 + 0.0737 \times \frac{92}{360}\right) \right] - 1 = 7.1214\%$$

9-month rate =

$$\left[\left(1 + 0.0625 \times \frac{91}{360}\right) \times \left(1 + 0.0659 \times \frac{91}{360}\right) \times \left(1 + 0.0716 \times \frac{91}{360}\right) \right]^{\frac{365}{273}} - 1 = 6.9325\%$$

$$6\text{-month rate} = \left[\left(1 + 0.0625 \times \frac{91}{360}\right) \times \left(1 + 0.0659 \times \frac{91}{360}\right) \right]^{\frac{365}{182}} - 1 = 6.6699\%$$

$$3\text{-month rate} = \left[\left(1 + 0.0625 \times \frac{91}{360}\right) \right]^{\frac{365}{91}} - 1 = 6.4891\%$$

- b. The 18-month par swap rate on a quarterly money market basis is then i , where the NPV of a par investment with quarterly money market coupon i is itself par:

$$1 = \frac{i \times \frac{91}{360}}{(1.064891)^{\frac{91}{365}}} + \frac{i \times \frac{91}{360}}{(1.066699)^{\frac{182}{365}}} + \frac{i \times \frac{91}{360}}{(1.069325)^{\frac{273}{365}}} + \frac{i \times \frac{92}{360}}{(1.071214)^{\frac{365}{365}}} + \frac{i \times \frac{91}{360}}{(1.072868)^{\frac{456}{365}}} + \frac{1 + i \times \frac{91}{360}}{(1.074470)^{\frac{547}{365}}}$$

This gives $1 = 0.248846i + 0.244769i + 0.240418i + 0.238566i + 0.231514i + 0.226981i + 0.897948$

Therefore $i = 7.13\%$

73. a. Discount the USD cashflows at 9% to an NPV (= USD 102.531 million). Convert to sterling at the current spot exchange rate (= GBP 66.149 million). Apply 11% to this amount to create a stream of

equivalent sterling flows (= GBP 7.276 million per year plus GBP 66.149 million at maturity). All flows below are in millions:

	(i) <i>Remaining bond cashflows</i> USD	(ii) <i>Equivalent of (i) discounted at 9%</i> USD	(iii) <i>Swap receipts</i> USD	(iv) <i>Swap payments</i> GBP
Year 1:	-10	-9.174	+10	-7.276
Year 2:	-10	-8.417	+10	-7.276
Year 3:	-10	-7.722	+10	-7.276
	-100	-77.218	+100	-66.149
NPV:		-102.531		

Your total cashflows are (i), (iii) and (iv); your net cashflows are (iv).

- b. To calculate the long-dated forward prices, remember that USD money market rates are on a 360-day basis. The forward rates (assuming there are actually 365 days in each year) are as follows:

$$1 \text{ year: } 1.55 \times \frac{(1 + 0.09 \times \frac{365}{360})}{(1 + 0.14 \times \frac{365}{365})} = 1.4837$$

$$2 \text{ years: } 1.55 \times \frac{(1 + 0.085 \times \frac{365}{360})^2}{(1 + 0.12 \times \frac{365}{365})^2} = 1.4578$$

$$3 \text{ years: } 1.55 \times \frac{(1 + 0.08 \times \frac{365}{360})^3}{(1 + 0.10 \times \frac{365}{365})^3} = 1.4715$$

At these rates, the cashflows can be converted as follows:

	(i) <i>Remaining bond cashflows</i> USD	(iv) <i>Cashflows converted at forward rates</i> GBP
Year 1:	-10	-6.740
Year 2:	-10	-6.860
Year 3:	-110	-74.754

- c. You might perhaps prefer the second cashflow profile because it defers the net cash outflows slightly. Apart from that, you would prefer the cashflow profile with the lower NPV. Unless the rate of discount is at least 23.4%, this is the first method.

74. The USD 10 million raised from the bond issue can be converted to CHF 15 million at the spot exchange rate. We therefore need to base the floating side of the swap on CHF 15 million:

Year	Bond cashflows	Swap cashflows	
1	− \$ 650,000	+ \$ 650,000	− (LIBOR − i) on CHF 15 mln
2	− \$ 650,000	+ \$ 650,000	− (LIBOR − i) on CHF 15 mln
3	− \$ 650,000	+ \$ 650,000	− (LIBOR − i) on CHF 15 mln
4	− \$ 650,000	+ \$ 650,000	− (LIBOR − i) on CHF 15 mln
5	− \$ 10,650,000	+ \$ 10,650,000	− (LIBOR − i) on CHF 15 mln − CHF 15 mln

The NPV of the USD flows in the swap (using 6.8%) is 9,876,333. This is equivalent to CHF 14,814,499 at the spot exchange rate. If $i = 0$, the CHF flows in the swap would have an NPV of CHF 15,000,000. In order for the two sides of the swap to match therefore, the NPV of $(i \times 15 \text{ million} \times \frac{1}{2} \times \frac{365}{360})$ each six months for 5 years must be $(15,000,000 - 14,814,499) = 185,501$.

Convert 4.5% per annum to an equivalent rate of 2.225% for a 6-monthly period ($\sqrt{1.045} = 1.02225$). Then, using the TVM function of an HP calculator:

$N = 10, I\%YR = 2.225, PV = 185,501, FV = 0$ gives $PMT = 20,895$

You therefore need $15,000,000 \times i \times \frac{1}{2} \times \frac{365}{360} = 20,895$. This gives $i = 0.27\%$. You can therefore achieve **(LIBOR − 27 basis points)** in CHF.

75. The remaining cashflows are as follows:

Date	USD	Swap	DEM
25 May 1998:	+10m × 8%		−15 m × 5.3% × $\frac{181}{360}$
25 Nov 1998:			−15 m × $L_1 \times \frac{184}{360}$
25 May 1999:	+10 m × 8% +10 m		−15 m × $L_2 \times \frac{181}{360}$ −15 m

where L_1 is LIBOR from 25 May 1998 to 25 November 1998
 L_2 is LIBOR from 25 November 1998 to 25 May 1999

Without upsetting the NPV valuation, add the cashflows for a DEM 15 million “FRN” which starts on 25 May 1998, matures on 25 May 1999 and pays LIBOR semi-annually. The resulting cashflows will then be:

Date	Swap USD	DEM	“FRN” DEM	Net DEM cashflows
25 May 1998:	+10 m × 8%	−15 m × 5.3% × $\frac{181}{360}$	−15 m	−15 m × $(1 + 5.3\% \times \frac{181}{360})$
25 Nov 1998:		−15 m × $L_1 \times \frac{184}{360}$	+15 m × $L_1 \times \frac{184}{360}$	
25 May 1999:	+10 m × 8% +10 m	−15 m × $L_2 \times \frac{181}{360}$ −15 m	+15 m × $L_2 \times \frac{181}{360}$ +15 m	

These cashflows can be valued using the discount factors to give:

$$\begin{aligned} & \text{USD } (800,000 \times 0.9850 + 10,800,000 \times 0.9300) \\ & - \text{DEM } (15,399,708 \times 0.9880) \\ & = \text{USD } 10,832,000 - \text{DEM } 15,214,912 \end{aligned}$$

Converted at 1.65 spot, this gives an NPV of USD 1,610,841

76. Receive 99.00 at the start, and transact a par swap based on 100. The bond and swap cashflows are then as follows:

<i>Date</i>	<i>Bond</i>	<i>Swap</i>	
Now:	+99		
6 months:			$-100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
1 year:	-7	+100 × 7.5%	$-100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
1½ years:			$-100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
2 years:	-7	+100 × 7.5%	$-100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
2½ years:			$-100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
3 years	-100 -7	+100 × 7.5%	$-100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$

You can provide a par / par liability swap based on 100, by eliminating, you need to eliminate the uneven fixed cashflows of -1 (= 99 - 100) now and +0.5 (= -7 + 7.5) each year. Discounting at 7.5%, these cashflows have an NPV of:

$$-1 + \frac{0.5}{1.075} + \frac{0.5}{(1.075)^2} + \frac{0.5}{(1.075)^3} = 0.3003$$

You can eliminate these cashflows if you replace them by a series of semi-annual cashflows with the same NPV. The rate of discount you are using is 7.5% (annual). You therefore need an interest rate *i* (money market basis) such that:

$$\begin{aligned} & \frac{100 \times i \times \frac{1}{2} \times \frac{365}{360}}{(1.075)^{0.5}} + \frac{100 \times i \times \frac{1}{2} \times \frac{365}{360}}{(1.075)} + \frac{100 \times i \times \frac{1}{2} \times \frac{365}{360}}{(1.075)^{1.5}} \\ & + \frac{100 \times i \times \frac{1}{2} \times \frac{365}{360}}{(1.075)^2} + \frac{100 \times i \times \frac{1}{2} \times \frac{365}{360}}{(1.075)^{2.5}} + \frac{100 \times i \times \frac{1}{2} \times \frac{365}{360}}{(1.075)^3} = 0.3003 \end{aligned}$$

The solution to this is: *i* = 0.11%

You can therefore replace the 30/360 annual swap inflows of (100 × 7.5%) by a combination of 30/360 annual swap inflows of (100 × 7%) and ACT/360 semi-annual inflows of (100 × 0.11%). This 0.11% can be deducted from the ACT/360 semi-annual swap outflows of (100 × LIBOR) which you already have.

The net effect is therefore a par / par liability swap based on an amount of 100, at (LIBOR – 11 basis points).

77.

Day	Price	Relative price change	LN (price change)	(Difference from mean) ²
1	1.6320			
2	1.6410	1.005515	0.005500	0.000040
3	1.6350	0.996344	-0.003663	0.000008
4	1.6390	1.002446	0.002443	0.000011
5	1.6280	0.993289	-0.006734	0.000035
6	1.6300	1.001229	0.001228	0.000004
7	1.6250	0.996933	-0.003072	0.000005
8	1.6200	0.996923	-0.003082	0.000005
9	1.6280	1.004938	0.004926	0.000033
10	1.6200	0.995086	-0.004926	0.000017
sum:			-0.007380	0.000158
mean = $\frac{\text{sum}}{9}$			-0.000820	
variance = $\frac{\text{sum}}{8}$				0.000020
standard deviation = $\sqrt{\text{variance}}$				0.004444

$$\text{volatility} = \text{annualized standard deviation} = \sqrt{252} \times 0.004444 = 7.1\%$$

78. USD put premium expressed in FRF = percentage of USD amount \times spot rate
 $= 1.5\% \times 5.7550 = 0.086325$

USD call premium = USD put premium + PV of (forward - strike)

$$= 0.086325 + \frac{5.7000 - 5.6000}{(1 + 0.05 \times \frac{182}{360})} = 0.18386$$

Converted to percentage of USD amount at spot rate:

$$\frac{0.18386}{5.7550} = 3.19\%$$

79. To calculate probabilities of up and down movement

Say there is a probability p of a 2% increase in price and a probability $(1 - p)$ of a 2% decrease.

Expected outcome after 1 month is:

$$p \times 100 \times 1.02 + (1 - p) \times \frac{100}{1.02}$$

$$= 3.960784p + 98.039216$$

This should equal the outcome of 100 invested at 12% for 1 month:

$$= 100 \times \left(1 + \frac{0.12}{12}\right) = 101$$

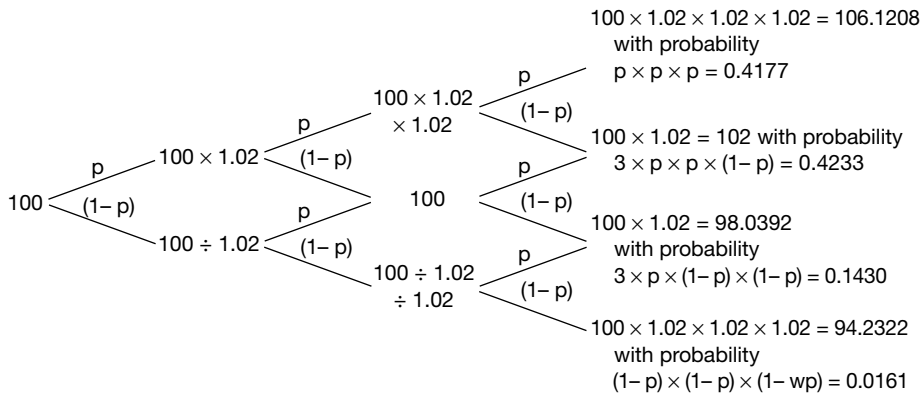
Therefore $3.960784p + 98.039216 = 101$

$$\text{Therefore } p = \frac{101 - 98.039216}{3.960784} = 0.7475$$

Therefore: probability of increase to 102 is 74.75%

probability of decrease to $\frac{100}{1.02}$ is 25.25%

Possible outcomes after three months



The expected value of the put option at the end of 3 months is therefore:

$$(101 - 98.0392) \times 0.1430 + (101 - 94.2322) \times 0.0161 = 0.5324$$

The premium for the option is therefore the present value of this expected value:

$$\frac{0.5324}{(1 + 0.12 \times \frac{3}{12})} = 0.517$$

ANSWERS TO PRACTICE EXAM

Part I

1. **BEF 829,374.89.** As there are 40 semi-annual periods, the TVM function of the calculator requires the interest for a semi-annual period, which is 5.25%.

FIN TVM
40 N
5.25 I%YR
50,000 PMT
0 FV1
PV

$$2. \frac{7}{(1+i)} + \frac{7}{(1+i)^2} + \frac{107}{(1+i)^3}$$

where i is the yield to maturity on an annual basis.

$$3. 10,000,000 \times \left(1 + \frac{0.07}{2}\right)^4 \times \left(1 + \frac{0.075}{4}\right)^{12} = 14,340,782.87$$

.07 ENTER 2 \div 1 + 4 $\square \wedge$
.075 ENTER 4 \div 1 + 12 $\square \wedge \times$
10,000,000 \times

$$4. \text{Forward-forward rate} = \left[\frac{\left(1 + \text{longer rate} \times \frac{\text{days}}{\text{year}}\right)}{\left(1 + \text{shorter rate} \times \frac{\text{days}}{\text{year}}\right)} - 1 \right] \times \frac{\text{year}}{\text{days in period}}$$

$$= \left[\frac{\left(1 + 0.09 \times \frac{275}{360}\right)}{\left(1 + 0.09 \times \frac{92}{360}\right)} - 1 \right] \times \frac{360}{183} = 8.80\%$$

.09 ENTER 275 \times 360 \div 1 +
.09 ENTER 92 \times 360 \div 1 + \div
1 - 360 \times 183 \div

You would **deposit now for only 3 months** at 9.00%. Depositing for 9 months now would be theoretically equivalent to depositing now for 3 months and depositing forward-forward from 3 months to 9 months. As the 3 v 9 rate implied by the current interest rate structure is lower than your forecast, this would be less advantageous than depositing now for 3 months and waiting.

5. Expressed in DEM:

GBP call premium = GBP put premium + PV of (forward – strike)

$$= 0.01 \times 2.6760 + \frac{(2.65 - 2.60)}{(1 + 0.04 \times \frac{91}{360})} = 0.0763$$

Therefore, expressed as percentage of GBP amount:

$$\text{GBP call premium} = \frac{0.0763}{2.676} = 2.85\%$$

2.65 ENTER 2.60 –
.04 ENTER 91 x 360 ÷ 1 + ÷
.01 ENTER 2.676 x +
2.676 ÷

6. a. $[(1 + 0.0835)^{\frac{1}{2}} - 1] \times 2 = 8.183\%$

b. $[(1 + 0.0835)^{\frac{1}{365}} - 1] \times 365 = 8.021\%$

1.0835 ENTER $\square \sqrt{x}$ 1 – 2 x
1.0835 ENTER 365 $\square \frac{1}{x}$ $\square \wedge$ 1 – 365 x

7. 98.48. Found by using an HP calculator. It would also be possible to calculate the price by discounting all 30 cashflows to an NPV.

FIN BOND TYPE SEMI EXIT
Enter any date, then SETT
Enter a date 15 years later, then MAT
10 CPN%
MORE 10.2 YLD%
PRICE

8. First interest payment plus reinvestment:

$$£1,000,000 \times \frac{0.062}{4} \times \left(1 + \frac{0.06}{4}\right)^3$$

Second interest payment plus reinvestment:

$$£1,000,000 \times \frac{0.062}{4} \times \left(1 + \frac{0.06}{4}\right)^2$$

Third interest payment plus reinvestment:

$$£1,000,000 \times \frac{0.062}{4} \times \left(1 + \frac{0.06}{4}\right)$$

Fourth interest payment:	$£1,000,000 \times \frac{0.062}{4}$
Principal amount	£1,000,000
Total:	£1,063,409.00

.06 ENTER 4 ÷ 1 + 3 $\square \wedge$
 .06 ENTER 4 ÷ 1 + 2 $\square \wedge +$
 .06 ENTER 4 ÷ 1 + +
 1 + .062 x 4 ÷
 1 + 1,000,000 x

9. $\left(1 + 0.085 \times \frac{57}{360}\right)^{\frac{365}{57}} - 1 = 8.94\%$

.085 ENTER 57 x 360 ÷ 1 + 365 ENTER 57 ÷ $\square \wedge$ 1 -

10. Converting all to a true yield on a 365-day basis:

a. 7.50%

b. $7.42\% \times \frac{365}{360} = 7.523\%$

c. $\frac{7.21\%}{(1 - 0.0721 \times \frac{182}{365})} = 7.479\%$

d. $\frac{7.18\%}{(1 - 0.0718 \times \frac{182}{360})} \times \frac{365}{360} = 7.554\%$

7.42 ENTER 365 x 360 ÷ (b)
 .0721 ENTER 182 x 365 ÷ 1 - +/- 7.21 $\square x \geq y \div$ (c)
 .0718 ENTER 182 x 360 ÷ 1 - +/- 7.18 $\square x \geq y \div$ 365 x 360 ÷ (d)

The best investment is (d).

11. Create a strip:

$$\left[\left(1 + 0.065 \times \frac{91}{360}\right) \times \left(1 + 0.066 \times \frac{92}{360}\right) \times \left(1 + 0.068 \times \frac{91}{360}\right) \times \left(1 + 0.07 \times \frac{91}{360}\right) - 1\right] \times \frac{360}{365} = 6.90\%$$

.065 ENTER $91 \times 360 \div 1 +$
 .066 ENTER $92 \times 360 \div 1 + x$
 .068 ENTER $91 \times 360 \div 1 + x$
 .07 ENTER $91 \times 360 \div 1 + x$
 $1 - 360 \times 365 \div$

12. a. **False.** This is only true on a coupon date. On other dates, there is a slight difference because the accrued interest is based on simple interest but the price / yield calculation is based on compound interest. Also, some bonds use different day/year counts for accrued interest and price / yield calculations.
- b. **False.** It generally has the highest; it is the bond which is most likely to be able to be used for a cash-and-carry arbitrage and hence to be financed more cheaply than its implied repo rate.
- c. **False.** The longer the duration, the higher the modified duration and hence the higher the sensitivity to yield changes.
- d. **False.** If interest rates are negative (which has happened occasionally in certain markets), the present value is higher than the future value.
- e. **False.** $\left(1 + \frac{0.0845}{2}\right)^2 - 1 = 8.63\%$, which is better than 8.60%.
- f. **False.** The lower the yield, the less the future cashflows are discounted to an NPV, so the higher the price.
- g. **False.** Out-of-the-money options have zero intrinsic value.

Part 2

Exercise 1

a. $\text{Accrued interest} = 8.0 \times \frac{270}{365} = 5.917808$

Dirty price = clean price + accrued interest
 $= 107.50 + 5.917808 = 113.417808$

Therefore:

$$113.417808 = \frac{8}{(1+i)^{\frac{93}{360}}} + \frac{8}{(1+i)^{1+\frac{93}{360}}} + \frac{8}{(1+i)^{2+\frac{93}{360}}} + \frac{8}{(1+i)^{3+\frac{93}{360}}} + \frac{8}{(1+i)^{4+\frac{93}{360}}} + \frac{108}{(1+i)^{5+\frac{93}{360}}}$$

where i = yield to maturity

This question is complicated by the fact that the calculation bases for accrued coupon and price are different. The bond calculation function on the HP calculator can be set to calculate on either a 30/360 basis or an ACT/ACT basis, but not a mixture. To make the calculation it is therefore necessary to “fool” the calculator as follows:

- Calculate the actual dirty price based on the true clean price as above:
113.417808
- Calculate what the accrued interest would be on a 30/360 basis:
 $8.0 \times \frac{267}{360} = 5.933333$
- Calculate what the clean price would be on this basis:
 $113.417808 - 5.933333 = 107.484475$
- Use the bond function on a 30/360 basis to calculate the yield, 6.274%

TIME CALC	
18.091997 DATE1	
15.061998 DATE2	
DAYS	(Actual calendar days for accrued interest)
365 ÷ 8 ×	(Accrued interest)
107.5 +	(Dirty price)
360D	
360 ÷ 8 ×	
–	(“Equivalent” clean price on a 30/360 basis)
EXIT EXIT	
FIN BOND MORE PRICE	
MORE TYPE 360 ANN EXIT	
15.061998 SETT	
18.092003 MAT	
8 CPN%	
MORE YLD%	

- b. Dirty price on purchase is 113.417808

$$\text{Dirty price on sale is } 106.40 + 8.0 \times \frac{337}{365} = 113.786301$$

$$\begin{aligned} \text{Simple rate of return} &= \left(\frac{\text{sale proceeds}}{\text{original investment}} - 1 \right) \times \frac{\text{year}}{\text{days held}} \\ &= \left(\frac{113.786301}{113.417808} - 1 \right) \times \frac{360}{67} = 1.75\% \end{aligned}$$

8 ENTER 337 × 365 ÷ 106.4 +	(Dirty price on sale)
8 ENTER 270 × 365 ÷ 107.5 +	(Dirty price on purchase)
÷ 1 - 360 × 67 ÷	(Rate of return)

$$\text{Effective rate of return} = \left(\frac{\text{sale proceeds}}{\text{original investment}} \right)^{\frac{365}{\text{days held}}} - 1$$

$$= \left(\frac{113.786301}{113.417808} \right)^{\frac{365}{67}} - 1 = 1.78\%$$

8 ENTER 337 × 365 ÷ 106.4 +
8 ENTER 270 × 365 ÷ 107.5 +
÷
365 ENTER 67 ÷ □ ∧ 1 -

c. Futures price =

$$\frac{[\text{bond price} + \text{accrued coupon now}] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] - (\text{accrued coupon at delivery of futures}) - (\text{intervening coupon} + \text{interest earned})}{\text{conversion factor}}$$

$$= \frac{(108 + 8.0 \times \frac{239}{365}) \times (1 + 0.05 \times \frac{35}{360}) - (8.0 \times \frac{274}{365})}{1.1000} = 97.98$$

8 ENTER 239 × 365 ÷ 108 +
.05 ENTER 35 × 360 ÷ 1 + x
8 ENTER 274 × 365 ÷ -
1.1 ÷

- d. The actual futures price is cheaper than the theoretical price. The arbitrage available can therefore be locked in by **buying the futures contract and selling the CTD bond**. This requires borrowing the bond through a reverse repo in order to deliver it to the bond buyer. There is some risk in this arbitrage, as the bond delivered on maturity of the futures contract will be the CTD bond at that time, which may not be the same as the CTD bond now.

Exercise 2

- a Duration is the weighted average life of a series of cashflows, using the present values of the cashflows as the weights:

$$\text{Duration} = \frac{\sum (\text{PV of cashflow} \times \text{time to cashflow})}{\sum (\text{PV of cashflow})}$$

Modified duration is a measure of a bond price's sensitivity to small changes in yield.

Approximately:

$$\text{Modified duration} = - \frac{\text{change in bond price} / \text{dirty price}}{\text{change in yield}}$$

The relationship is:

$$\text{Modified duration} = \frac{\text{duration}}{\left(1 + \frac{\text{yield to maturity}}{\text{coupon frequency}}\right)}$$

- b. For a zero-coupon bond, duration = maturity = 3 years

Using the HP calculator, the yield of the four-year bond is 15.00%

```
FIN TVM
4 N
114.27 +/- PV
20 PMT
100 FV
I%YR
```

The bond's duration is therefore:

$$\frac{1 \times \frac{20}{(1.15)} + 2 \times \frac{20}{(1.15)^2} + 3 \times \frac{20}{(1.15)^3} + 4 \times \frac{120}{(1.15)^4}}{114.27} = 3.164 \text{ years}$$

The zero-coupon bond therefore has the shorter duration.

```
20 ENTER 1.15 ÷
1.15 ENTER 2 [^] 20 [x] y ÷ 2 x +
1.15 ENTER 3 [^] 20 [x] y ÷ 3 x +
1.15 ENTER 4 [^] 120 [x] y ÷ 4 x +
114.27 ÷
```

- c. Using the HP calculator, the yield of the zero-coupon bond is 12.00%

```
FIN TVM
3 N
71.18 +/- PV
0 PMT
100 FV
I%YR
```

The modified duration is therefore $\frac{3}{1.12} = 2.68$

For the four-year bond, the modified duration is $\frac{3.164}{1.15} = 2.75$

- d. Approximate change in price of zero-coupon bond is:

$$-(0.10\%) \times 2.68 \times 71.18 = -0.191$$

Approximate change in price of other bond is:

$$-(0.10\%) \times 2.75 \times 114.27 = -0.314$$

Therefore loss is approximately:

$$\left(30 \text{ million} \times \frac{0.191}{100}\right) + \left(10 \text{ million} \times \frac{0.314}{100}\right) = 88,700$$

3 ENTER 1.12 ÷	(Modified duration of first bond)
.001 × 71.18 ×	(Fall in price of first bond)
100 ÷ 30,000,000 ×	(Loss on first holding)
3.164 ENTER 1.15 ÷	(Modified duration of second bond)
.001 × 114.27 ×	(Fall in price of second bond)
100 ÷ 10,000,000 ×	(Loss on second holding)
+	

- e. Number of contracts to sell =

$$\frac{\text{value of portfolio}}{\text{size of futures contract}} \times \frac{100}{\text{dirty price of CTD bond}} \times \frac{\text{modified duration of portfolio}}{\text{modified duration of CTD}} \times \frac{\text{conversion factor of CTD}}{\left(1 + \text{funding rate} \times \frac{\text{days to futures delivery}}{\text{year}}\right)}$$

Exercise 3

- a. Calculate the 2-year zero-coupon rate by constructing a synthetic 2-year zero-coupon structure from the 1-year and 2-year rates, by investing for 2 years and borrowing for 1 year as follows:

	2-year investment	1-year borrowing	Net cashflows
Now	-104.500	+ $\frac{8.000}{1.05}$	-96.881
1 year	+8.000	-8.000	
2 years	+108.000		+108.000

The 2-year zero-coupon rate is $\left(\frac{108}{96.881}\right)^{\frac{1}{2}} - 1 = 5.583\%$

The 2-year discount factor is $\frac{96.881}{108} = 0.8970$

Calculate the 3-year zero-coupon rate similarly by constructing a synthetic 3-year zero-coupon structure:

	3-year investment	2-year zero-coupon borrowing	1-year borrowing	Net cashflows
Now	-98.70	$+5.500 \times 0.8970$	$+ \frac{5.500}{1.05}$	-88.528
1 year	+5.500		-5.500	
2 years	+5.500	-5.500		
3 years	+105.500			+105.500

The 3-year zero-coupon rate is $\left(\frac{105.500}{88.528}\right)^{\frac{1}{3}} - 1 = 6.021\%$

The 3-year discount factor is $\frac{88.528}{105.500} = 0.8391$

- b. 1-year discount factor is $\frac{1}{1.05} = 0.9524$

From the discount factors:

$$1 \text{ year v } 2 \text{ year forward-forward} = \frac{0.9524}{0.8970} - 1 = 6.17\%$$

$$2 \text{ year v } 3 \text{ year forward-forward} = \frac{0.8970}{0.8391} - 1 = 6.90\%$$

- c. Discounting the cashflows of the bond using the discount factors gives a price of:

$$12 \times 0.9524 + 12 \times 0.8970 + 112 \times 0.8391 = 116.172$$

Using the TVM function, this gives a yield to maturity of 5.95%.

- d. If i is the 2-year par yield, then:

$$1 = i \times 0.9524 + (1 + i) \times 0.8970$$

$$\text{Therefore } i = \frac{1 - 0.8970}{0.9524 + 0.8970} = 5.57\%$$

$$\text{Similarly the 3-year par yield} = \frac{1 - 0.8391}{0.9524 + 0.8970 + 0.8391} = 5.98\%$$

Exercise 4

a. The remaining cashflows are as follows:

$$2 \text{ June } 1999: \quad - \left(50 \text{ million} \times 8.5\% \times \frac{182}{360} \right) + \left(50 \text{ million} \times 10\% \times \frac{360}{360} \right)$$

$$2 \text{ December } 1999: \quad - \left(50 \text{ million} \times L_1 \times \frac{183}{360} \right)$$

$$2 \text{ June } 2000: \quad - \left(50 \text{ million} \times L_2 \times \frac{183}{360} \right) + \left(50 \text{ million} \times 10\% \times \frac{360}{360} \right)$$

Where L_1 is 6-month LIBOR for 2 June 1999
and L_2 is 6-month LIBOR for 2 December 1999

Method 1

In order to value the unknown interest payments consistently with the discount factors, calculate forward-forward rates as follows:

$$L_1 = \left(\frac{0.9885}{0.9459} - 1 \right) \times \frac{360}{183} = 8.85963\%$$

$$L_2 = \left(\frac{0.9459}{0.9064} - 1 \right) \times \frac{360}{183} = 8.57292\%$$

The mark-to-market value of the remaining cashflows is then their NPV:

$$\begin{aligned} & \left[- \left(50 \text{ million} \times 8.5\% \times \frac{182}{360} \right) + \left(50 \text{ million} \times 10\% \times \frac{360}{360} \right) \right] \times 0.9885 &= 2,818,598 \\ & \left[- \left(50 \text{ million} \times 8.85963\% \times \frac{183}{360} \right) \right] \times 0.9459 &= -2,129,999 \end{aligned}$$

$$\begin{aligned} & \left[- \left(50 \text{ million} \times 8.57292\% \times \frac{183}{360} \right) + \left(50 \text{ million} \times 10\% \times \frac{360}{360} \right) \right] \times 0.9064 &= \frac{2,556,999}{3,245,598} \end{aligned}$$

Method 2

As an alternative to using forward-forward rates, the unknown cashflows can be eliminated by adding an appropriate structure of cashflows which itself has a zero NPV. The appropriate structure is an FRN structure as follows:

$$2 \text{ June } 1999: \quad -50 \text{ million}$$

$$2 \text{ December } 1999: \quad + \left(50 \text{ million} \times L_1 \times \frac{183}{360} \right)$$

$$2 \text{ June } 2000: \quad + \left(50 \text{ million} \times L_2 \times \frac{183}{360} \right) + 50 \text{ million}$$

Adding these cashflows to the whole original structure and netting the result gives the following:

$$2 \text{ June } 1999: \quad - \left(50 \text{ million} \times 8.5\% \times \frac{182}{360} \right) + \left(50 \text{ million} \times 10\% \times \frac{360}{360} \right) - (50 \text{ million})$$

$$2 \text{ June } 2000: \quad + \left(50 \text{ million} \times 10\% \times \frac{360}{360} \right) + 50 \text{ million}$$

The NPV of these cashflows can be calculated as before to give + 3,245,598

- b. Invest 103.50 and transact a par swap based on 100. The bond and swap cashflows are then as follows:

<i>Date</i>	<i>Bond</i>	<i>Swap</i>	
Now:	-103.5		
6 months:			$+100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
1 year:	+9	$-100 \times 7.5\%$	$+100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
1½ years:			$+100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
2 years:	+9	$-100 \times 7.5\%$	$+100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
2½ years:			$+100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$
3 years	-100 +9	$-100 \times 7.5\%$	$+100 \times \text{LIBOR} \times \frac{1}{2} \times \frac{365}{360}$

In order to create a par / par asset swap, you need to eliminate the uneven fixed cashflows of -3.5 now and (9 - 7.5) each year. These cashflows have an NPV of:

$$-3.5 + 1.5 \times 0.9300 + 1.5 \times 0.8650 + 1.5 \times 0.8050 = +0.40$$

Suppose that a series of semi-annual cashflows of $(100 \times i \times \frac{1}{2} \times \frac{365}{360})$ has the same NPVs. This means that:

$$\begin{aligned} &(100 \times i \times \frac{182.5}{360} \times .9650) + (100 \times i \times \frac{182.5}{360} \times .9300) \\ &+ (100 \times i \times \frac{182.5}{360} \times .8970) + (100 \times i \times \frac{182.5}{360} \times .8650) \\ &+ (100 \times i \times \frac{182.5}{360} \times .8350) + (100 \times i \times \frac{182.5}{360} \times .8050) = +0.40 \end{aligned}$$

The solution to this is: $i = +0.15\%$

We can therefore add cashflows of + 3.5 now and -1.5 each year (NPV = -0.40) if we also add 0.15% to LIBOR each six months (NPV = +0.40). We can therefore achieve a par / par asset swap of LIBOR + 15 basis points.

Appendices

1 Using an HP calculator

2 A summary of market day/year conventions

3 A summary of calculation procedures

4 Glossary

5 ISO/SWIFT codes for currencies

APPENDIX I

Using an HP calculator

INTRODUCTION

We have not tried to give full instructions here for using Hewlett Packard calculators, but have set out only those operations necessary for the examples and exercises in this book.

The HP calculators generally used fall into three categories:

The HP12C calculator

It is essential first to understand the logic of this calculator, which is called “reverse Polish notation” (RPN). On a traditional calculator, the steps are entered in the same order as on a piece of paper. Thus, for example, the calculation $4 \times 5 + 6 = 26$ would be performed by entering 4, \times , 5, $+$, 6 and $=$ in that order. The result appears as 26.

With an HP12C using RPN, however, it is necessary instead to enter 4, ENTER, 5, \times , 6 and $+$, in that order. The first number is generally followed by ENTER; thereafter, each number is followed (rather than preceded as is traditional) by its operator. In this example, we are multiplying by 5. The \times therefore follows the 5 instead of the other way round.

The HP17BII and HP19BII

With these calculators, the user can choose whether to use RPN or the more traditional algebraic logic, using the calculator’s MODES function. The labels shown on the calculator’s keys for the various operation functions are also different in a few cases from the HP12C.

The HP17B and HP19B

With these calculators, the user can use only the traditional “algebraic” logic.

In this Appendix, we show the essential steps for the HP12C and for the HP19BII in each mode. This would be very cumbersome for all the examples in the book, however, and we have therefore shown steps elsewhere in the text for the HP19BII in RPN mode only. This will not give a problem to a user of the HP12C and HP17BII in RPN, as the steps are the same even though a few keys are labelled differently. The steps in algebraic mode are in any case more familiar.

Basic operations

In the steps shown below, “f” and “g” refer to the yellow and blue shift keys marked “f” and “g” on the HP12C and “□” refers to the yellow shift key on the HP19BII. Also, the key “^” may alternatively be marked “y^x”.

Switching between algebraic and RPN modes

To switch the HP19BII to algebraic mode: □ MODES MORE ALG

To switch the HP19BII to RPN mode: □ MODES MORE RPN

Deleting an incorrect entry

To delete the current entry without affecting the calculation so far:

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
CLx	□ CLEAR	□ CLEAR

To delete the entire calculation so far:

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
f CLEAR REG	□ CLEAR DATA	□ CLEAR DATA

To correct the current entry before pressing another key:

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
<i>not available</i>	←	←
	<i>works as a backspace key</i>	

Number of decimal places

The number of decimal places displayed can be adjusted, without affecting the accuracy of the calculation. For example, to display four decimal places:

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
f 4	DISP FIX 4 INPUT	DISP FIX 4 INPUT

Addition

Example: 5 + 4 = 9

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
5 ENTER 4 +	5 ENTER 4 +	5 + 4 =

Example: $-5 + 4 = -1$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
5 ENTER CHS 4 +	5 ENTER +/- 4 +	$-5 + 4 =$
<i>(Because the first entry in RPN must be a number rather than an operator, it is necessary to enter the 5 and then change its sign from positive to negative)</i>		

Subtraction**Example:** $7 - 3 = 4$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
7 ENTER 3 -	7 ENTER 3 -	$7 - 3 =$

Multiplication, division**Example:** $8 \div 2 \times 5 = 20$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
8 ENTER 2 \div 5 \times	8 ENTER 2 \div 5 \times	$8 \div 2 \times 5 =$

Example: $3 \times 4 + 7 = 19$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
3 ENTER 4 \times 7 +	3 ENTER 4 \times 7 +	$3 \times 4 + 7 =$

Example: $3 + 4 \times 7 = 31$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
4 ENTER 7 \times 3 +	4 ENTER 7 \times 3 +	$3 + (4 \times 7) =$

Note that in the expression “ $3 + 4 \times 7$ ”, you must do the multiplication *before* the addition. It is a convention of the way mathematical formulas are written that any exponents (5^4 , x^2 , $4 \cdot 2^{\frac{1}{4}}$, etc.) must be done first, followed by multiplication and division (0.08×92 , $x \div y$, $\frac{17}{38}$ etc.) and addition and subtraction last. This rule is applied first to anything inside brackets (...) and then to everything else.

Exponents

Example: $3^5 = 243$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
3 ENTER 5 y^x	3 ENTER 5 $\square \wedge$	$3 \square \wedge 5 =$

Chaining

“Chaining” is performing a series of calculations in succession without the need to keep stopping and starting or using memory stores.

Example: $\left[\frac{(1 + 0.4 \times \frac{78}{360})}{(1 + 0.5 \times \frac{28}{360})} - 1 \right] \times \frac{360}{50} = 0.3311$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
.4 ENTER 78 \times 360 \div 1 + .5 ENTER 28 \times 360 \div 1 + \div 1 – 360 \times 50 \div	.4 ENTER 78 \times 360 \div 1 + .5 ENTER 28 \times 360 \div 1 + \div 1 – 360 \times 50 \div	$((1 + (.4 \times 78 \div 360)) \div$ $(1 + (.5 \times 28 \div 360))$ $-1) \times 360 \div 50 =$
<i>(Note that the \div in the fifth line divides the result of the fourth line into the result of the second line without the need to re-enter any interim results)</i>		

Reversing the order of the current operation

This can be useful in the middle of chaining.

Example: $\frac{85}{1 + 8 \times 2} = 5$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
8 ENTER 2 \times 1 + 85 $x\rightleftharpoons y \div$ <i>(“$x\rightleftharpoons y$” switches the order, so that 85 is divided by the result of “1 + 8 \times 2”, instead of the other way round)</i>	8 ENTER 2 \times 1 + 85 $\square x\rightleftharpoons y \div$ <i>(“$\square x\rightleftharpoons y$” switches the order, so that 85 is divided by the result of “1 + 8 \times 2”, instead of the other way round)</i>	Not available

Square roots

Example: $\sqrt{(9 + 7)}$ [the same as $(9 + 7)^{\frac{1}{2}} = 4$]

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
9 ENTER 7 + g \sqrt{x}	9 ENTER 7 + $\square \sqrt{x}$	$9 + 7 = \square \sqrt{x}$

Reciprocals

Example: $\frac{1}{16} = 0.625$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
16 $1/x$	16 $\square 1/x$	16 $\square 1/x$

Example: $\frac{1}{4 \times 5} = 0.05$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
4 ENTER 5 $\times 1/x$	4 ENTER 5 $\times \square 1/x$	$4 \times 5 = \square 1/x$

Example: $(21 + 43)^{\frac{1}{6}} = 2$

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
21 ENTER 43 + 6 $1/x y^x$	21 ENTER 43 + 6 $\square 1/x \square \wedge$	$21 + 43 = \square \wedge 6 \square 1/x =$

Function menus

Various functions on the HP19BII are available through special menus and sub-menus accessed by pressing the calculator's top row of keys. Pressing "EXIT" moves from the current level menu to the previous level. Pressing " \square MAIN" moves from the current level to the highest level menu.

Date calculations

The calculator can be switched to accept dates in either European format (e.g. 18 August 1998 entered as 18.081998) or American format (e.g. 18 August 1998 entered as 08.181998). This switching is done as follows:

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
g D.MY (for European format) or g M.DY (for American format)	Select TIME menu Select SET menu Select M/D (This switches from the existing format to the other one) <input type="checkbox"/> MAIN	Select TIME menu Select SET menu Select M/D (This switches from the existing format to the other one) <input type="checkbox"/> MAIN

All our examples are shown using the European format.

Example: What is the number of true calendar days between 18 August 1998 and 12 December 1998?

Answer: 116

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
18.081998 ENTER 12.121998 g ΔDYS	Select TIME menu Select CALC menu 18.081998 DATE1 12.121998 DATE2 DAYS <input type="checkbox"/> MAIN	Select TIME menu Select CALC menu 18.081998 DATE1 12.121998 DATE2 DAYS <input type="checkbox"/> MAIN

Example: What is the number of days between 18 August 1998 and 12 December 1998 on a 30(A)/360 basis? (For an explanation of this, see the section “Day/year conventions” in Chapter 5.)

Answer: 114

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
Repeat the steps above, followed by $x \geq y$ or R↓	Repeat the steps above, but type 360D instead of DAYS	Repeat the steps above, but type 360D instead of DAYS

(Note that the HP calculators can perform date calculations based on the American 30(A)/360 convention but not on the very similar European / Eurobond 30(E)/360 convention.)

Example: What date is 180 calendar days after 18 August 1998?

Answer: 14 February 1999

HP12C	HP19BII (RPN mode)	HP19BII (algebraic)
18.081998 ENTER 180 g DATE	<i>Select TIME menu</i> <i>Select CALC menu</i> 18.081998 DATE1 180 DAYS DATE2 <input type="checkbox"/> MAIN	<i>Select TIME menu</i> <i>Select CALC menu</i> 18.081998 DATE1 180 DAYS DATE2 <input type="checkbox"/> MAIN

Other operations

Hewlett Packard calculators – particularly the HP17II and HP19II – have a number of more sophisticated inbuilt functions on which the calculators' manuals are the best source of information. Several of these functions are, however, described elsewhere in this book in the appropriate chapters as they become relevant. These are:

- **Time value of money:** covered in Chapter 1.
- **Irregular cashflows:** covered in Chapter 1.
- **Bond calculations:** covered in Chapter 5.
- **Maths functions:** examples of use are given in Chapter 1.
- **Solving equations:** an example is given in Chapter 5.

APPENDIX 2

A Summary of Market Day/Year Conventions for Money Markets and Government Bond Markets

<i>Instrument</i>	<i>Day/year basis</i>	<i>Yield or discount</i>
Australia		
Money market	ACT/365	Y
Bond (semi-annual coupons)	ACT/ACT	
Austria		
Money market	ACT/360	Y
Bond (annual coupons)	30(E)/360	
Belgium		
Money market	ACT/365	Y
OLO (annual coupons)	30(E)/360	
Canada		
Money market	ACT/365	Y
Bond (semi-annual coupons)	ACT/365 (accrued coupon) ACT/ACT (dirty price calculation)	
Denmark		
Money market	ACT/360	Y
Bond (annual coupons)	30(E)/360	
Eire		
Money market	ACT/365	Y
Bond (annual and semi-annual coupons)	30(E)/360 ¹	
Finland		
Money market	ACT/360	Y
Bond (annual coupons)	30(E)/360	

France		
Money market	ACT/360	Y
OAT, BTAN (annual coupons)	ACT/ACT	
Germany		
Money market	ACT/360	Y
Bund, OBL (annual coupons)	30(E)/360	
Italy		
Money market	ACT/365	Y
BTP (semi-annual coupons)	30(E) ² /360 (accrued coupon) ACT/365 (dirty price calculation)	
Japan		
Money market	ACT/365	Y
JGB (semi-annual coupons)	ACT ³ /365	
Netherlands		
Money market	ACT/360	Y
Bond (annual coupons)	30(E)/360	
Norway		
T-bills	ACT/365 ⁴	Y
Other money market	ACT/360	Y
Bond (annual coupons)	ACT/365	
Spain		
Money market	ACT/360	Y
Bono (annual coupons)	ACT/ACT (accrued coupon) ACT/365 (dirty price calculation)	
Sweden		
T-bills	30(E)/360	Y
Other money market	ACT/360	Y
Bond	30(E)/360	
Switzerland		
Money market	ACT/360	Y
Bond (annual coupons)	30(E)/360	
UK		
Depo / CD / £CP / T-bill (ECU)	ACT/365	Y
BA / T-bill (£)	ACT/365	D
Gilt (semi-annual coupons)	ACT/ACT ⁵	

USA

Depo / CD	ACT/360	Y
BA / \$CP / T-bill	ACT/360	D
T-bond / note		
(semi-annual coupons)	ACT/ACT	
Federal agency & corporate bonds	30(A)/360	

Euro (*the single “domestic” currency for certain European Union countries from 1999*)

Money market	ACT/360	Y
Bond	ACT/ACT	

Euromarket (*non-domestic markets generally*)

Money market	ACT/360 (exceptions using ACT/365 include GBP, IEP, BEF / LUF, PTE, GRD, SGD, HKD, MYR, TWD, THB, ZAR)	Y
Eurobond (annual coupons)	30(E)/360	

Notes

1. Some older bonds still use ACT/365.
2. Accrued coupon calculation adds one day to the usual calculation (the start date and end date are counted inclusively).
3. One day's extra coupon is accrued in the first coupon period.
4. Quoted as an effective (annual equivalent) yield rather than a simple rate.
5. ACT/365 for accrued coupon until late 1998.

APPENDIX 3

A Summary of Calculation Procedures

Notation

The following general notation is used throughout these formulas, unless something different is specifically mentioned:

D	=	discount rate
FV	=	future value, or future cashflow
i	=	interest rate or yield per annum
n	=	number of times per year an interest payment is made
N	=	number of years or number of coupon periods
P	=	price (dirty price for a bond)
PV	=	present value, or cashflow now
r	=	continuously compounded interest rate
R	=	coupon rate paid on a security
year	=	number of days in the conventional year
z_k	=	zero-coupon yield for k years

Financial arithmetic basics

Effective and nominal rates

If the interest rate with n payments per year is i, the effective rate (equivalent annual rate) i^* is:

$$i^* = \left[\left(1 + \frac{i}{n} \right)^n - 1 \right]$$

Similarly:

$$i = \left[(1 + i^*)^{\frac{1}{n}} - 1 \right] \times n$$

$$\text{Effective rate} = \left(1 + i \times \frac{\text{days}}{\text{year}} \right)^{\frac{365}{\text{days}}} - 1$$

$$\text{Effective rate on a 360-day basis} = \text{effective rate} \times \frac{360}{365}$$

Continuously compounded interest rate

$$r = \frac{365}{\text{days}} \times \text{LN} \left(1 + i \times \frac{\text{days}}{\text{year}} \right)$$

$$\text{Discount factor} = e^{r \times \frac{\text{days}}{365}}$$

where i is the nominal rate for that number of days

In particular, when i is the effective rate:

$$r = \text{LN}(1 + i)$$

and:

$$i = \frac{\text{year}}{\text{days}} \times \left(e^{r \times \frac{\text{days}}{365}} - 1 \right)$$

Short-term investments

$$\text{FV} = \text{PV} \times \left(1 + i \times \frac{\text{days}}{\text{year}} \right)$$

$$\text{PV} = \frac{\text{FV}}{\left(1 + i \times \frac{\text{days}}{\text{year}} \right)}$$

$$i = \left(\frac{\text{FV}}{\text{PV}} - 1 \right) \times \frac{\text{year}}{\text{days}}$$

$$\text{Effective yield} = \left(\frac{\text{FV}}{\text{PV}} \right)^{\frac{365}{\text{days}}} - 1$$

$$\text{Discount factor} = \frac{1}{1 + i \times \frac{\text{days}}{\text{year}}}$$

Long-term investments over N years

$$\text{FV} = \text{PV} \times (1 + i)^N$$

$$\text{PV} = \frac{\text{FV}}{(1 + i)^N}$$

$$i = \left(\frac{\text{FV}}{\text{PV}} \right)^{\frac{1}{N}} - 1$$

$$\text{Discount factor} = \left(\frac{1}{1 + i} \right)^N$$

NPV and internal rate of return

NPV = sum of all the present values

The internal rate of return is the interest rate which discounts all the future cashflows to a given NPV. This is equivalent to the interest rate which discounts all the cashflows *including* any cashflow *now* to zero.

Interpolation and extrapolation

$$i = i_1 + (i_2 - i_1) \times \frac{(d - d_1)}{(d_2 - d_1)}$$

where: i is the rate required for d days
 i_1 is the known rate for d_1 days
 i_2 is the known rate for d_2 days

The money market

$$\text{Interest rate on 360-day basis} = \text{interest rate on 365-day basis} \times \frac{360}{365}$$

$$\text{Interest rate on 365-day basis} = \text{interest rate on 360-day basis} \times \frac{365}{360}$$

Certificate of deposit

$$\text{Maturity proceeds} = \text{face value} \times$$

$$\left(1 + \text{coupon rate} \times \frac{\text{days from issue to maturity}}{\text{year}} \right)$$

$$\text{Secondary market proceeds} = \frac{\text{maturity proceeds}}{(1 + \text{market yield} \times \frac{\text{days left to maturity}}{\text{year}})}$$

$$\text{Return on holding a CD} =$$

$$\left[\frac{(1 + \text{purchase yield} \times \frac{\text{days from purchase to maturity}}{\text{year}})}{(1 + \text{sale yield} \times \frac{\text{days from sale to maturity}}{\text{year}})} - 1 \right] \times \frac{\text{year}}{\text{days held}}$$

Discount instrument

$$\text{Maturity proceeds} = \text{face value}$$

$$\text{Secondary market proceeds} = \frac{\text{face value}}{(1 + \text{market yield} \times \frac{\text{days left to maturity}}{\text{year}})}$$

Instruments quoted on a discount rate

$$\text{Rate of true yield} = \frac{\text{discount rate}}{(1 - \text{discount rate} \times \frac{\text{days}}{\text{year}})}$$

$$\text{Discount rate} = \frac{\text{rate of true yield}}{(1 + \text{yield} \times \frac{\text{days}}{\text{year}})}$$

$$\text{Amount of discount} = \text{face value} \times \text{discount rate} \times \frac{\text{days}}{\text{year}}$$

$$\text{Amount paid} = \text{face value} \times (1 - \text{discount rate} \times \frac{\text{days}}{\text{year}})$$

Instruments quoted on a discount rate:

US:	T-bill	UK:	T-bill (£)
	BA		BA
	CP		

Medium-term CD

$$P = F \times \left[\frac{1}{A_N} + \left(\frac{R}{\text{year}} \times \sum_{k=1}^N \left[\frac{d_{k-1;k}}{A_k} \right] \right) \right]$$

$$\text{where: } A_k = (1 + i \times \frac{d_{p1}}{\text{year}}) \times (1 + i \times \frac{d_{12}}{\text{year}}) \times (1 + i \times \frac{d_{23}}{\text{year}}) \\ \dots \times (1 + i \times \frac{d_{k-1;k}}{\text{year}})$$

N = number of coupons not yet paid

$d_{k-1;k}$ = number of days between $(k-1)^{\text{th}}$ coupon and k^{th} coupon

d_{p1} = number of days between purchase and first coupon

Forward-forwards and forward rate agreements

For periods up to one year:

$$\text{Forward-forward rate} = \left[\frac{\left(1 + i_L \times \frac{d_L}{\text{year}} \right)}{\left(1 + i_S \times \frac{d_S}{\text{year}} \right)} - 1 \right] \times \left(\frac{\text{year}}{d_L - d_S} \right)$$

where: i_L = interest rate for longer period

i_S = interest rate for shorter period

d_L = number of days in longer period

d_S = number of days in shorter period

$$\text{FRA settlement amount} = \text{principal} \times \frac{(f - L) \times \frac{\text{days}}{\text{year}}}{(1 + L \times \frac{\text{days}}{\text{year}})}$$

where: f = FRA rate
 L = LIBOR at the beginning of the FRA period
 days = number of days in the FRA period

For periods longer than a year but less than 2 years:

$$\text{FRA settlement} = \text{principal} \times \frac{(f - L) \times \frac{d_1}{\text{year}}}{(1 + L \times \frac{d_1}{\text{year}})} + \frac{(f - L) \times \frac{d_2}{\text{year}}}{(1 + L \times \frac{d_1}{\text{year}}) \times (1 + L \times \frac{d_2}{\text{year}})}$$

where: d_1 = number of days in the first year of the FRA period
 d_2 = number of days from d_1 until the end of the FRA period

Constructing a strip

The interest rate for a longer period up to one year =

$$\left[\left(1 + i_1 \times \frac{d_1}{\text{year}} \right) \times \left(1 + i_2 \times \frac{d_2}{\text{year}} \right) \times \left(1 + i_3 \times \frac{d_3}{\text{year}} \right) \times \dots - 1 \right] \times \frac{\text{year}}{\text{total days}}$$

where: i_1, i_2, i_3, \dots are the cash interest rate and forward-forward rates for a series of consecutive periods lasting d_1, d_2, d_3, \dots days.

Interest rate futures

Price = 100 – (implied forward-forward interest rate \times 100)

Profit / loss on long position in a 3 month contract =

$$\text{notional contract size} \times \frac{(\text{sale price} - \text{purchase price})}{100} \times \frac{1}{4}$$

Basis = implied cash price – actual futures price

Theoretical basis = implied cash price – theoretical futures price

Value basis = theoretical futures price – actual futures price

Bond market calculations

General dirty price formula

$$P = \text{NPV of all future cashflows} = \sum_k \frac{C_k}{\left(1 + \frac{i}{n} \right)^{\frac{d_k \times n}{\text{year}}}}$$

where: C_k = the k^{th} cashflow arising
 d_k = number of days until C_k
 i = yield on the basis of n interest payments per year

Conventional dirty price formula

$$P = \frac{100}{(1 + \frac{i}{n})^W} \left[\frac{R}{n} \times \frac{(1 - \frac{1}{(1 + \frac{i}{n})^N})}{(1 - \frac{1}{(1 + \frac{i}{n})})} + \frac{1}{(1 + \frac{i}{n})^{N-1}} \right]$$

where: R = the annual coupon rate paid n times per year
 W = the fraction of a coupon period between purchase and the next coupon to be received
 N = the number of coupon payments not yet made
 i = yield per annum based on n payments per year

$$\text{Accrued coupon} = 100 \times \text{coupon rate} \times \frac{\text{days since last coupon}}{\text{year}}$$

For ex-dividend prices, accrued coupon is negative:

$$\text{Accrued coupon} = -100 \times \text{coupon rate} \times \frac{\text{days to next coupon}}{\text{year}}$$

$$\text{Clean price} = \text{dirty price} - \text{accrued coupon}$$

Price falls as yield rises and vice versa.

Generally, if a bond's price is greater than par, its yield is less than the coupon rate and vice versa – except that, on a non-coupon date, if a bond is priced at par, the yield is slightly lower than the coupon rate.

Other yields

$$\text{Current yield} = \frac{\text{coupon rate}}{\text{clean price}/100}$$

$$\text{Simple yield to maturity} = \frac{\text{coupon rate} + \left(\frac{\text{redemption amount} - \text{clean price}}{\text{years to maturity}} \right)}{\text{clean price}/100}$$

Alternative yield in final coupon period (simple)

$$i = \left[\frac{\text{total final cashflow including coupon}}{\text{dirty price}} - 1 \right] \times \frac{\text{year}}{\text{days to maturity}}$$

where days and year are measured on the relevant bond basis

Bond equivalent yield for US T-bill

If 182 days or less to maturity:

$$i = \frac{D}{1 - D \times \frac{\text{days}}{360}} \times \frac{365}{360}$$

If more than 182 days to maturity:

$$i = \frac{-\frac{\text{days}}{365} + \left(\left(\frac{\text{days}}{365} \right)^2 + 2 \times \left(\left(\frac{\text{days}}{365} - \frac{1}{2} \right) \times \left(\frac{1}{1 - D \times \frac{\text{days}}{360}} - 1 \right) \right)^{\frac{1}{2}}}{\left(\frac{\text{days}}{365} - \frac{1}{2} \right)}$$

If 29 February falls in the 12-month period starting on the purchase date, replace 365 by 366.

Money market yield

$$P = \frac{100}{\left(1 + \frac{i}{n} \times W\right)} \left[\frac{R}{n} \times \frac{\left(1 - \frac{1}{\left(1 + \frac{i}{n} \times \frac{365}{360}\right)^N}\right)}{\left(1 - \frac{1}{\left(1 + \frac{i}{n} \times \frac{365}{360}\right)}\right)} + \frac{1}{\left(1 + \frac{i}{n} \times \frac{365}{360}\right)^{N-1}} \right]$$

where i and W are the yield and fraction of a coupon period on a money market basis rather than a bond basis.

Moosmüller yield

$$P = \frac{100}{\left(1 + \frac{i}{n} \times W\right)} \left[\frac{R}{n} \times \frac{\left(1 - \frac{1}{\left(1 + \frac{i}{n}\right)^N}\right)}{\left(1 - \frac{1}{\left(1 + \frac{i}{n}\right)}\right)} + \frac{1}{\left(1 + \frac{i}{n}\right)^{N-1}} \right]$$

where i and W are the yield and fraction of a coupon period on a bond basis.

Duration and convexity

$$(\text{Macaulay}) \text{ duration} = \frac{\sum (\text{present value of cashflow} \times \text{time to cashflow})}{\sum (\text{present value of cashflow})}$$

$$\text{Modified duration} = -\frac{dP}{di} / \text{dirty price} = \frac{\text{duration}}{\left(1 + \frac{i}{n}\right)}$$

$$DV01 = \text{modified duration} \times \text{dirty price} \times 0.0001$$

Approximation

$$\text{Change in price} \approx -\text{dirty price} \times \text{change in yield} \times \text{modified duration}$$

Convexity =

$$-\frac{d^2P}{di^2} / \text{dirty price} = \sum_k \left[\frac{C_k}{\left(1 + \frac{i}{n}\right)^{n \frac{d_k}{\text{year}} + 2}} \times \frac{d_k}{\text{year}} \left(\frac{d_k}{\text{year}} + \frac{1}{n} \right) \right] / \text{dirty price}$$

where d_k is the number of days to cashflow C_k

Better approximation

$$\text{Change in price} \approx - \text{dirty price} \times \text{modified duration} \times \text{change in yield} + \frac{1}{2} \text{dirty price} \times \text{convexity} \times (\text{change in yield})^2$$

Approximations for a portfolio

$$\text{Duration} = \frac{\Sigma(\text{duration of investment} \times \text{value of each investment})}{\text{value of portfolio}}$$

$$\text{Modified duration} = \frac{\Sigma(\text{mod. dur. of each investment} \times \text{value of each investment})}{\text{value of portfolio}}$$

$$\text{Convexity} = \frac{\Sigma(\text{convexity of each investment} \times \text{value of each investment})}{\text{value of portfolio}}$$

Bond futures

Conversion factor = clean price at delivery for one unit of the deliverable bond, at which that bond's yield equals the futures contract notional coupon rate

Theoretical bond futures price =

$$\left([\text{bond price} + \text{accrued coupon now}] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] \right) - \frac{(\text{accrued coupon at delivery}) - (\text{intervening coupon reinvested})}{\text{conversion factor}}$$

where i = short-term funding rate

Forward bond price =

$$\left([\text{bond price} + \text{accrued coupon now}] \times \left[1 + i \times \frac{\text{days}}{\text{year}} \right] \right) - (\text{accrued coupon at delivery}) - (\text{intervening coupon reinvested})$$

where i = short-term funding rate

Generally, a forward bond price is at a premium (discount) to the cash price

if the short-term funding cost is greater than (less than) $\frac{\text{coupon rate}}{\text{cash price}/100}$.

Hedge ratio

$$\frac{\text{notional amount of futures contract required to hedge a position in bond A}}{\text{face value of bond A}} =$$

$$\frac{\text{dirty price of bond A}}{\text{dirty price of CTD bond}} \times \frac{\text{modified duration of bond A}}{\text{modified duration of CTD bond}} \times$$

$$\frac{\text{conversion factor for CTD bond}}{\left(1 + i \times \frac{\text{days to futures delivery}}{\text{year}}\right)}$$

where i = short-term funding rate

Cashflows in a classic repo

$$\text{Cash paid at the beginning} = \frac{\text{nominal bond amount} \times (\text{clean price} + \text{accrued coupon})}{100}$$

$$\text{Cash repaid at the end} = \frac{\text{cash consideration at the beginning} \times \left(1 + \text{repo rate} \times \frac{\text{days}}{\text{year}}\right)}{100}$$

Implied repo rate =

$$\left[\frac{(\text{futures price} \times \text{conversion factor}) + (\text{accrued coupon at delivery of futures}) + (\text{interim coupon reinvested})}{(\text{bond price} + \text{accrued coupon now})} - 1 \right] \times \frac{\text{year}}{\text{days}}$$

Cash-and-carry arbitrage

Assume the arbitrage is achieved by buying the cash bond and selling the futures:

$$\text{Cash cost at start} = \frac{\text{nominal bond amount} \times (\text{cash bond price} + \text{accrued coupon at start})}{100}$$

$$\text{Total payments} = \frac{\text{cash cost at start} \times \left(1 + \text{repo rate} \times \frac{\text{days to futures delivery}}{\text{year}}\right)}{100}$$

$$\text{Total receipts} = \frac{\text{nominal bond amount} \times (\text{futures price} \times \text{conversion factor} + \text{accrued coupon at delivery of futures})}{100}$$

$$\text{Profit} = \text{total receipts} - \text{total payments}$$

For each futures, the bond amount above is $\frac{\text{notional contract size}}{\text{conversion factor}}$

$$\text{Basis} = \text{bond price} - \text{futures price} \times \text{conversion factor}$$

$$\text{Net cost of carry} = \text{coupon income} - \text{financing cost}$$

$$\text{Net basis} = \text{basis} - \text{net cost of carry}$$

Zero-coupon rates and yield curves

$$\text{Par yield for } N \text{ years} = \frac{1 - df_N}{\sum_{k=1}^N df_k}$$

where df_k = zero-coupon discount factor for k years

Forward-forward zero-coupon yield from k years to m years =

$$\left[\frac{(1 + z_m)^m}{(1 + z_k)^k} \right]^{\frac{1}{(m-k)}} - 1$$

In particular:

Forward-forward yield from k years to $(k + 1)$ years =

$$\frac{(1 + z_{k+1})^{k+1}}{(1 + z_k)^k} - 1$$

Creating a strip

$$z_k = [(1 + i_1) \times (1 + i_2) \times (1 + i_3) \times \dots \times (1 + i_k)]^{\frac{1}{k}} - 1$$

where $i_1, i_2, i_3 \dots i_k$ are the 1-year cash interest rate and the 1-year v 2-year, 2-year v 3-year, ..., $(k - 1)$ -year v k -year forward-forward rates

Conversion between yield curves

To create a zero-coupon yield from coupon-bearing yields: **bootstrap**

To calculate the yield to maturity on a non-par coupon-bearing bond from zero-coupon yields: **calculate the NPV of the bond using the zero-coupon yields, then calculate the yield to maturity of the bond from this dirty price**

To create a par yield from zero-coupon yields: **use the formula above**

To create a forward-forward yield from zero-coupon yields: **use the formula above**

To create a zero-coupon yield from forward-forward yields: **create a strip of the first cash leg with a series of forward-forwards**

Foreign exchange

To calculate cross-rates from dollar rates

Between two indirect rates or two direct rates against the dollar: divide opposite sides of the dollar exchange rates.

Between one indirect rate and one direct rate against the dollar: multiply the same sides of the dollar exchange rates.

In general:

Given two exchange rates A/B and A/C, the cross-rates are:

$$B/C = A/C \div A/B$$

$$C/B = A/B \div A/C$$

Given two exchange rates B/A and A/C, the cross-rates are:

$$B/C = B/A \times A/C$$

$$C/B = 1 \div (B/A \times A/C)$$

When dividing, use opposite sides. When multiplying, use the same sides.

Forwards

$$\text{Forward outright} = \text{spot} \times \frac{(1 + \text{variable currency interest} \times \frac{\text{days}}{\text{year}})}{(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{year}})}$$

$$\text{Forward swap} = \text{spot} \times$$

$$\frac{(\text{variable currency interest rate} \times \frac{\text{days}}{\text{year}} - \text{base currency interest rate} \times \frac{\text{days}}{\text{year}})}{(1 + \text{base currency interest rate} \times \frac{\text{days}}{\text{year}})}$$

$$\text{Forward outright} = \text{spot} + \text{forward swap}$$

Approximations

$$\text{Forward swap} \approx \text{spot} \times \text{interest rate differential} \times \frac{\text{days}}{\text{year}}$$

$$\text{Interest rate differential} \approx \frac{\text{forward swap}}{\text{spot}} \times \frac{\text{year}}{\text{days}}$$

Premiums and discounts

1. The currency with higher interest rates (= the currency at a “discount”) is worth less in the future.
The currency with lower interest rates (= the currency at a “premium”) is worth more in the future.
2. The bank quoting the price buys the base currency / sells the variable currency on the far date on the left.
The bank quoting the price sells the base currency / buys the variable currency on the far date on the right.
3. For outright later than spot, if the swap price is larger on the right than the left, add it to the spot price. If the swap price is larger on the left than the right, subtract it from the spot price.
4. For outright later than spot, the right-hand swap price is added to (or subtracted from) the right-hand spot price; the left-hand swap price is added to (or subtracted from) the left-hand spot price.

5. For outright deals earlier than spot, calculate as if the swap price is reversed and follow (3) and (4).
6. Of the two prices available, the customer gets the worse one. Thus if the swap price is 3 / 2 and the customer knows that the points are “in his/her favour” (the outright will be better than the spot), the price will be 2. If he/she knows that the points are “against him/her” (the outright will be worse than the spot), the price will be 3.
7. The effect of combining the swap points with the spot price will always be to widen the spread, never to narrow it.

A forward dealer expecting the interest rate differential to move in favour of the base currency (for example, base currency interest rates rise or variable currency interest rates fall) will “buy and sell” the base currency. This is equivalent to borrowing the base currency and depositing in the variable currency. And vice versa.

Covered interest arbitrage

Variable currency rate created =

$$\left[\left(1 + \text{base currency rate} \times \frac{\text{days}}{\text{base year}} \right) \times \frac{\text{outright}}{\text{spot}} - 1 \right] \times \frac{\text{variable year}}{\text{days}}$$

Base currency rate created =

$$\left[\left(1 + \text{variable currency rate} \times \frac{\text{days}}{\text{variable year}} \right) \times \frac{\text{spot}}{\text{outright}} - 1 \right] \times \frac{\text{base year}}{\text{days}}$$

Forward-forward price after spot

Left side = (left side of far-date swap) – (right side of near-date swap)

Right side = (right side of far-date swap) – (left side of near-date swap)

Time option

A time option price is the best for the bank / worst for the customer over the time option period.

Long-dated forwards

$$\text{Forward outright} = \text{spot} \times \frac{(1 + \text{variable interest rate})^N}{(1 + \text{base interest rate})^N}$$

SAFEs

FXA settlement amount =

$$A_2 \times \left[\frac{(\text{OER} - \text{SSR} + (\text{CFS} - \text{SFS}))}{(1 + L \times \frac{\text{days}}{\text{year}})} \right] - A_1 \times [\text{OER} - \text{SSR}]$$

paid by the “seller” to the “buyer” of the FXA (or vice versa if it is a negative amount), where the “buyer” is the party which buys the base currency on the first date and sells it on the second date.

where: A_1 = the base currency amount transacted at the beginning of the swap
 A_2 = the base currency amount transacted at the end of the swap
 OER = the outright exchange rate, when the FXA is transacted, to the beginning of the swap period
 SSR = the settlement spot rate two working days before the swap period
 CFS = the contract forward spread – that is, the swap price agreed when the FXA is transacted
 SFS = the settlement forward spread – that is, the swap price used for settlement two working days before the swap period
 L = variable currency LIBOR for the swap period, two working days before the swap period
 days = the number of days in the swap period
 year = the year basis for the variable currency

$$\text{ERA settlement amount} = A \times \left(\frac{\text{CFS} - \text{SFS}}{1 + L \times \frac{\text{days}}{\text{year}}} \right)$$

Buy FRA in currency A = buy FRA in currency B + buy FXA in rate A/B

Buy FRA in currency A + sell FRA in currency B + sell FXA in rate A/B = 0

Covered interest arbitrage (forward)

Variable currency FRA rate =

$$\left[\left(1 + \text{base currency FRA} \times \frac{\text{days}}{\text{year}} \right) \times \frac{\text{outright to far date}}{\text{outright to near date}} - 1 \right] \times \frac{\text{variable year}}{\text{days}}$$

Base currency FRA rate =

$$\left[\left(1 + \text{variable currency FRA} \times \frac{\text{days}}{\text{variable year}} \right) \times \frac{\text{outright to near date}}{\text{outright to far date}} - 1 \right] \times \frac{\text{base year}}{\text{days}}$$

Forward-forward swap =

$$\text{outright to near date} \times \frac{(\text{variable currency FRA} \times \frac{\text{days}}{\text{year}} - \text{base currency FRA} \times \frac{\text{days}}{\text{year}})}{(1 + \text{base currency FRA} \times \frac{\text{days}}{\text{year}})}$$

Interest rate swaps and currency swaps

Pricing interest rate swaps from futures or FRAs

- For each successive futures maturity, create a strip to generate a discount factor
- Use the series of discount factors to calculate the yield of a par swap

Valuing swaps

To value a swap, calculate the NPV of the cashflows, preferably using zero-coupon swap yields or the equivalent discount factors.

To value floating-rate cashflows, superimpose offsetting floating-rate cashflows known to have an NPV of zero – effectively an FRN.

To value cashflows in a different currency, convert the resulting NPV at the spot exchange rate.

A swap at current rates has an NPV of zero.

If a current swap involves an off-market fixed rate, this is compensated by a one-off payment or by an adjustment to the other side of the swap, so as to maintain the NPV at zero.

The current swap rate for a swap based on an irregular or forward-start notional principal is again the rate which gives the swap an NPV of zero.

Options

Price quotation

Currency option price expressed as points of the variable currency =
(price expressed as percentage of the base currency amount) \times spot exchange rate.

Currency option price expressed as percentage of base currency amount =
(price expressed as points of the variable currency) \div spot exchange rate.

Basic statistics

Mean (μ) = sum of all the values divided by the number of values.

Variance (σ^2) = mean of (difference from mean)²

When estimating the variance from only a sample of the data rather than all the data, divide by one less than the number of values used.

Standard deviation (σ) = $\sqrt{\text{variance}}$

Historic volatility

Historic volatility = $\frac{\text{standard deviation of LN(relative price movement)}}{\sqrt{\text{frequency of data per year}}}$

Take a series of n price data.

Divide each price by the previous day's price to give the relative price change – so that you now have only (n–1) data.

Take the natural logarithms (LN) of these (n–1) relative price changes. These are now the data from which to calculate the annualised standard deviation (= volatility). This is calculated as above:

- Calculate the mean
- Calculate the differences from the mean
- Square the differences
- Add these squares and divide by (n–2)
- Calculate the square root

To annualize volatility, multiply by the square root of the frequency of data per year.

Black-Scholes

Option-pricing formula for a non-dividend-paying asset:

$$\text{Call premium} = \text{spot price} \times N(-d_1) - \text{strike price} \times N(d_2) \times e^{-rt}$$

$$\begin{aligned} \text{Put premium} &= -\text{spot price} \times N(-d_1) + \text{strike price} \times N(-d_2) \times e^{-rt} \\ &= \text{call premium} + \text{strike price} \times e^{-rt} - \text{spot price} \end{aligned}$$

$$\text{where: } d_1 = \frac{\text{LN}\left(\frac{\text{spot} \times e^{rt}}{\text{strike}}\right) + \frac{\sigma^2 t}{2}}{\sqrt{\sigma} \quad t}$$

$$d_2 = \frac{\text{LN}\left(\frac{\text{spot} \times e^{rt}}{\text{strike}}\right) - \frac{\sigma^2 t}{2}}{\sqrt{\sigma} \quad t}$$

t = the time to expiry of the option as a proportion of a year

σ = the annualized volatility

r = the continuously compounded interest rate

N(d) = the standardized normal cumulative probability distribution

The normal distribution function can be approximated by:

$$N(d) = 1 - \frac{\frac{0.4361836}{1+0.33267d} - \frac{0.1201676}{(1+0.33267d)^2} + \frac{0.937298}{(1+0.33267d)^3}}{\sqrt{2\pi} e^{\frac{d^2}{2}}} \quad \text{when } d \geq 0 \text{ and}$$

$$N(d) = 1 - N(-d) \quad \text{when } d < 0$$

Currency option pricing formula:

$$\text{Call premium} = (\text{forward price} \times N(d_1) - \text{strike price} \times N(d_2)) \times e^{-rt}$$

$$\begin{aligned} \text{Put premium} &= (-\text{forward price} \times N(-d_1) + \text{strike price} \times N(-d_2)) \times e^{-rt} \\ &= \text{call premium} + (\text{strike price} - \text{forward price}) \times e^{-rt} \end{aligned}$$

where: the option is a call on a unit of the base currency (that is, a put on the variable currency) and the premium is expressed in units of the variable currency.

$$d_1 = \frac{\text{LN}\left(\frac{\text{forward}}{\text{strike}}\right) + \frac{\sigma^2 t}{2}}{\sigma \sqrt{t}}$$

$$d_2 = \frac{\text{LN}\left(\frac{\text{forward}}{\text{strike}}\right) - \frac{\sigma^2 t}{2}}{\sigma \sqrt{t}}$$

r = the continuously compounded interest rate for the variable currency

Put-call relationship

$$\begin{aligned} \text{Call premium} &= \text{put premium} + \text{spot} - \text{strike} \times e^{-rt} \\ &= \text{put premium} + (\text{forward} - \text{strike}) \times e^{-rt} \end{aligned}$$

$$\begin{aligned} \text{Put premium} &= \text{call premium} - \text{spot} - \text{strike} \times e^{-rt} \\ &= \text{call premium} - (\text{forward} - \text{strike}) \times e^{-rt} \end{aligned}$$

where: r = continuously compounded interest rate
 t = time to expiry as a proportion of a year

In particular, when the strike is the same as the simple forward, the call and put premiums are equal (put – call parity).

The expressions e^{rt} and e^{-rt} in the various formulas above can be replaced by $(1 + i \times t)$ and $\frac{1}{(1+i \times t)}$ respectively, where i is the simple interest rate for the period.

Synthetic forwards:

Buy forward = buy call *plus* sell put
 Sell forward = sell call *plus* buy put

Risk reversal

Buy call = buy put *plus* buy forward
 Sell call = sell put *plus* sell forward
 Buy put = buy call *plus* sell forward
 Sell put = sell call *plus* buy forward

Option price sensitivities

$$\text{delta } (\Delta) = \frac{\text{change in option's value}}{\text{change in underlying's value}}$$

$$\text{gamma } (\Gamma) = \frac{\text{change in delta}}{\text{change in underlying's value}}$$

$$\text{vega} = \frac{\text{change in option's value}}{\text{change in volatility}}$$

$$\text{theta } (\theta) = - \frac{\text{change in option's value}}{\text{change in time}}$$

$$\text{rho } (\rho) = \frac{\text{change in option's value}}{\text{change in interest rate}}$$

based on the Black–Scholes formula:

$$\Delta = N(d_1) \text{ for a call} \\ \text{or } -N(-d_1) \text{ for a put}$$

$$\Gamma = \frac{1}{S\sigma\sqrt{2\pi t} e^{\frac{d_1^2}{2}}} \text{ for a call or a put}$$

$$\text{vega} = \frac{S\sqrt{2\pi}}{d_1^2} e^{\frac{d_1^2}{2}} \text{ for a call or a put}$$

$$\text{theta } (\Theta) = \frac{S\sigma}{2\sqrt{2\pi t}} e^{-\frac{d_1^2}{2}} - Kre^{-rt}N(d_2) \text{ for a call} \\ \text{or } - \frac{S\sigma}{2\sqrt{2\pi t}} e^{-\frac{d_1^2}{2}} + Kre^{-rt}N(-d_2) \text{ for a put}$$

$$\text{rho } (\rho) = Kte^{-rt}N(d_2) \text{ for a call} \\ \text{or } -Kte^{-rt}N(-d_2) \text{ for a put}$$

where: S = spot price for the asset

K = strike price

APPENDIX 4

Glossary

30/360	(Or 360/360). A day/year count convention assuming 30 days in each calendar month and a “year” of 360 days; adjusted in America for certain periods ending on 31st day of the month.
360/360	Same as 30/360.
Accreting	An accreting principal is one which increases during the life of the deal. <i>See</i> amortizing , bullet .
Accrued interest	The proportion of interest or coupon earned on an investment from the previous coupon payment date until the value date .
Accumulated value	Same as future value .
ACT/360	A day/year count convention taking the number of calendar days in a period and a “year” of 360 days.
ACT/365	(Or ACT/365 fixed). A day/year count convention taking the number of calendar days in a period and a “year” of 365 days. Under the ISDA definitions used for interest rate swap documentation, ACT/365 means the same as ACT/ACT.
ACT/365 fixed	See ACT/365.
ACT/ACT	A day/year count convention taking the number of calendar days in a period and a “year” equal to the number of days in the current coupon period multiplied by the coupon frequency. For an interest rate swap , that part of the interest period falling in a leap year is divided by 366 and the remainder is divided by 365.
American	An American option is one which may be exercised at any time during its life. <i>See</i> European .
Amortizing	<p>An amortizing principal is one which decreases during the life of the deal, or is repaid in stages during a loan.</p> <p>Amortizing an amount over a period of time also means accruing for it pro rata over the period. <i>See</i> accreting, bullet.</p>
Annuity	An investment providing a series of (generally equal) future cashflows.

Appreciation	An increase in the market value of a currency in terms of other currencies. <i>See depreciation, revaluation.</i>
Arbitrage	Arbitrage is the simultaneous operation in two different but related markets in order to take advantage of a discrepancy between them which will lock in a profit. The arbitrage operation itself usually tends to cause the different markets to converge. <i>See covered interest arbitrage.</i>
Asian	An Asian option depends on the average value of the underlying over the option's life.
Ask	<i>See offer.</i>
Asset-backed security	A security which is collateralized by specific assets – such as mortgages – rather than by the intangible creditworthiness of the issuer.
Asset swap	An interest rate swap or currency swap used in conjunction with an underlying asset such as a bond investment. <i>See liability swap.</i>
At-the-money	(Or ATM). An option is at-the-money if the current value of the underlying is the same as the strike price . <i>See in-the-money, out-of-the-money.</i>
ATM	<i>See at-the-money.</i>
Backwardation	The situation when a forward or futures price for something is lower than the spot price (the same as forward discount in foreign exchange). <i>See contango.</i>
Band	The Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) links the currencies of Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain in a system which limits the degree of fluctuation of each currency against the others within a band of 15 percent either side of an agreed par value.
Banker's acceptance	<i>See bill of exchange.</i>
Barrier option	A barrier option is one which ceases to exist, or starts to exist, if the underlying reaches a certain barrier level. <i>See knock out / in.</i>
Base currency	Exchange rates are quoted in terms of the number of units of one currency (the variable or counter currency) which corresponds to one unit of the other currency (the base currency).
Basis	The underlying cash market price minus the futures price . In the case of a bond futures contract, the futures price must be multiplied by the conversion factor for the cash bond in question.

Basis points	In interest rate quotations, 0.01 percent.
Basis risk	The risk that the prices of two instruments will not move exactly in line – for example, the price of a particular bond and the price of a futures contract being used to hedge a position in that bond.
Basis swap	An interest rate swap where both legs are based on floating rate payments.
Basis trade	Buying the basis means selling a futures contract and buying the commodity or instrument underlying the futures contract. Selling the basis is the opposite.
Bear spread	A spread position taken with the expectation of a fall in value in the underlying .
Bearer security	A security where the issuer pays coupons and principal to the holders of the security from time to time, without the need for the holders to register their ownership; this provides anonymity to investors.
Bid	In general, the price at which the dealer quoting a price is prepared to buy or borrow. The bid price of a foreign exchange quotation is the rate at which the dealer will buy the base currency and sell the variable currency. The bid rate in a deposit quotation is the interest rate at which the dealer will borrow the currency involved. The bid rate in a repo is the interest rate at which the dealer will borrow the collateral and lend the cash. <i>See offer.</i>
Big figure	In a foreign exchange quotation, the exchange rate omitting the last two decimal places. For example, when USD/DEM is 1.5510 / 20, the big figure is 1.55. <i>See points.</i>
Bill of exchange	A short-term, zero-coupon debt issued by a company to finance commercial trading. If it is guaranteed by a bank, it becomes a banker's acceptance .
Binomial tree	A mathematical model to value options , based on the assumption that the value of the underlying can move either up or down a given extent over a given short time. This process is repeated many times to give a large number of possible paths (the “tree”) which the value could follow during the option's life.
Black–Scholes	A widely used option pricing formula devised by Fischer Black and Myron Scholes.
Bond basis	An interest rate is quoted on a bond basis if it is on an ACT/365 , ACT/ACT or 30/360 basis. In the short term (for accrued interest , for example), these three are different. Over a whole (non-leap) year, however, they all

	equate to 1. In general, the expression “bond basis” does not distinguish between them and is calculated as ACT/365. <i>See money-market basis.</i>
Bond-equivalent yield	The yield which would be quoted on a US treasury bond which is trading at par and which has the same economic return and maturity as a given treasury bill .
Bootstrapping	Building up successive zero-coupon yields from a combination of coupon-bearing yields .
Bräss/Fangmeyer	A method for calculating the yield of a bond similar to the Moosmüller method but, in the case of bonds which pay coupons more frequently than annually, using a mixture of annual and less than annual compounding.
Break forward	A product equivalent to a straightforward option , but structured as a forward deal at an off-market rate which can be reversed at a penalty rate.
Broken date	(Or odd date). A maturity date other than the standard ones (such as 1 week, 1, 2, 3, 6 and 12 months) normally quoted.
Bull spread	A spread position taken with the expectation of a rise in value in the underlying .
Bullet	A loan / deposit has a bullet maturity if the principal is all repaid at maturity. <i>See amortizing.</i>
Buy / sell-back	Opposite of sell / buy-back.
Cable	The exchange rate for sterling against the US dollar.
Calendar spread	The simultaneous purchase / sale of a futures contract for one date and the sale / purchase of a similar futures contract for a different date. <i>See spread.</i>
Call option	An option to purchase the commodity or instrument underlying the option. <i>See put.</i>
Cap	A series of borrower’s IRGs , designed to protect a borrower against rising interest rates on each of a series of dates.
Capital market	Long-term market (generally longer than one year) for financial instruments. <i>See money market.</i>
Cash	<i>See cash market.</i>
Cash-and-carry	A round trip (arbitrage) where a dealer buys bonds, repos them out for cash to fund their purchase, sells bond futures and delivers the bonds to the futures buyer at maturity of the futures contract.
Cash market	The market for trading an underlying financial instrument, where the whole value of the instrument will

	potentially be settled on the normal delivery date – as opposed to contracts for differences , futures , options , etc. (where the cash amount to be settled is not intended to be the full value of the underlying) or forwards (where delivery is for a later date than normal). <i>See derivative.</i>
CD	<i>See certificate of deposit.</i>
Ceiling	Same as cap .
Certificate of deposit	(Or CD). A security, generally coupon-bearing, issued by a bank to borrow money.
Cheapest to deliver	(Or CTD). In a bond futures contract, the one underlying bond among all those that are deliverable , which is the most price-efficient for the seller to deliver.
Classic repo	(Or repo or US-style repo). Repo is short for “sale and repurchase agreement” – a simultaneous spot sale and forward purchase of a security , equivalent to borrowing money against a loan of collateral . A reverse repo is the opposite. The terminology is usually applied from the perspective of the repo dealer. For example, when a central bank does repos, it is lending cash (the repo dealer is borrowing cash from the central bank).
Clean deposit	Same as time deposit .
Clean price	The price of a bond excluding accrued coupon. The price quoted in the market for a bond is generally a clean price rather than a dirty price .
Collar	The simultaneous sale of a put (or call) option and purchase of a call (or put) at different strikes – typically both out-of-the-money .
Collateral	(Or security). Something of value, often of good creditworthiness such as a government bond, given temporarily to a counterparty to enhance a party’s creditworthiness. In a repo , the collateral is actually sold temporarily by one party to the other rather than merely lodged with it.
Commercial paper	A short-term security issued by a company or bank, generally with a zero coupon .
Compound interest	When some interest on an investment is paid before maturity and the investor can reinvest it to earn interest on interest, the interest is said to be compounded. Compounding generally assumes that the reinvestment rate is the same as the original rate. <i>See simple interest.</i>
Contango	The situation when a forward or futures price for something is higher than the spot price (the same as forward premium in foreign exchange). <i>See backwardation.</i>

Continuous compounding	A mathematical, rather than practical, concept of compound interest where the period of compounding is infinitesimally small.
Contract date	The date on which a transaction is negotiated. <i>See value date.</i>
Contract for differences	A deal such as an FRA and some futures contracts , where the instrument or commodity effectively bought or sold cannot be delivered; instead, a cash gain or loss is taken by comparing the price dealt with the market price, or an index, at maturity.
Conversion factor	(or price factor). In a bond futures contract , a factor to make each deliverable bond comparable with the contract's notional bond specification. Defined as the price of one unit of the deliverable bond required to make its yield equal the notional coupon . The price paid for a bond on delivery is the futures settlement price times the conversion factor.
Convertible currency	A currency that may be freely exchanged for other currencies.
Convexity	A measure of the curvature of a bond's price / yield curve (mathematically, $\frac{d^2P}{di^2}$ /dirty price).
Corridor	Same as collar .
Cost of carry	The net running cost of holding a position (which may be negative) – for example, the cost of borrowing cash to buy a bond less the coupon earned on the bond while holding it.
Counter currency	<i>See variable currency.</i>
Coupon	The interest payment(s) made by the issuer of a security to the holders , based on the coupon rate and face value .
Coupon swap	An interest rate swap in which one leg is fixed-rate and the other floating-rate. <i>See basis swap.</i>
Cover	To cover an exposure is to deal in such a way as to remove the risk – either reversing the position, or hedging it by dealing in an instrument with a similar but opposite risk profile.
Covered call / put	The sale of a covered call option is when the option writer also owns the underlying . If the underlying rises in value so that the option is exercised , the writer is protected by his position in the underlying. Covered puts are defined analogously. <i>See naked.</i>

Covered interest arbitrage	Creating a loan / deposit in one currency by combining a loan / deposit in another with a forward foreign exchange swap .
CP	<i>See</i> commercial paper .
Cross	<i>See</i> cross-rate .
Cross-rate	Generally an exchange rate between two currencies, neither of which is the US dollar. In the American market, spot cross is the exchange rate for US dollars against Canadian dollars in its direct form.
CTD	<i>See</i> cheapest to deliver .
Cum-dividend	When (as is usual) the next coupon or other payment due on a security is paid to the buyer of a security. <i>See</i> ex-dividend .
Currency swap	An agreement to exchange a series of cashflows determined in one currency, possibly with reference to a particular fixed or floating interest payment schedule, for a series of cashflows based in a different currency. <i>See</i> interest rate swap .
Current yield	Bond coupon as a proportion of clean price per 100; does not take principal gain / loss or time value of money into account. <i>See</i> yield to maturity , simple yield to maturity .
Cylinder	Same as collar .
DAC – RAP	Delivery against collateral – receipt against payment. Same as DVP .
Deliverable bond	One of the bonds which is eligible to be delivered by the seller of a bond futures contract at the contract's maturity, according to the specifications of that particular contract.
Delta (Δ)	The change in an option 's value relative to a change in the underlying 's value.
Depreciation	A decrease in the market value of a currency in terms of other currencies. <i>See</i> appreciation , devaluation .
Derivative	Strictly, any financial instrument whose value is derived from another, such as a forward foreign exchange rate, a futures contract , an option , an interest rate swap , etc. Forward deals to be settled in full are not always called derivatives, however.
Devaluation	An official one-off decrease in the value of a currency in terms of other currencies. <i>See</i> revaluation , depreciation .
Direct	An exchange rate quotation against the US dollar in which the dollar is the variable currency and the other currency is the base currency .
Dirty price	The price of a security including accrued coupon . <i>See</i> clean price .

Discount	<p>The amount by which a currency is cheaper, in terms of another currency, for future delivery than for spot, is the forward discount (in general, a reflection of interest rate differentials between two currencies). If an exchange rate is “at a discount” (without specifying to which of the two currencies this refers), this generally means that the variable currency is at a discount. <i>See premium.</i></p> <p>To discount a future cashflow means to calculate its present value.</p>
Discount rate	<p>The method of market quotation for certain securities (US and UK treasury bills, for example), expressing the return on the security as a proportion of the face value of the security received at maturity – as opposed to a yield which expresses the return as a proportion of the original investment.</p>
Duration	<p>(Or Macaulay duration). A measure of the weighted average life of a bond or other series of cashflows, using the present values of the cashflows as the weights. <i>See modified duration.</i></p>
DVP	<p>Delivery versus payment, in which the settlement mechanics of a sale or loan of securities against cash is such that the securities and cash are exchanged against each other simultaneously through the same clearing mechanism and neither can be transferred unless the other is.</p>
Effective rate	<p>An effective interest rate is the rate which, earned as simple interest over one year, gives the same return as interest paid more frequently than once per year and then compounded. <i>See nominal rate.</i></p>
End-end	<p>A money market deal commencing on the last working day of a month and lasting for a whole number of months, maturing on the last working day of the corresponding month.</p>
Epsilon (ϵ)	<p>Same as vega.</p>
Equivalent life	<p>The weighted average life of the principal of a bond where there are partial redemptions, using the present values of the partial redemptions as the weights.</p>
ERA	<p><i>See exchange rate agreement.</i></p>
Eta (η)	<p>Same as vega.</p>
Euro	<p>The name for the proposed currency of the European Monetary Union.</p>
Eurocurrency	<p>A Eurocurrency is a currency owned by a non-resident of the country in which the currency is legal tender.</p>

Euromarket	The international market in which Eurocurrencies are traded.
European	A European option is one that may be exercised only at expiry . <i>See American.</i>
Exchange controls	Regulations restricting the free convertibility of a currency into other currencies.
Exchange rate agreement	(Or ERA). A contract for differences based on the movement in a forward-forward foreign exchange swap price. Does not take account of the effect of spot rate changes as an FXA does. <i>See SAFE.</i>
Exchange-traded	Futures contracts are traded on a futures exchange, as opposed to forward deals which are OTC . Some option contracts are similarly exchange traded rather than OTC .
Ex-dividend	When the next coupon or other payment due on a security is paid to the seller of a security after he/she has sold it, rather than to the buyer, generally because the transaction is settled after the record date . <i>See cum-dividend.</i>
Exercise	To exercise an option (by the holder) is to require the other party (the writer) to fulfil the underlying transaction. Exercise price is the same as strike price.
Expiry	An option's expiry is the time after which it can no longer be exercised .
Exposure	Risk to market movements.
Extrapolation	The process of estimating a price or rate for a particular value date, from other known prices, when the value date required lies outside the period covered by the known prices. <i>See interpolation.</i>
Face value	(Or nominal value). The principal amount of a security, generally repaid ("redeemed") all at maturity, but sometimes repaid in stages, on which the coupon amounts are calculated.
Fence	Same as collar .
Floating rate	In interest rates, an instrument paying a floating rate is one where the rate of interest is refixed in line with market conditions at regular intervals such as every three or six months. In the currency market, an exchange rate determined by market forces with no government intervention.
Floating rate CD	(Or FRCD). CD on which the rate of interest payable is refixed in line with market conditions at regular intervals (usually six months).

Floating rate note	(Or FRN). Capital market instrument on which the rate of interest payable is refixed in line with market conditions at regular intervals (usually six months).
Floor	A series of lender's IRGs , designed to protect an investor against falling interest rates on each of a series of dates.
Forward	In general, a deal for value later than the normal value date for that particular commodity or instrument. In the foreign exchange market, a forward price is the price quoted for the purchase or sale of one currency against another where the value date is at least one month after the spot date. <i>See short date.</i>
Forward exchange agreement	(Or FXA). A contract for differences designed to create exactly the same economic result as a foreign exchange cash forward-forward deal. <i>See ERA, SAFE.</i>
Forward-forward	An FX swap, loan or other interest-rate agreement starting on one forward date and ending on another.
Forward rate agreement	(Or FRA). A contract for differences based on a forward-forward interest rate.
FRA	<i>See forward rate agreement.</i>
FRCd	<i>See floating rate CD.</i>
FRN	<i>See floating rate note.</i>
Funds	The USD/CAD exchange rate for value on the next business day (standard practice for USD/CAD in preference to spot).
Future value	The amount of money achieved in the future, including interest, by investing a given amount of money now. <i>See time value of money, present value.</i>
Futures contract	A deal to buy or sell some financial instrument or commodity for value on a future date. Unlike a forward deal, futures contracts are traded only on an exchange (rather than OTC), have standardized contract sizes and value dates, and are often only contracts for differences rather than deliverable.
FXA	<i>See forward exchange agreement.</i>
Gamma (Γ)	The change in an option's delta relative to a change in the underlying's value.
Gross redemption yield	The same as yield to maturity ; "gross" because it does not take tax effects into account.
GRY	<i>See gross redemption yield.</i>

Hedge ratio	The ratio of the size of the position it is necessary to take in a particular instrument as a hedge against another, to the size of the position being hedged.
Hedging	Protecting against the risks arising from potential market movements in exchange rates, interest rates or other variables. <i>See cover, arbitrage, speculation.</i>
Historic rate rollover	A forward swap in FX where the settlement exchange rate for the near date is based on a historic off-market rate rather than the current market rate. This is prohibited by many central banks.
Historic volatility	The actual volatility recorded in market prices over a particular period.
Holder	The holder of an option is the party that has purchased it.
Immunization	The construction of a portfolio of securities so as not to be adversely affected by yield changes, provided it is held until a specific time.
Implied repo rate	The break-even interest rate at which it is possible to sell a bond futures contract , buy a deliverable bond , and repo the bond out. <i>See cash-and-carry.</i>
Implied volatility	The volatility used by a dealer to calculate an option price; conversely, the volatility implied by the price actually quoted.
Index swap	Sometimes the same as a basis swap . Otherwise a swap like an interest rate swap where payments on one or both of the legs are based on the value of an index – such as an equity index, for example.
Indirect	An exchange rate quotation against the US dollar in which the dollar is the base currency and the other currency is the variable currency .
Initial margin	<i>See margin.</i>
Interest rate guarantee	(Or IRG). An option on an FRA .
Interest rate swap	(Or IRS). An agreement to exchange a series of cashflows determined in one currency, based on fixed or floating interest payments on an agreed notional principal, for a series of cashflows based in the same currency but on a different interest rate. May be combined with a currency swap .
Internal rate of return	(Or IRR). The yield necessary to discount a series of cashflows to an NPV of zero.
Interpolation	The process of estimating a price or rate for value on a particular date by comparing the prices actually quoted for value dates either side. <i>See extrapolation.</i>

Intervention	Purchases or sales of currencies in the market by central banks in an attempt to reduce exchange rate fluctuations or to maintain the value of a currency within a particular band, or at a particular level. Similarly, central bank operations in the money markets to maintain interest rates at a certain level.
In-the-money	A call (put) option is in-the-money if the underlying is currently more (less) valuable than the strike price . <i>See at-the-money, out-of-the-money.</i>
IRG	<i>See interest rate guarantee.</i>
IRR	<i>See internal rate of return.</i>
IRS	<i>See interest rate swap.</i>
Iteration	The repetitive mathematical process of estimating the answer to a problem, by trying how well this estimate fits the data, adjusting the estimate appropriately and trying again, until the fit is acceptably close. Used, for example, in calculating a bond's yield from its price.
Kappa (κ)	Same as vega .
Knock out / in	A knock out (in) option ceases to exist (starts to exist) if the underlying reaches a certain trigger level. <i>See barrier option.</i>
Lambda (λ)	Same as vega .
Liability swap	An interest rate swap or currency swap used in conjunction with an underlying liability such as a borrowing. <i>See asset swap.</i>
LIBID	<i>See LIBOR.</i>
LIBOR	London inter-bank offered rate, the rate at which banks are willing to lend to other banks of top creditworthiness. The term is used both generally to mean the interest rate at any time, and specifically to mean the rate at a particular time (often 11:00 am) for the purpose of providing a benchmark to fix an interest payment such as on an FRN . LIBID is similarly London inter-bank bid rate. LIMEAN is the average between LIBID and LIBOR .
LIMEAN	<i>See LIBOR.</i>
Limit up / down	Futures prices are generally not allowed to change by more than a specified total amount in a specified time, in order to control risk in very volatile conditions. The maximum movements permitted are referred to as limit up and limit down .
Lognormal	A variable's probability distribution is lognormal if the logarithm of the variable has a normal distribution.

Long	A long position is a surplus of purchases over sales of a given currency or asset, or a situation which naturally gives rise to an organization benefiting from a strengthening of that currency or asset. To a money market dealer, however, a long position is a surplus of borrowings taken in over money lent out, (which gives rise to a benefit if that currency weakens rather than strengthens). <i>See short.</i>
Macaulay duration	<i>See duration.</i>
Margin	<p>Initial margin is collateral, placed by one party with a counterparty at the time of a deal, against the possibility that the market price will move against the first party, thereby leaving the counterparty with a credit risk.</p> <p>Variation margin is a payment or extra collateral transferred subsequently from one party to the other because the market price has moved. Variation margin payment is either in effect a settlement of profit/loss (for example, in the case of a futures contract) or the reduction of credit exposure (for example, in the case of a repo). In gilt repos, variation margin refers to the fluctuation band or threshold within which the existing collateral's value may vary before further cash or collateral needs to be transferred.</p> <p>In a loan, margin is the extra interest above a benchmark (e.g. a margin of 0.5 percent over LIBOR) required by a lender to compensate for the credit risk of that particular borrower.</p>
Margin call	A call by one party in a transaction for variation margin to be transferred by the other.
Margin transfer	The payment of a margin call .
Mark-to-market	Generally, the process of revaluing a position at current market rates.
Mean	Average.
Modified duration	A measure of the proportional change in the price of a bond or other series of cashflows, relative to a change in yield. (Mathematically $-\frac{dP}{di}$ /dirty price.) <i>See duration.</i>
Modified following	The convention that if a value date in the future falls on a non-business day, the value date will be moved to the next following business day, unless this moves the value date to the next month, in which case the value date is moved back to the last previous business day.
Money market	Short-term market (generally up to one year) for financial instruments. <i>See capital market.</i>

Money-market basis	An interest rate quoted on an ACT/360 basis is said to be on a money-market basis. <i>See</i> bond basis .
Moosmüller	A method for calculating the yield of a bond.
Naked	A naked option position is one not protected by an off-setting position in the underlying . <i>See</i> covered call / put .
Negotiable	A security which can be bought and sold in a secondary market is negotiable.
Net present value	(Or NPV). The net present value of a series of cashflows is the sum of the present values of each cashflow (some or all of which may be negative).
Nominal amount	Same as face value of a security.
Nominal rate	A rate of interest as quoted, rather than the effective rate to which it is equivalent.
Normal	A normal probability distribution is a particular distribution assumed to prevail in a wide variety of circumstances, including the financial markets. Mathematically, it corresponds to the probability density function $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\phi^2}$.
Notional	In a bond futures contract, the bond bought or sold is a standardized non-existent notional bond, as opposed to the actual bonds which are deliverable at maturity. Contracts for differences also require a notional principal amount on which settlement can be calculated.
NPV	<i>See</i> net present value .
O/N	<i>See</i> overnight .
Odd date	<i>See</i> broken date .
Offer	(Or ask). In general, the price at which the dealer quoting a price is prepared to sell or lend. The offer price of a foreign exchange quotation is the rate at which the dealer will sell the base currency and buy the variable currency . The offer rate in a deposit quotation is the interest rate at which the dealer will lend the currency involved. The offer rate in a repo is the interest rate at which the dealer will lend the collateral and borrow the cash.
Off-market	A rate which is not the current market rate.
Open interest	The quantity of futures contracts (of a particular specification) which have not yet been closed out by reversing. Either all long positions or all short positions are counted but not both.
Option	An option is the right, without any obligation, to undertake a particular deal at predetermined rates. <i>See</i> call, put .

Option forward	<i>See</i> time option.
OTC	<i>See</i> over the counter.
Out-of-the-money	A call (put) option is out-of-the-money if the underlying is currently less (more) valuable than the strike price. <i>See</i> at-the-money , in-the-money .
Outright	An outright (or forward outright) is the sale or purchase of one foreign currency against another for value on any date other than spot. <i>See</i> spot , swap , forward , short date .
Over the counter	(Or OTC). An OTC transaction is one dealt privately between any two parties, with all details agreed between them, as opposed to one dealt on an exchange – for example, a forward deal as opposed to a futures contract .
Overborrowed	A position in which a dealer's liabilities (borrowings taken in) are of longer maturity than the assets (loans out).
Overlent	A position in which a dealer's assets (loans out) are of longer maturity than the liabilities (borrowings taken in).
Overnight	(Or O/N or today/tomorrow). A deal from today until the next working day ("tomorrow").
Par	In foreign exchange, when the outright and spot exchange rates are equal, the forward swap is zero or par. When the price of a security is equal to the face value, usually expressed as 100, it is said to be trading at par. A par swap rate is the current market rate for a fixed interest rate swap against LIBOR .
Par yield curve	A curve plotting maturity against yield for bonds priced at par.
Parity	The official rate of exchange for one currency in terms of another which a government is obliged to maintain by means of intervention.
Participation forward	A product equivalent to a straightforward option plus a forward deal, but structured as a forward deal at an off-market rate plus the opportunity to benefit partially if the market rate improves.
Path-dependent	A path-dependent option is one which depends on what happens to the underlying throughout the option's life (such as an American or barrier option) rather than only at expiry (a European option).
Pips	<i>See</i> points.
Plain vanilla	<i>See</i> vanilla.

Points	The last two decimal places in an exchange rate. For example, when USD/DEM is 1.5510 / 1.5520, the points are 10 / 20. <i>See</i> big figure .
Premium	The amount by which a currency is more expensive, in terms of another currency, for future delivery than for spot, is the forward premium (in general, a reflection of interest rate differentials between two currencies). If an exchange rate is “at a premium” (without specifying to which of the two currencies this refers), this generally means that the variable currency is at a premium. <i>See</i> discount . An option premium is the amount paid up-front by the purchaser of the option to the writer .
Present value	The amount of money which needs to be invested now to achieve a given amount in the future when interest is added. <i>See</i> time value of money , future value .
Price factor	<i>See</i> conversion factor .
Primary market	The primary market for a security refers to its original issue. <i>See</i> secondary market .
Probability distribution	The mathematical description of how probable it is that the value of something is less than or equal to a particular level.
Put	A put option is an option to sell the commodity or instrument underlying the option. <i>See</i> call .
Quanto swap	A swap where the payments on one or both legs are based on a measurement (such as the interest rate) in one currency but payable in another currency.
Quasi-coupon date	The regular date for which a coupon payment would be scheduled if there were one. Used for price / yield calculations for zero-coupon instruments.
Range forward	A zero-cost collar where the customer is obliged to deal with the same bank at spot if neither limit of the collar is breached at expiry .
Record date	A coupon or other payment due on a security is paid by the issuer to whoever is registered on the record date as being the owner. <i>See</i> ex-dividend , cum-dividend .
Redeem	A security is said to be redeemed when the principal is repaid.
Reinvestment rate	The rate at which interest paid during the life of an investment is reinvested to earn interest-on-interest, which in practice will generally not be the same as the original yield quoted on the investment.

Repo	(Or RP). Usually refers in particular to classic repo . Also used as a term to include classic repos, buy/sell-backs and securities lending .
Repurchase agreement	<i>See</i> repo.
Revaluation	An official one-off increase in the value of a currency in terms of other currencies. <i>See</i> devaluation .
Reverse	<i>See</i> reverse repo.
Reverse repo	(Or reverse). The opposite of a repo.
Rho (ρ)	The change in an option 's value relative to a change in interest rates.
Risk reversal	Changing a long (or short) position in a call option to the same position in a put option by selling (or buying) forward , and vice versa.
Rollover	<i>See</i> tom/next. Also refers to renewal of a loan.
RP	<i>See</i> repo.
Running yield	Same as current yield .
S/N	<i>See</i> spot/next.
S/W	<i>See</i> spot-a-week.
SAFE	<i>See</i> synthetic agreement for forward exchange.
Secondary market	The market for buying and selling a security after it has been issued. <i>See</i> primary market .
Securities lending	(Or stock lending). When a specific security is lent against some form of collateral.
Security	A financial asset sold initially for cash by a borrowing organization (the "issuer"). The security is often negotiable and usually has a maturity date when it is redeemed. Same as collateral .
Sell / buy-back	Simultaneous spot sale and forward purchase of a security , with the forward price calculated to achieve an effect equivalent to a classic repo .
Short	A short position is a surplus of sales over purchases of a given currency or asset, or a situation which naturally gives rise to an organization benefiting from a weakening of that currency or asset. To a money market dealer, however, a short position is a surplus of money lent out over borrowings taken in (which gives rise to a benefit if that currency strengthens rather than weakens). <i>See</i> long .

Short date	A deal for value on a date other than spot but less than one month after spot.
Simple interest	When interest on an investment is paid all at maturity or not reinvested to earn interest on interest, the interest is said to be simple. <i>See</i> compound interest .
Simple yield to maturity	Bond coupon plus principal gain / loss amortized over the time to maturity, as a proportion of the clean price per 100. Does not take time value of money into account. <i>See</i> yield to maturity , current yield .
Speculation	A deal undertaken because the dealer expects prices to move in his favour, as opposed to hedging or arbitrage .
Spot	A deal to be settled on the customary value date for that particular market. In the foreign exchange market, this is for value in two working days' time. A spot curve is a yield curve using zero-coupon yields.
Spot-a-week	(Or S/W). A transaction from spot to a week later.
Spot/next	(Or S/N). A transaction from spot until the next working day.
Spread	The difference between the bid and offer prices in a quotation. Also a strategy involving the purchase of an instrument and the simultaneous sale of a similar related instrument, such as the purchase of a call option at one strike and the sale of a call option at a different strike.
Square	A position in which sales exactly match purchases, or in which assets exactly match liabilities. <i>See</i> long , short .
Standard deviation (σ)	A measure of how much the values of something fluctuate around its mean value. Defined as the square root of the variance .
Stock lending	<i>See</i> securities lending .
Straddle	A position combining the purchase of both a call and a put at the same strike for the same date. <i>See</i> strangle .
Strangle	A position combining the purchase of both a call and a put at different strikes for the same date. <i>See</i> straddle .
Street	The "street" is a nickname for the market. The street convention for quoting the price or yield for a particular instrument is the generally accepted market convention.
Strike	(Or exercise price). The strike price or strike rate of an option is the price or rate at which the holder can insist on the underlying transaction being fulfilled.

Strip	<p>A strip of futures is a series of short-term futures contracts with consecutive delivery dates, which together create the effect of a longer term instrument (for example, four consecutive 3-month futures contracts as a hedge against a one-year swap). A strip of FRAs is similar.</p> <p>To strip a bond is to separate its principal amount and its coupons and trade each individual cashflow as a separate instrument (“separately <i>traded</i> and <i>registered</i> for <i>interest</i> and <i>principal</i>”).</p>
Swap	<p>A foreign exchange swap is the purchase of one currency against another for delivery on one date, with a simultaneous sale to reverse the transaction on another value date.</p> <p><i>See also</i> interest rate swap, currency swap.</p>
Swaption	An option on an interest rate swap or currency swap .
Synthetic	A package of transactions which is economically equivalent to a different transaction (for example, the purchase of a call option and simultaneous sale of a put option at the same strike is a synthetic forward purchase).
Synthetic agreement for forward exchange	(or SAFE). A generic term for ERAs and FXAs .
T/N	<i>See tom/next.</i>
Tail	<p>The exposure to interest rates over a forward-forward period arising from a mismatched position (such as a two-month borrowing against a three-month loan).</p> <p>A forward foreign exchange dealer’s exposure to spot movements.</p>
Term	The time between the beginning and end of a deal or investment.
Theta (Θ)	The change in an option ’s value relative to a change in the time left to expiry .
Tick	The minimum change allowed in a futures price.
Time deposit	A non-negotiable deposit for a specific term .
Time option	(Or option forward). A forward currency deal in which the value date is set to be within a period rather than on a particular day. The customer sets the exact date two working days before settlement.
Time value of money	The concept that a future cashflow can be valued as the amount of money which it is necessary to invest now in order to achieve that cashflow in the future. <i>See</i> present value , future value .
Today/tomorrow	<i>See</i> overnight .

Tom/next	(Or T/N or rollover). A transaction from the next working day (“tomorrow”) until the day after (“next” day – i.e. spot in the foreign exchange market).
Treasury bill	A short-term security issued by a government, generally with a zero coupon .
True yield	The yield which is equivalent to the quoted discount rate (for a US or UK treasury bill, for example).
Tunnel	Same as collar .
Underlying	The underlying of a futures or option contract is the commodity or financial instrument on which the contract depends. Thus the underlying for a bond option is the bond; the underlying for a short-term interest rate futures contract is typically a three-month deposit.
Value date	(Or settlement date or maturity date). The date on which a deal is to be consummated. In some bond markets, the value date for coupon accruals can sometimes differ from the settlement date.
Vanilla	A vanilla transaction is a straightforward one.
Variable currency	(Or counter currency). Exchange rates are quoted in terms of the number of units of one currency (the variable or counter currency) which corresponds to one unit of the other currency (the base currency).
Variance (σ^2)	A measure of how much the values of something fluctuate around its mean value. Defined as the average of $(\text{value} - \text{mean})^2$. <i>See</i> standard deviation .
Variation margin	<i>See</i> margin .
Vega	(Or epsilon (ϵ), eta (η), kappa (κ) or lambda (λ)). The change in an option ’s value relative to a change in the underlying ’s volatility.
Volatility	The standard deviation of the continuously compounded return on the underlying . Volatility is generally annualized. <i>See</i> historic volatility , implied volatility . Also the price sensitivity of a bond as measured by modified duration .
Warrant	An option , generally referring to a call option – often a call option on a security where the warrant is purchased as part of an investment in another or the same security.
Writer	Same as “seller” of an option .
Yield	The interest rate which can be earned on an investment, currently quoted by the market or implied by the current market price for the investment – as opposed to the

coupon paid by an issuer on a security, which is based on the coupon rate and the face value.

For a bond, generally the same as **yield to maturity** unless otherwise specified.

Yield to equivalent life The same as **yield to maturity** for a bond with partial redemptions.

Yield to maturity (Or YTM). The **internal rate of return** of a bond – the yield necessary to discount all the bond's cashflows to an NPV equal to its current price. *See simple yield to maturity, current yield.*

YTM *See yield to maturity.*

Zero-cost collar A collar where the premiums paid and received are equal, giving a net zero cost.

Zero-coupon A zero-coupon security is one that does not pay a **coupon**. Its price is correspondingly less to compensate for this.

A zero-coupon **yield** is the yield which a zero-coupon investment for that term would have if it were consistent with the **par yield curve**.

APPENDIX 5

ISO (SWIFT) currency codes

<i>Country</i>	<i>Currency</i>	<i>Code</i>
Abu Dhabi	UAE dirham	AED
Afghanistan	afghani	AFA
Ajman	UAE Dirham	AED
Albania	lek	ALL
Algeria	dinar	DZD
Andorra	French franc	ADF
Andorra	peseta	ADP
Angola	kwanza	AON
Anguilla	E Caribbean dollar	XCD
Antigua	E Caribbean dollar	XCD
Argentina	peso	ARS
Armenia	dram	AMD
Aruba	florin	AWG
Australia	Australian dollar	AUD
Austria	Schilling	ATS
Azerbaijan	manat	AZM
Azores	Portugese escudo	PTE
Bahamas	Bahama dollar	BSD
Bahrain	dinar	BHD
Bangladesh	taka	BDT
Barbados	Barbados dollar	BBD
Belarus	rouble	BYR
Belgium	Belgian Franc	BEF
Belize	Belize dollar	BZD
Benin	CFA franc	XOF
Bermuda	Bermuda dollar	BMD
Bhutan	ngultrum	BTN
Bolivia	boliviano	BOB
Botswana	pula	BWP
Brazil	real	BRL
Brunei	Brunei dollar	BND
Bulgaria	lev	BGL
Burkina Faso	CFA franc	XOF
Burundi	Burundi franc	BIF
Cambodia	riel	KHR
Cameroon	CFA franc	XAF
Canada	Canadian dollar	CAD
Canary Islands	Spanish peseta	ESP

<i>Country</i>	<i>Currency</i>	<i>Code</i>
Cape Verde	escudo	CVE
Cayman Islands	CI dollar	KYD
Central African Republic	CFA franc	XAF
Chad	CFA franc	XAF
Chile	peso	CLP
China	renmimbi yuan	CNY
Colombia	peso	COP
Congo	CFA franc	XAF
Congo Dem. Rep.	New zaire/Congolese franc	ZRN
Cormoros	franc	KMF
Costa Rica	colon	CRC
Croatia	kuna	HRK
Cuba	peso	CUP
Cyprus	Cyprus pound	CYP
Czech Republic	koruna	CZK
Denmark	krone	DKK
Djibouti	Djibouti franc	DJF
Dominica	E Caribbean dollar	XCD
Dominican Republic	peso	DOP
Dubai	UAE dirham	AED
Ecuador	sucre	ECS
Egypt	Egyptian pound	EGP
El Salvador	colon	SVC
Equatorial Guinea	CFA franc	XAF
Estonia	kroon	EEK
Ethiopia	birr	ETB
European Union	European currency unit	ECU
Falkland Islands	pound	FKP
Faroe Islands	Danish krone	DKK
Fiji Islands	Fiji dollar	FJD
Finland	markka	FIM
France	franc	FRF
French Pacific Islands	CFP franc	XPF
Fujairah	UAE dirham	AED
Gabon	CFA franc	XAF
Gambia	dalasi	GMD
Germany	Mark	DEM
Ghana	cedi	GHC
Gibraltar	Gibraltar pound	GIP
Great Britain	pound	GBP
Greece	drachma	GRD
Greenland	Danish krone	DKK
Grenada	E Caribbean dollar	XCD
Guadeloupe	French franc	FRF
Guatemala	quetzal	GTQ

<i>Country</i>	<i>Currency</i>	<i>Code</i>
Guinea Republic	Guinean franc	GNF
Guinea-Bissau	CFA franc	XAF
Guyana	Guyana dollar	GYD
Haiti	gourde	HTG
Honduras	lempira	HNL
Hong Kong	Hong Kong dollar	HKD
Hungary	forint	HUF
Iceland	krona	ISK
India	rupee	INR
Indonesia	rupiah	IDR
Iran	rial	IRR
Iraq	dinar	IQD
Irish Republic	punt	IEP
Israel	shekel	ILS
Italy	lira	ITL
Ivory Coast	CFA franc	XOF
Jamaica	Jamaican dollar	JMD
Japan	yen	JPY
Jordan	dinar	JOD
Kazakhstan	tenge	KZT
Kenya	shilling	KES
Kiribati	Australian dollar	AUD
Korea (North)	won	KPW
Korea (South)	won	KRW
Kuwait	dinar	KWD
Kyrgyzstan	som	KGS
Laos	kip	LAK
Latvia	lat	LVL
Lebanon	Lebanese pound	LBP
Lesotho	loti	LSL
Liberia	Liberian dollar	LRD
Libya	dinar	LYD
Liechtenstein	Swiss franc	CHF
Lithuania	litas	LTL
Luxembourg	Luxembourg franc	LUF
Macao	pataca	MOP
Macedonia	denar	MKD
Madeira	Portugese escudo	PTE
Malagasy Republic	franc	MGF
Malawi	kwacha	MWK
Malaysia	ringgitt	MYR
Maldives	rufiyaa	MVR
Mali	CFA franc	XOF
Malta	lira	MTL
Martinique	French franc	FRF

<i>Country</i>	<i>Currency</i>	<i>Code</i>
Mauritania	ouguiya	MRO
Mauritius	rupee	MUR
Mexico	peso nuevo	MXN
Moldova	leu	MDL
Monaco	French franc	FRF
Mongolia	tugrik	MNT
Montserrat	E Caribbean dollar	XCD
Morocco	dirham	MAD
Mozambique	metical	MZM
Myanmar	kyat	MMK
Namibia	rand	NAD
Nauru Isles	dollar	
Nepal	rupee	NPR
Netherlands	guilder	NLG
Netherlands Antilles	guilder	ANG
New Caledonia	CFP franc	XPF
New Zealand	NZ dollar	NZD
Nicaragua	cordoba	NIO
Niger	CFA franc	XOF
Nigeria	naira	NGN
Norway	krone	NOK
Oman	riyal	OMR
Pakistan	rupee	PKR
Panama	balboa	PAB
Papua New Guinea	kina	PGK
Paraguay	guarani	PYG
Peru	sol nuevo	PEN
Philippines	peso	PHP
Poland	zloty	PLN
Portugal	escudo	PTE
Qatar	riyal	QAR
Ras Al Khaimah	UAE dirham	AED
Reunion Island	French franc	FRF
Romania	leu	ROL
Russia	rouble	RUR
Rwanda	franc	RWF
São Tomé	dobra	STD
Saudi Arabia	riyal	SAR
Senegal	CFA franc	XOF
Seychelles	rupee	SCR
Sharjah	UAE dirham	AED
Sierra Leone	leone	SLL
Singapore	Singapore dollar	SGD
Slovakia	koruna	SKK
Slovenia	tolar	SIT

<i>Country</i>	<i>Currency</i>	<i>Code</i>
Solomon Islands	dollar	SBD
Somalia	shilling	SOS
South Africa	rand	ZAR
Spain	peseta	ESP
Sri Lanka	rupee	LKR
St Christopher	E Caribbean dollar	XCD
St Helena	pound	SHP
St Kitts & Nevis	E Caribbean dollar	XCD
St Lucia	E Caribbean dollar	XCD
St Pierre et Miquelon	French franc	FRF
St Vincent	E Caribbean dollar	XCD
Sudan	dinar	SDD
Sudan	Sudanese pound	SDP
Surinam	Surinam guilder	SRG
Swaziland	lilangeni	SZL
Sweden	krona	SEK
Switzerland	Swiss franc	CHF
Syria	Syrian pound	SYF
Taiwan	dollar	TWD
Tajikistan	sum	TJS
Tanzania	shilling	TZS
Thailand	baht	THB
Togo	CFA franc	XOF
Tonga	pa' anga	TOP
Trinidad & Tobago	TT dollar	TTD
Tunisia	dinar	TND
Turkey	lira	TRL
Turkmenistan	manat	TMM
Uganda	shilling	UGX
Ukraine	hryvna	UAH
Um Al Quwain	UAE dirham	AED
United Arab Emirates	dirham	AED
Uruguay	peso	UYF
USA	US dollar	USD
Uzbekistan	sum	UZS
Vanuatu	vatu	VUV
Venezuela	bolivar	VEB
Vietnam	dong	VND
Virgin Islands (USA)	US dollar	USD
Wallis & Futuna Islands	CFP franc	XPF
Western Samoa	tala	WST
Yemen	rial	YER
Yugoslavia	dinar	YUM
Zambia	kwacha	ZMK
Zimbabwe	Zimbabwe dollar	ZWD

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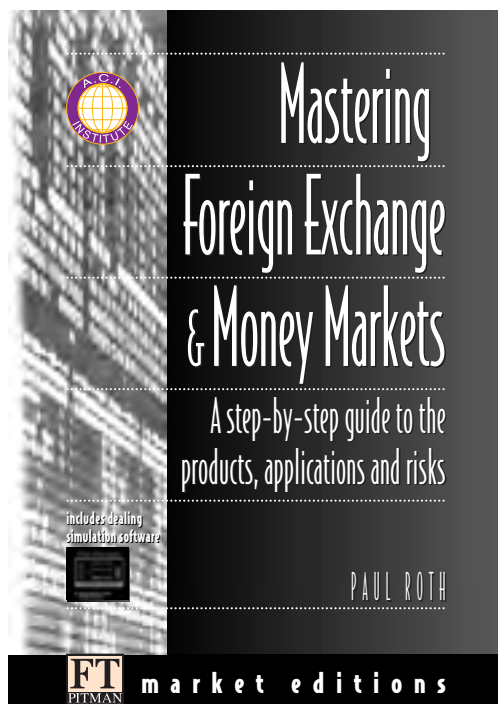
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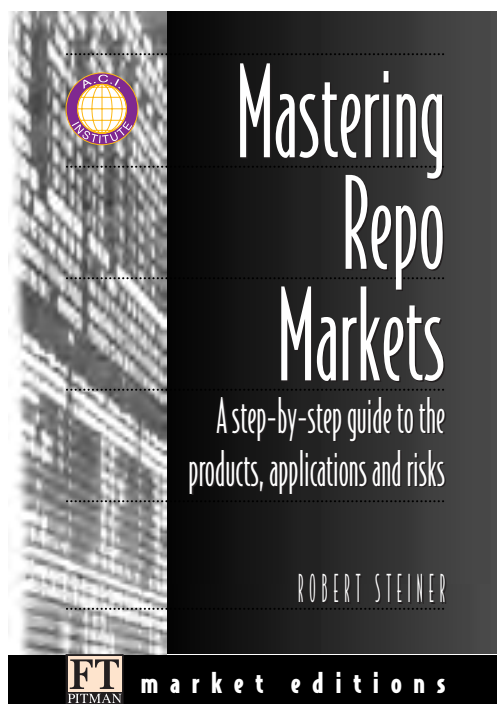
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